Module 8

Reactor Power Response to Changes in Reactivity

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Notes & References

8.1 MODULE OVERVIEW

This module deals with a topic of direct operational significance—the way the reactor responds to an imposed change in reactivity.

Period

Prompt jump

We start by showing that when a small positive step change in reactivity is imposed on a critical reactor, the neutron flux will increase ***exponentially.*** This exponential rise is characterized by a ***period*** which depends on the neutron lifetime in the system. We show that the delayed neutrons are crucially important in slowing down the rate of power rise to an extent that makes it possible to control the reactor. We then look at the way the detailed time response of the power depends on individual contributions of prompt and delayed neutrons, showing that this consists of a ***prompt jump*** followed by a much slower ***long-term*** increase. The existence of the prompt jump is of direct importance in designing the control system to limit the power increases to which the core may be subjected.

The discussion of power rise is then extended to the case of larger reactivity injections, including those which would be large enough to take the reactor into the prompt critical condition. The response of the reactor to a given change in reactivity depends on the changing composition of the fuel as the core moves from the fresh to the equilibrium fuel state. Finally, we consider the way power decreases when a large ***negative*** reactivity is inserted, as in reactor shutdown.

8.2 MODULE OBJECTIVES

After studying this module, you should be able to:

i) Define the reactor period, and write an expression, in terms of the period, which describes how power increases after a reactivity change.

ii) Given that, in a very simple treatment, the period is equal to $\ell/\Delta k$, describe how ℓ may be modified to take account of the presence of delayed neutrons.

iii) Describe a model which allows one to take account of the individual response of prompt and delayed neutrons to a change in reactivity. State why the power rises rapidly immediately after the change, and then settles to a much slower rise.

iv) Given the formula which includes both the prompt jump and the stable period, calculate the power at any time after the reactivity change.

v) Define prompt criticality and explain how and why the reactivity step required to attain this condition changes with the degree of fuel burnup.

vi) Describe in general terms how neutron power decreases after a large negative insertion of reactivity.

8.3 FACTORS AFFECTING THE REACTIVITY OF A REACTOR UNDER OPERATIONAL CONDITIONS

Under operational conditions, the reactivity of the reactor (see Section 5.6) can be affected in a variety of ways, the most important of which are listed below:

i) operation of the reactor control systems;

ii) changes in temperature of reactor components produced, for example, by raising or lowering reactor power;

iii) build-up of fission products in the fuel;

iv) changes in the composition of the fuel due to burnup.

All of these changes are time-dependent. They are each discussed in separate modules. In this module, we will examine the response of the reactor to a change in reactivity induced, for example, by operation of the control system.

8.4 THE RESPONSE OF REACTOR FLUX (OR POWER) TO IMPOSED CHANGES OF REACTIVITY

8.4.1 Step change of reactivity

Consider a reactor operating in a steady state with k = 1. Suppose that the multiplication factor is suddenly increased to a value of 1 + Δk and then held fixed. In other words, we have imposed a ***step change*** in reactivity of amount Δk. In practice, because of the finite rate at which reactivity can be changed by the control system, the kind of variation likely to be achieved is more like a ***ramp change*** (see Figure 8.1), but we will limit ourselves to step changes, because they are easier to treat mathematically. ***We will also initially restrict the value of Δk to be much less than the delayed neutron fraction, β.***

Step change

Ramp change

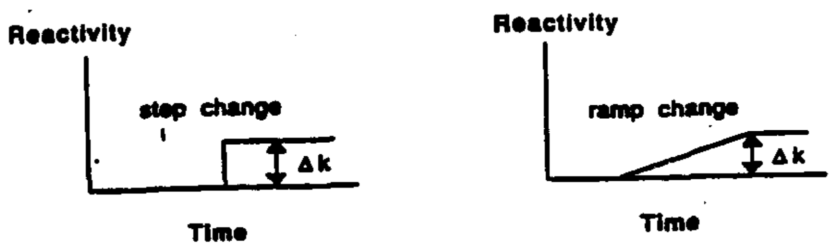

Figure 8.1: Step and ramp changes of reactivity

Reactor kinetics

The study of how reactor power varies with time after a change in reactivity is known as ***reactor kinetics***. We might start by reminding ourselves that because the quantities neutron density (n), neutron flux (ϕ) and power (P) are proportional to one another, they will all change the same way. The relationships between the three quantities are:

$$\phi = n\text{v}$$

where v is the average speed of the thermal neutrons, and

$$P = \Sigma_f \phi V E$$

where: Σ_f is the macroscopic fission cross-section;

V is the volume of the reactor;

E is the average energy release per fission.

Neutron power

We should note that power as defined above is more accurately described as ***neutron power***, that is, power generated by fission reactions induced by neutrons in the core. In a reactor which has operated for some time, a significant fraction of the total power is produced by the radioactive decay of fission products built up in the fuel. This point will be discussed in detail in Module 10, but for the moment we will avoid this complication and deal only with neutron power.

When the reactor is operating at high power, the response to an imposed reactivity is complicated by ***feedback effects*** caused by the resulting power change. If a small positive reactivity is injected, for example, power will start to rise, causing changes in the temperatures of various reactor components. This will, in turn, affect the reactivity of the reactor (as discussed in Module 12). The existence of these feedback effects makes the analysis much more complicated. We will ignore them at this point and instead assume that the reactivity remains at the value originally injected by the control system.

Notes & References

The ***multiplication factor*** was defined in Section 4.3 as

$$k = \frac{\textit{number of neutrons in one generation}}{\textit{number of neutrons in the previous generation}}$$

The length of time between generations is the average neutron lifetime (ℓ), that is, the average time between the birth of a fission neutron and its absorption following its slowing down and diffusion through the reactor. From the definition of k, the change of the neutron density over one generation is

$$\Delta n = kn - n$$

where n is the neutron density in one generation

kn is the neutron density in the next generation.

The time during which this change takes place is the neutron lifetime, ℓ. Hence, the ***rate of change*** is

$$\frac{\Delta n}{\Delta t} = \frac{kn - n}{\ell}$$

or

Rate of change of neutron density

$$\frac{dn}{dt} = \frac{kn - n}{\ell} = \frac{n}{\ell}(k - 1) = \frac{n}{\ell}\Delta k$$

The ***solution*** to this equation is known to be

$$n(t) = n_0 e^{\frac{\Delta k}{\ell}t}$$

where n_0 is the neutron density at t = 0

n(t) is the neutron density at time t.

Since both neutron flux and neutron power are proportional to the neutron density, the equation for the time variation of the power is

$$P(t) = P_0 e^{\frac{\Delta k}{\ell} t} \tag{8.1}$$

This equation illustrates that power will change exponentially with time, and its rate of increase will depend on the ratio of the reactivity (Δk) to the neutron lifetime (ℓ).

8.4.2 Reactor period

Period

It is convenient to define a parameter which indicates how long power takes to change by some given factor. The parameter most often quoted for CANDU reactors is the ***reactor period***, which is the time taken for the power (or the flux) to increase by a factor of e, the base for natural logarithms (e = 2.7183). For the period, we will use the symbol τ (tau).

In the model considered above, we can easily derive the reactor period. I f we take t in equation (8.1) to be equal to one reactor period ($t = \tau$), we know that over that time, the power will have increased from P_0 to eP_0 (that is, $P(\tau)$ in equation 8.1 is equal to eP_0).

Hence

$$eP_0 = P_0 e^{\frac{\Delta k}{\ell} \tau}$$

$$e = e^{\frac{\Delta k}{\ell} \tau}$$

It follows that

$$\frac{\Delta k}{\ell}\tau = 1$$

so that the expression for the reactor period in a supercritical reactor is

$$\tau = \frac{\ell}{\Delta k} \quad (8.2)$$

Expression for period

Note that equation 8.1 can also be written in the general form

$$P(t) = P_o e^{t/\tau} \quad (8.3)$$

where, in this case, $\tau = \ell / \Delta k$.

The ability to control a reactor is crucially dependent on the fact that a small fraction of the neutrons produced in fission is delayed, as noted in Section 2.8.5. To illustrate the importance of the delayed neutrons, let's look at what equation 8.1 would predict ***if all the neutrons were prompt***, that is, if they appeared instantaneously after fission occurred. The time between neutron generations would then be simply the average time required for a neutron to slow down to thermal energy plus the average time it spends wandering around as a thermal neutron before being absorbed, that is, the sum of its slowing-down and diffusion times. For a heavy water system, this is approximately 1 millisecond (0.001 s).

Suppose that a step reactivity of 0.5 mk were imposed on the critical reactor. According to equation (8.2), the corresponding reactor period would be

$$\tau = \frac{\ell}{\Delta k} = \frac{0.001}{0.0005} = 2s$$

In one second, reactor power would increase by a factor of $e^{0.5} = 1.65$. If the reactivity imposed had been +5 mk, the corresponding power increase per second would have been e^5, or a factor of 148.

These two examples illustrate how rapid the power increases would be, even for relatively small reactivity increases in the order of 1 mk, if it were really the case that all fission neutrons were prompt ones. Fortunately, despite the fact that only a small fraction of fission neutrons is delayed, this effect makes a big difference to the rate of power change. We can demonstrate this, in a rather simplified way, by replacing the neutron lifetime in equation (8.1) by a lifetime averaged over both prompt and delayed neutrons. Using the data in Table 2.3 of Section 2.8.5, we can show that the average lifetime for a ***delayed neutron*** to emerge after fission has taken place is approximately 13 seconds.

Average waiting time

The average "waiting time" for ***all*** neutrons, prompt plus delayed, is then obtained by multiplying the waiting times for prompt and delayed neutrons by their relative proportions in the fission process (0.993 and 0.0070 respectively). Since the waiting time for prompt neutrons is essentially zero, the average waiting time is equal to

$$(0.993 \times 0) + (0.0070 \times 13) = 0.091\ s.$$

This represents the average length of time for a neutron to be "born". Thus, the lifetime of a neutron generation is this average lifetime plus the slowing-down time and the diffusion time in the moderator. The sum of the latter two times is about 0.001 s, so that the time between neutron generations becomes $\ell = 0.092$ s when the effect of delayed neutrons is included.

Using this new value results in a very much slower response to an imposed reactivity. For a reactivity of +0.5 mk, for example, the period is now

$$\tau = \frac{\ell}{\Delta k} = \frac{0.092}{0.0005} = 184s$$

so that after one second, the power from equation (8.3) has risen to

$$P = P_0\, e^{(\frac{1}{184})} = 1.005\, P_0$$

This represents an increase of only 0.5% per second. Even for the very high reactivity injection of +5 mk, the rate of power rise would only be 5% per second. Consequently, power increases are much less rapid than if all neutrons were prompt, and regulation and protection become practical realities.

8.5 DETAILED ANALYSIS OF THE EFFECT OF DELAYED NEUTRONS

While using a neutron lifetime weighted to include both prompt and delayed neutrons offers a much better description of the longer-term rate of power increase after a reactivity change, the oversimplification involved means that the power variation ***immediately following the change*** is not properly predicted. To understand this, we must consider the time behavior of the prompt and delayed neutrons in greater detail, rather than simply correcting for the presence of delayed neutrons by using a single weighted lifetime in equation (8.1). We will still simplify things, however, by amalgamating all six delayed neutron groups into a single group with a lifetime of 13 seconds.

We can illustrate what happens by using a simple model of the neutron cycle, shown in Figure 8.2. In this cycle, where the multiplication factor is equal to k, the N neutrons of one generation give rise to a total of kN in the next generation. A fraction β of these is delayed, so that a number kNβ may be regarded as being fed into a "bank" of delayed neutron precursors, while the remaining kN(1-β) appear immediately as prompt neutrons.

Model for delayed neutrons

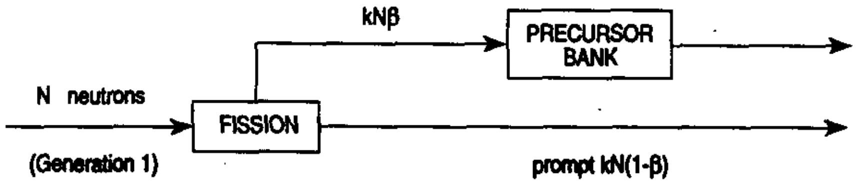

Figure 8.2: Prompt and delayed neutrons in multiplication process

To simplify the numbers involved, we'll take a numerical example using a greatly exaggerated value of β (0.1 instead of 0.0070). This is done only for numerical simplicity and in no way alters the qualitative arguments. ***We start with the reactor in a steady critical condition***, where the number of neutrons in each generation is the same as in the preceding one. We first draw a block diagram showing how this constancy of neutron density from one generation to another is achieved. For simplicity, we assume a value of 1,000 for neutron density. Figure 8.3 shows the neutron balance ***for the critical reactor.***

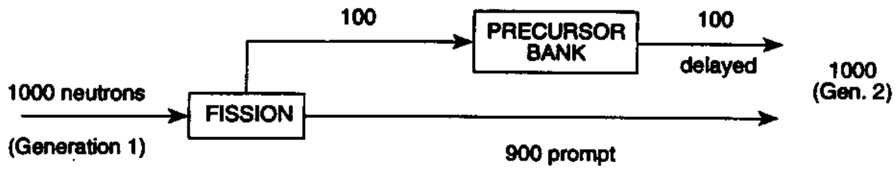

Figure 8.3: Neutron balance for the critical reactor

Of the 1,000 neutrons of the second generation, 900 will be prompt neutrons from fission and the other 100 will come from the precursor bank. At the same time, 100 of the fissions caused by neutrons of the first generation will lead to the creation of precursors, that is, we can consider 100 neutrons as being stored in the precursor bank, to appear at some later time. The concentration of the precursor bank therefore remains constant, since the 100 neutrons that it feeds into the second generation are replaced by the 100 "potential neutrons" going into the bank.

Now let's see what happens if we suddenly put some positive reactivity into the reactor. To make the numbers easier, we'll take $\Delta k = 0.05$ (in other words, k goes from 1.00 to 1.05). Since k is now greater than 1, the total number of neutrons produced by fission will rise to kN (where N is the number of neutrons in the first generation). As shown in Figure 8.2, only the number $kN(1-\beta)$ will appear as prompt fission neutrons; the remaining $kN\beta$ can be visualized as being stored in the precursor bank, to be released into the chain reaction some time later.

Note that the rate at which the precursor bank feeds neutrons into the system will not change over as short a time as one neutron lifetime (1 ms) because ***the neutrons coming from it reflect a precursor concentration accumulated before the change in reactivity took place.*** Because of the relatively long time constant of the precursor bank (average delayed neutron lifetime of about 13 s), it will take some time (many generations) before the output from the bank starts to reflect the increased rate at which potential neutrons are being fed into it. With this in mind, let's go back to assuming that N = 1,000 neutrons and numerically follow what happens over the first few generations after the reactivity change occurs (Figure 8.4).

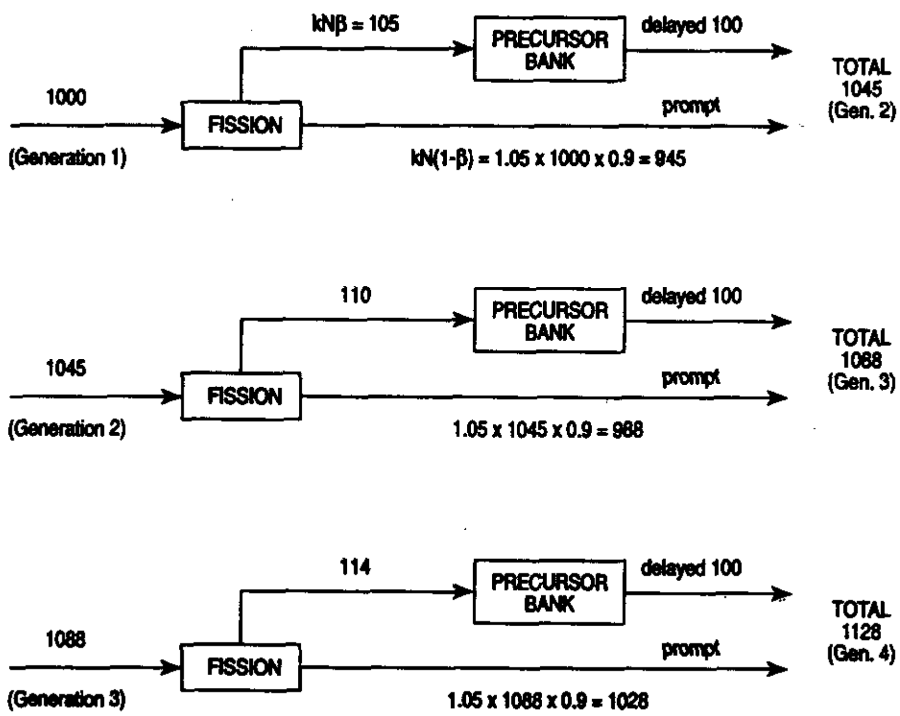

Figure 8.4: Initial response to a change in reactivity

Notice two things:

1. Although the input to the precursor bank continues to increase, the ***output*** remains constant (the time occupied by the whole sequence of the three diagrams is only 3 ms).

2. Although the neutron density rises rapidly, the ***rate*** it rises is falling off (increases of 45, 43, and 40 in successive generations). The reason it falls off is that, at each generation, progressively more of the neutrons produced by fission are being stored in the bank rather than being released immediately into the chain reaction.

   The result is that the rate of increase of the neutron density will have fallen off to zero after, say, 1 second (1,000 generations), if we assume that there is no increase in the output of the precursor bank during that time. The situation after 1,000 generations would then be approximately as shown in Figure 8.5.

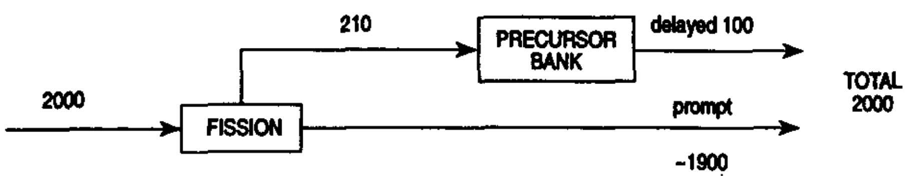

Figure 8.5 Situation after many generations

Notes & References

Prompt jump

In other words, the first effect is a rapid increase in power by a factor of two (in this case). This initial rise is so fast that it is customary to assume that it is instantaneous; this is known as the ***prompt jump approximation*** (see Figure 8.6). Mathematical analysis shows that, in general, the factor by which the flux rises (***the prompt jump factor***) is given by the ratio

$$\frac{P}{P_0} = \frac{\beta}{\beta - \Delta k} \qquad \textbf{(8.4)}$$

(which, in this case, is equal to $\frac{0.10}{0.10 - 0.05} = 2.00$)

Long-term jump

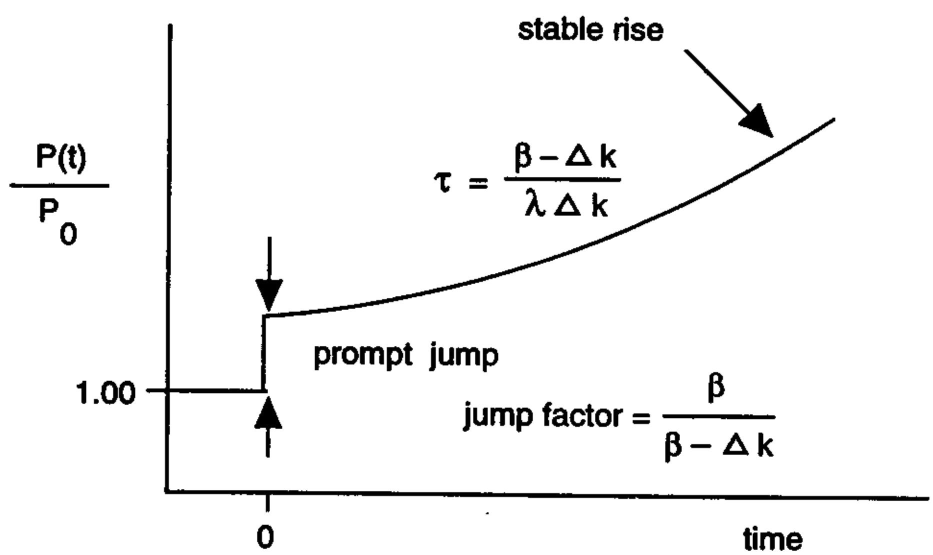

Figure 8.6: The prompt jump approximation

Following this rapid increase, the power tends to level off until the output from the precursor bank starts to reflect the increase in the precursor density. Again, mathematical analysis shows that a more accurate expression for the period (τ) with which the ***long-term*** increase in power takes place is

$$\tau = \frac{\beta - \Delta k}{\lambda \Delta k} \tag{8.5}$$

where λ is the average decay constant of delayed neutron precursors which can be shown to be approximately 0.08 s^{-1}. In the present case, the period would therefore be

$$\tau = \frac{0.10 - 0.05}{0.08 \times 0.05} = 12.5s$$

If the initial power is equal to P_0, the power level at any time t after the reactivity change has taken place is given by multiplying P_0 by the prompt jump factor and the exponential term which describes the long-term rise at period τ, that is

$$P(t) = P_0 \frac{\beta}{\beta - \Delta k} e^{t/\tau} \tag{8.6}$$

or

$$P(t) = P_0 \frac{\beta}{\beta - \Delta k} e^{\frac{\lambda \Delta k}{\beta - \Delta k} t} \tag{8.7}$$

As an example of the use of the prompt-jump approximation, take the case of a critical freshly-fuelled reactor into which we inject a reactivity step of +0.5 mk. If the initial steady power level is 5 kW, what is it 10 seconds after the change in reactivity?

The reactor period is

$$\tau = \frac{\beta - \Delta k}{\lambda \Delta k} = \frac{0.0070 - 0.0005}{(0.08) \times (0.0005)} = 162s$$

In this case, the prompt jump factor is

$$\frac{\beta}{\beta - \Delta k} = \frac{0.0070}{0.0070 - 0.0005} = 1.077$$

The power level at t = 10 s is therefore, from equation (8.6),

$$P = 5 \times 1.077 \times e^{10/162} = 5.385 \times 1.064 = 5.73\ kW$$

The existence of the prompt jump has important implications for the safety of the reactor when it is operating at full power, since the sudden injection of even a modest amount of positive reactivity could lead to an undesirably high rise in power output. For this reason, the rate of reactivity insertion is restricted to low levels by physically limiting the speed of drives, etc., so that one has slow ramp rather than prompt jump increases.

Before we leave this issue, it may be worth comparing the predictions of several different models of the reactor kinetics. Figure 8.7 shows such a comparison for the case $\Delta k = 0.5$ mk that we have just considered. The upper diagram shows a comparison of the power rise predicted by the prompt jump approximation with a model which essentially uses the formula derived in Section 8.4 for the "all-prompt" neutron case, but with the average lifetime of the neutrons adjusted so that it represents a weighted average of prompt and delayed neutrons. Note that the "average-lifetime approximation" predicts the right sort of stable period, but fails to predict the prompt jump.

Notes & References

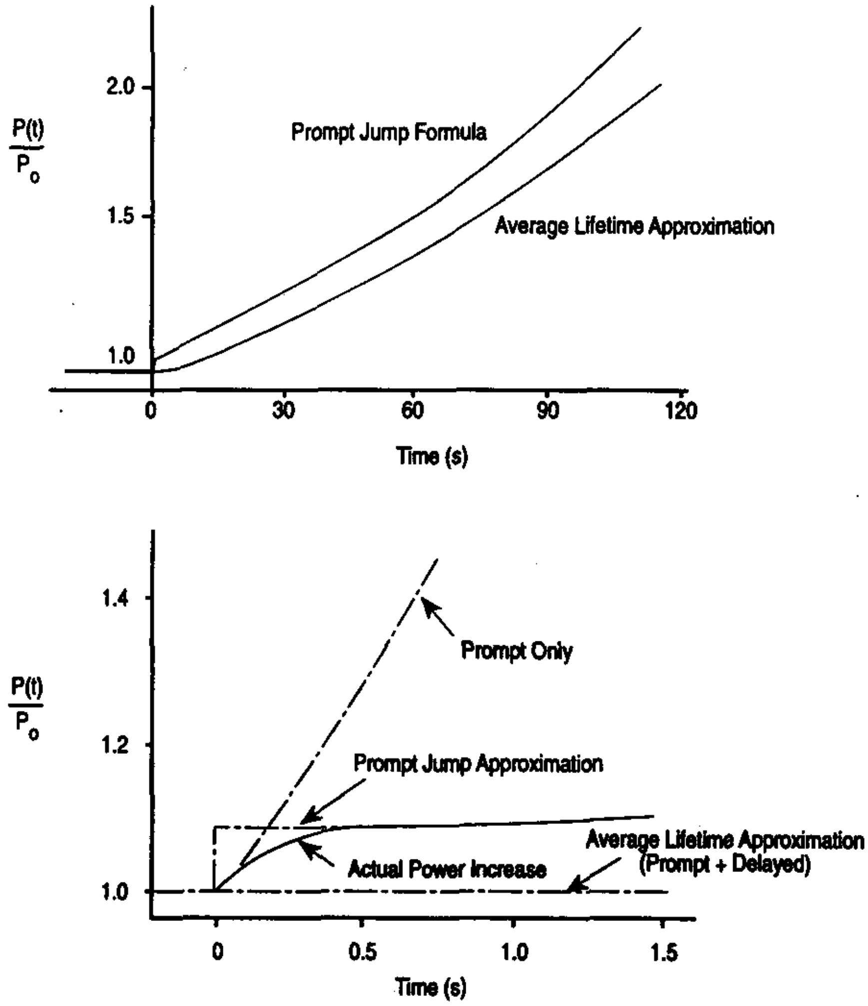

Figure 8.7: Power versus time for a step insertion of 0.5 mk (assuming fresh fuel)

The lower diagram in Figure 8.7 shows the earlier part of the transient in more detail. The full line illustrates how power ***actually*** rises. The prompt jump approximation overestimates the rate of the initial rise, but agrees with the actual rise from t = 0.5 s onwards. The diagram also includes a "prompt only" curve which assumes that all fission neutrons are prompt; this curve, of course, bears no relation to what really happens to the power level.

Before continuing, let's summarize the results of the model which takes into account the detailed behavior of prompt and delayed neutrons. Following the injection of positive reactivity, there is a rapid power rise, terminated by the increasing loss of potential neutrons to the precursor bank. This, in turn, is followed by a gradual rise as the enhanced precursors begin to increase the input of delayed neutrons to the system. We should note that the initial rapid rise in power following a step reactivity injection is an important factor which must be taken into account in the design of all reactivity mechanisms. Because of the undesirability of having significant rises in power taking place on a very short time scale, all mechanisms must be designed to limit the magnitude and speed of any possible reactivity injection that could occur.

8.6 PROMPT CRITICALITY

To this point, we have restricted the reactivity addition Δk to the range $\Delta k \ll \beta$. What happens if a positive step change of reactivity greater than β is applied to the reactor? Let's go back to our example, where $\beta = 0.1$, and insert a reactivity step $\Delta k = +0.15$. In this case (assuming, as before, 1,000 neutrons in generation 1), the number of prompt fission neutrons in generation 2 is:

$$kn(1-\beta) = 1.15 \times 1000 \times 0.90 = 1035$$

The reactor becomes supercritical on prompt neutrons alone, without having to wait for the delayed neutrons. A reactor which is subjected to a reactivity increase $\Delta k = \beta$ is said to be prompt critical. For a fresh core, with U-235 as the fissile material, the reactor will go prompt critical when a reactivity addition $\Delta k = 0.0070$ (7.0 mk) is applied to the critical reactor. Under these circumstances, as shown in Figure 8.8, the reactor period becomes less than 1 second. Note, incidentally, that no abrupt change in the curve of period versus reactivity takes place as we move through the prompt critical condition; as the applied Δk approaches β, the reactor becomes less and less dependent on the delayed neutrons so that the period decreases smoothly up to and through the prompt critical state. The control rods shutdown system (SDS1) trip (period = 10 s) and the poison injection shutdown system (SDS2) trip (period = 4 s) are also shown in Figure 8.8. These, of course, are set to trip at considerably slower rates than those associated with prompt critical.

Prompt critical

Figure 8.8 also illustrates the reactivity-period relationship for a core with Pu-239 as the fissile material. The reactivity addition required to take a Pu-239 core prompt critical is about one third of that needed with U-235, since β for Pu-239 is only 0.0023 (2.3 mk). This is important in our reactors. As Pu-239 builds up with irradiation, the effective delayed neutron fraction, which is an appropriately weighted mean of those of U-235 and Pu-239, will gradually decrease. For the equilibrium core, the effective value of β is approximately 0.005, compared with the 0.0070 of the fresh core. The reactor becomes more sensitive to reactivity changes as burnup proceeds, and the protection against positive reactivity changes is therefore based on the equilibrium fuel case.

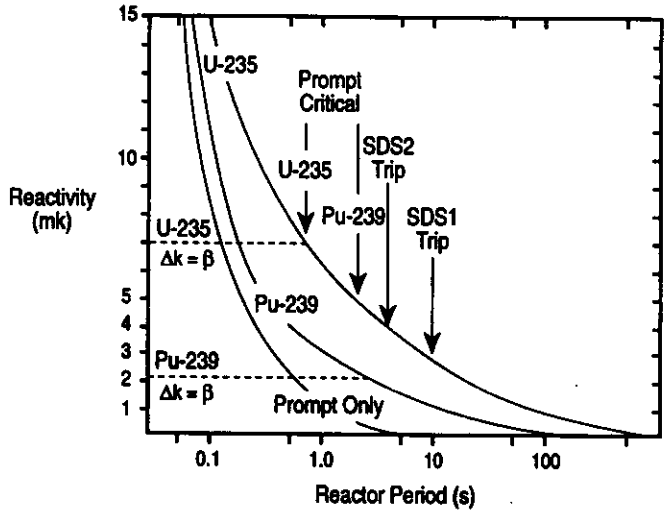

Figure 8.8: Reactivity-period relationships for U-235 and Pu-239

8.7 LARGE NEGATIVE REACTIVITIES (REACTOR TRIPS)

Prompt drop

The equations describing the prompt jump are equally valid when the reactivity inserted is a negative one; in this case, we have a prompt ***drop*** followed by a stable ***negative*** period. The ratio of the power level P immediately after the step insertion of reactivity to the original level P_0 is still represented by equation (8.4) but in this case Δk is negative. For example, if 5 mk of negative reactivity is injected into the critical (freshly-fuelled) reactor, the power immediately after the step change is, with Δk = -0.005

$$P = P_o \frac{\beta}{\beta - \Delta k} = P_o \frac{0.0070}{0.0070 + 0.005} = 0.58P_o$$

Following the prompt drop, the power will settle down to decaying with a negative period. As mentioned in Section 8.4.1, the equation for the reactor period

$$\tau = \frac{\beta - \Delta k}{\lambda \Delta k} \qquad (8.8)$$

was developed on the assumption that Δk was very much less than the delayed neutron fraction, β. Provided this is so, it can still be used to give the period for negative reactivities. For example, the period for a reactivity insertion of -0.2 mk into a fresh core would be

$$\tau = \frac{0.0070-(-0.0002)}{0.08 \times (-0.0002)} = -450\,s$$

This result differs by about 3% from one that would be obtained with a more rigorous treatment. For larger Δk, the equation for the period becomes increasingly inaccurate. If Δk is >>β, the period becomes approximately

$$\tau = -\frac{1}{\lambda}$$

Negative period

Thus, the stable period for a large negative reactivity insertion is determined by the decay constant of the precursors. Considering the fact that there are actually six precursor groups, the longest-lived having a half-life of 55 s (Table 2.3 of Section 2.8.5), the power, after the shorter-lived groups have decayed, will decrease with a period of τ of about 80 s. ($\tau = 1/\lambda$ and $\lambda = 0.693/t_{1/2}$). As discussed in Module 10, the rate of power decrease will eventually be slowed even further by the presence of photoneutrons, which have longer half-lives than the delayed neutrons.

A final cautionary note: remember that this module deals only with the **neutron** power. When the reactor is shut down from a high power, the **thermal power** after a shutdown is not at all proportional to the neutron flux, owing to the heat contributed by the accumulated fission products. This, too, will be considered in detail in Module 10

ASSIGNMENT

1. Define the reactor ***period*** and write an equation showing how power varies as a function of time when it is rising with a period τ

2. A positive step change of magnitude Δk is applied to a freshly-fuelled CANDU reactor. ***Some time after the step change has taken place,*** it is observed that the power rises from 50 kW to 80 kW in a time of 100 s. Calculate the reactor period.

3. When a positive step change of reactivity is applied to a critical reactor, power rises very rapidly for a very short time and then at a much slower rate. Explain why the rate of power increase falls off so rapidly after the initial jump.

4. The expression for the power at time t after an injection of positive reactivity Δk into a critical reactor running at a power P_0 is

$$P(t) = P_o \frac{\beta}{\beta - \Delta k} e^{\frac{\lambda \Delta k}{\beta - \Delta k} t}$$

(where $\lambda = 0.08 s^{-1}$)

Use this expression to calculate the following for a freshly-fuelled reactor ($\beta = 0.0070$) initially running at a power of 1 kW, when a sudden reactivity change of $\Delta k = +1$ mk takes place:

a) the magnitude of the prompt jump

b) the power level 120 s after the change in reactivity

c) the steady period

d) the time required to go from 10 kW to 20 kW

5. a) The effect of delayed neutrons on the kinetics of a freshly-fuelled reactor becomes insignificant as the reactor becomes supercritical by an amount greater than the delayed neutron fraction. Explain why.

b) Assuming that it were somehow possible to increase the reactivity of a CANDU reactor so that is became prompt critical, would the period suddenly change as it went through this point? Explain.