SUPPLEMENT TO CHAPTER 11
OF
REACTOR PHYSICS FUNDAMENTALS

This supplement reviews the text material from a slightly different point of view. It also extends the mathematical description outlined in the text. You should be familiar with the text material before studying this supplement.

Feedback Reactivity Effects due to Fission Product Poisons

Buildup of Iodine and Xenon

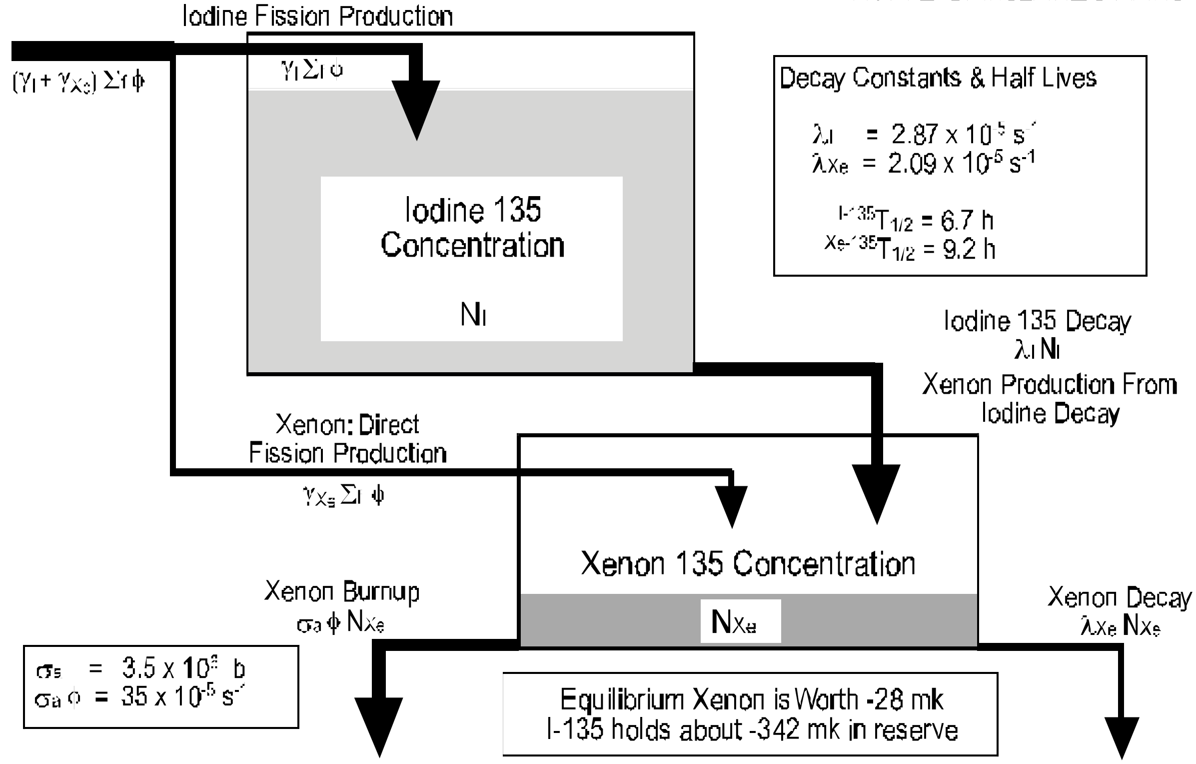
Bulk Xenon

Transient Xenon

Samarium and Other Fission Products

John L. Groh

Xenon Production and Loss Mechanisms



<u>**Xenon & Iodine**</u>

* *Tank Model for Xenon and Iodine Production and Removal*

The figure opposite can be used to construct most of the relationships described in the text. Numerical values in the figure are typical CANDU values, but they do differ a little from station to station. Table 1 gives up to date values. Numerical estimates in this supplement generally use the old values, for consistency with numbers in the text.

TABLE 1 **PARAMETERS**

λ_I = 2.93 × 10^{-5} s^{-1}	(G.E. Nuclear Chart 1996)
λ_{Xe} = 2.11 × 10^{-5} s^{-1}	(G.E. Nuclear Chart 1996)
σ_a^{Xe} = 3.5 x 10^6 b = 3.5 × 10^{-18} cm^2	(New Transent value is 3.1 × 10^{-18} cm^2)
γ_I = 6.3% (New Transent value ≈ 6.4 % for equilibrium fuel & 6.3% for U-235 fissions.)	
γ_{Xe} = 0.3% (New Transent value ≈ 0.6 % for equilibrium fuel & 0.24% for U-235 fissions.)	
Σ_f ≈ 0.1 cm^{-1} (fresh CANDU fuel) Σ_f ≈ 0.089 cm^{-1} (equilibrium fuelling) is burnup dependent	
$\phi_{F.P.}$ = 9.1 × 10^{13} n cm^{-2} s^{-1} (fuel flux at full power/equilibrium fuelling: BNGSB Xe predictor)	
$\phi_{F.P.}$ = 1.0 × 10^{14} n cm^{-2} s^{-1} is a convenient value for calculation, and close enough.	
time constants for ϕ_{final} = full power flux (for equivalent half lives multiply by $ln2$ = 0.693):	
$\left(\sigma_a^{Xe}\phi_{final} + \lambda_{Xe}\right)^{-1} \approx 49.1\,minutes$	(half time 34 minutes)
$\left[\sigma_a^{Xe}\phi_{final} - \left(\lambda_I - \lambda_{Xe}\right)\right]^{-1} \approx 53.7\,minutes$	(half time 37 minutes)
$1/\lambda_I$ = 569 minutes	(half life 6.6 hours)
$1/\lambda_{Xe}$ =790 minutes	(half life 9.1 hours)
$1/(\lambda_I-\lambda_{Xe})$ = 2032 min = 33.9 hrs	(half time 23.5 hours)
To convert from number concentration to mk worth of xenon-135, take 1 mk ≈ 6 × 10^{16} atoms	

* *Calculate the fraction of mass 135 fission fragments that are xenon and the fraction that are iodine.*

6.6% of fissions produce either xenon-135 or iodine-135.
Direct production by fission is 0.003/0.066 = 4.5% xenon
and 0.063/0.066 = 95.5% iodine
The newest numbers in the table give about 4% xenon, 96% iodine for fresh fuel and 8.6% xenon, 91.4% iodine for equilibrium fuel.

*   *Equilibrium Steady State Conditions for Xenon and Iodine*
Show that the % of production of xenon once equilibrium is achieved is almost 95% from iodine decay and 5% direct fission production.

Once equilibrium is achieved, iodine decay = iodine production so the same per cents apply as the fission fractions above.

*   *Show that the removal of xenon at normal full power flux conditions is more than 90% by burnout and almost 10% by decay.*

Calculate ratio of burnout to total removal: $[N\sigma\phi/(N\sigma\phi + \lambda N)] = [1 + (\lambda/\sigma\phi)]^{-1}$
$\lambda/\sigma = 2.09 \times 10^{-5}/3.5 \times 10^{-18}\ cm^{-2}s^{-1} = 6 \times 10^{12}\ cm^{-2}s^{-1}$ and take $\phi \approx 1 \times 10^{14}$
$[1 + (\lambda/\sigma\phi)]^{-1} = 1.06^{-1}$ (94%)   (text values of 90%/10% use much lower flux)

*   *Develop formula for equilibrium iodine concentration and show that equilibrium iodine concentration is proportional to steady state flux.*

Equate Iodine tank inflow = outflow at equilibrium to get the text equation. Solve for N to get equation 11.3 in the text; $N_{I\ eq} = \gamma_I\Sigma_f\phi/\lambda_I$

*   *Develop formula for equilibrium xenon concentration and show that the* <u>*Xenon Load*</u> *at equilibrium is nearly flux independent for a high flux reactor*

Equate the two inflow terms in the xenon tank to the two outflow terms to get the text equation Solve for N to get equation 11.4 in the text and rearrange:

$$N_{Xe(eq)} = \frac{(\gamma_I + \gamma_{xe})}{\lambda_{xe} + \sigma_a^{xe}\phi}\Sigma_f\phi = \frac{(\gamma_I + \gamma_{xe})}{\sigma_a^{xe}\phi\left(1 + \frac{\lambda_{xe}}{\sigma_a^{xe}\phi}\right)}\Sigma_f\phi = \frac{(\gamma_I + \gamma_{xe})}{\left(1 + \frac{\lambda_{xe}}{\sigma_a^{xe}\phi}\right)}\frac{\Sigma_f}{\sigma_a^{xe}}$$

For $\phi >> \lambda/\sigma$ this equation is the same as the high flux equation in the text.

*   *What power level constitutes "high flux"?*

$\lambda/\sigma = 2.09 \times 10^{-5}/3.5 \times 10^{-18}\ cm^{-2}s^{-1} = 6 \times 10^{12}\ cm^{-2}s^{-1} \approx 6\%$ full power flux.
“High power” is much greater than this, perhaps 60% F.P. or higher. Above this power equilibrium xenon reactivity worth is always close to 28 mk.

* *Determine the relative concentrations of iodine and xenon and use equilibrium xenon mk worth = 28 mk absorption to calculate the reserve of xenon stored as iodine (<u>Iodine Load</u>).*

$$\frac{N_{I(eq)}}{N_{Xe(eq)}} = \frac{\gamma_I}{(\gamma_I + \gamma_{xe})} \frac{(\lambda_{xe} + \sigma_a^{xe}\phi)}{\lambda_I} \approx 0.95 \times (2.11 + 35)/2.93) = 12$$

Equilibrium xenon is worth 28 mk, so the reactivity "bank" of iodine is ≈ 12 × 28 mk = 336 mk worth. Lower flux gives a smaller number. ($N_{Ieq} \propto \phi$)

* *Build-up of Iodine and Xenon to Equilibrium*
  Describe the time behaviour of Iodine as it builds up.
  Describe the time behaviour of Xenon during its build-up.

At any instant, the rate of change of iodine concentration is given by the difference between inflow and outflow to the iodine tank. Similarly the rate of change of xenon is given by the inflow and outflow difference in the xenon tank. These differences give the differential equations 11.1 and 11.2 in the text. These can be solved for the initial build-up of iodine and xenon.

INITIAL BUILDUP

The time dependent expression for the initial buildup of I-135 in a fresh core is:

$$N_I(t) = N_{I(eq)}\left(1 - e^{-\lambda_I t}\right) \qquad (1)$$

$$N_{I(eq)} = \frac{\gamma_I \Sigma_f \phi}{\lambda_I} \qquad (2)$$

The time dependent expression for the initial buildup of Xe-135 in a fresh core is more complicated. It can be written as two terms:

$$N_{Xe}(t) = N_{Xe(eq)}\left(1 - e^{-\lambda_I t}\right) - N_{Xe(eq)}\left[\frac{\lambda_I}{\sigma_a^{Xe}\phi - (\lambda_I - \lambda_{Xe})} \cdot \frac{N_{I(eq)}}{N_{Xe(eq)}} - 1\right]\left(e^{-\lambda_I t}\right)\left(1 - e^{-\left[\sigma_a^{Xe}\phi - (\lambda_I - \lambda_{Xe})\right]}\right) \qquad (3)$$

$$N_{Xe(eq)} = \frac{(\gamma_I + \gamma_{Xe})\Sigma_f}{\sigma_a^{Xe}\left(1 + \frac{\lambda_{Xe}}{\sigma_a^{Xe}\phi}\right)} \qquad (4)$$

Equations 1 and 3 are graphed in the text as figures 11.2 and 11.3, respectively.

The formulas above calculate concentration in number density (atoms/cm^3). A conversion from number density to mk worth requires that the flux and mk worth be specified.

The following give results that agree, within roundoff, with Ontario Hydro's CANDU reactor Xenon Predictor equations used by the Bruce Station operations staff.

1. $\phi = \phi_{F.P.} \times P$
where power, P, varies such that $0 < P < 1$ and $\phi_{F.P.} \approx 9.1 \times 10^{13}$ cm^{-2} s^{-1}
2. 1 mk requires 5.6×10^{13} xenon-135 atoms per cm^3.

Using these values and parameters from table 1 into (2) and (4) gives;

$N_{I(eq)} = -\ 322 \times P\ mk$ worth of iodine in reserve.

$$N_{Xe(eq)} = \frac{-28mk \times P}{0.94P + 0.06} = \frac{-28mk}{\left(0.94 + \dfrac{0.06}{P}\right)} \qquad (5)$$

This relation for equilibrium xenon is graphed in figure 11.4 in the text.

BRIEF DISCUSSION OF THE EQUATIONS

The first term in (3), taken by itself, shows the same time dependence for xenon buildup as for I-135 buildup in (1). The second long messy term is a small correction for Xe-135 "holdup". Xenon comes mostly from iodine decay so must wait for iodine to be produced. During the first few iodine half lives the "holdup" term reduces xenon concentration a little.

The holdup term is zero for t = 0 and disappears again after a few iodine half lives. The square bracket is numerically about 0.09 for full power flux and the peak value of the product of time dependent brackets is about 0.725 at 2.2 hours, so the maximum reduction is approximately -28 mk × 0.09 × 0.725 ≈
-1.8 mk at 2.2 hours.

TRIP FROM FULL POWER EQUILIBRIUM CONDITIONS

The equation for the time dependent xenon concentration after a trip from equilibrium steady state is simpler than the one for Xe-135 buildup in a freshly fuelled core:

$$N_{Xe}(t) = N_{Xe(eq)} e^{-\lambda_{Xe} t} + \frac{\lambda_I}{\lambda_I - \lambda_{Xe}} \cdot N_{I(eq)} \cdot \left\{ e^{-\lambda_{Xe} t} - e^{-\lambda_I t} \right\} \qquad (6)$$

The numerical equivalent, in mk, is $Xe(t) = 28e^{-\lambda_{Xe} t} + 3.6 \times 322 \cdot \left\{ e^{-\lambda_{Xe} t} - e^{-\lambda_I t} \right\}$

By taking a time derivative of $N_{Xe}(t)$ and equating to zero one can solve for the time from the trip to the peak of the transient

$$t_{peak} = \frac{1}{\lambda_I - \lambda_{Xe}} ln\left[\frac{\lambda_I}{\lambda_{Xe}}\right] - \frac{1}{\lambda_I - \lambda_{Xe}} ln\left[1 + \frac{(\lambda_I - \lambda_{Xe})}{\lambda_I} \cdot \frac{N_{Xe(eq)}}{N_{I(eq)}}\right] \qquad (7)$$

Using Table 1 and equations (5), equation (7) can be written numerically:

t_{peak} (hours) = 11.1 - 33.9 ln[1 + 0.024/(0.94 × P + 0.06)]

This gives t_{peak} = 11.1 hrs - 0.8 hrs = 10.3 hours.

For very high flux reactors the time to the peak approaches 11.1 hours. For trips from lower power it takes less time to reach the peak.

The size of the peak at 10.3 hours after a trip from full power is, using (6):
28 mk×0.46 + [2.93/(2.93 - 2.11)]×322mk×{0.457 - 0.337} = 13 + 3.57×322×0.12 = 151 mk

A simpler numerical method can be used to estimate the peak size. At the peak, xenon decay = iodine decay, so $\lambda_I (N_{Ieq} e^{-\lambda t}) = \lambda_{Xe} N_{Xe}^{peak}$, giving, using equation (2):

$$N_{Xe}^{peak} = \frac{\lambda_I}{\lambda_{Xe}} \cdot \left(\frac{\gamma_I \Sigma_f \phi}{\lambda_I}\right) e^{-\lambda_I t_{peak}}$$

This explicitly shows the peak size is proportional to pre-shutdown flux.

Numerically, $N_{xe}^{peak} = (\lambda_I/\lambda_{Xe}) \times 322\ mk \times 2^{-(10.3/6.6)}$

The estimate of the xenon peak size from this gives N_{Xe} near 152 mk.

* *Xenon Transient Behaviour after a Trip from Full Power.* *poison override time* and *decision and action time*

The net xenon production rate immediately after a trip from full power is, ($\lambda_I \times 322$ mk - $\lambda_{Xe} \times 28$ mk) = (9.4 - 0.6) $\times 10^{-3}$ mk/s ≈ 0.5 mk/min. (λN gives the number of decays per second). Build-up continues at this fairly steady pace immediately following the trip.

The reactor control system (adjuster rod removal) can add + 15 mk of reactivity to offset the loss. Approximately 30 minutes after the trip the xenon load will have increased by 15 mk (30 min × 0.5 mk/min) , making recovery impossible. This time interval is called the ***poison override time***.

Since removing adjuster rods takes at least 5 to 10 minutes, the trip must be reset within about 20 minutes, the ***decision and action time***. In most cases investigation of the cause of the trip takes longer than this. Most trips result in a full reactor shutdown with restart in about 1_ days, after the iodine is gone and xenon decreases from the peak to near its equilibrium value.

* *Samarium*

Use the tank model to find equilibrium samarium concentration. Show it is independent of flux.

The same methods can be used to analyze Promethium and Samarium build-up to equilibrium and after a trip. Here we will, but simply present some results.

PROMETHIUM & SAMARIUM EQUATIONS

Promethium and Samarium obey the same set of differential equations as Iodine and Xenon, only the parameters are different. The differential equations and initial conditions in a fresh core are identical for Pm-149 and I-135, so the equations for initial buildup of promethium are:

$$N_{Pm}(t) = N_{Pm(eq)}\left(1 - e^{-\lambda^{Pm} t}\right) \qquad \text{(S1)}$$

with

$$N_{Pm(eq)} = \frac{\gamma_{Pm}\Sigma_f \phi}{\lambda^{Pm}} \qquad \text{(S2)}$$

TABLE 2

$\lambda_I \rightarrow \lambda_{Pm} = 3.63 \times 10^{-6}\ s^{-1}$	(G.E. Nuclear Chart 1989)
$\lambda_{Xe} \rightarrow \lambda_{Sm} = 0$	
$\sigma_a^{Xe} \rightarrow \sigma_a^{Sm} = 4.2 \times 10^4\ b = 4.2 \times 10^{-20}\ cm^2$	(New Transent value is $4.38 \times 10^{-20}\ cm^2$)
$\gamma_I \rightarrow \gamma_{Pm} = 1.13\%$	(Nuc. Theory notes)
$\gamma_{Xe} \rightarrow \gamma_{Sm} = 0$	
$\Sigma_f \approx 0.1\ cm^{-1}$ (fresh CANDU fuel)	
$\Sigma_f \approx 0.089\ cm^{-1}$ (equilibrium fuelling) is burnup dependent	
$\phi_{F.P.} = 9.1 \times 10^{13}\ n\ cm^{-2}\ s^{-1}$ (fuel flux at full power/equilibrium fuelling: BNGSB Xe predictor)	
time constants for ϕ_{final} = full power flux (for equivalent half lives multiply by $ln2 = 0.693$):	
$\left(\sigma_a^{Sm}\phi_{final}\right)^{-1} \approx 72.6 hrs = 3.0 days$	(half time 2.1 days)
$\left[\sigma_a^{Sm}\phi_{final} - \lambda_{Pm}\right]^{-1} \approx 1375 hours = 57.3 days$	(half time 39.7 days)
$1/\lambda_{Pm}$ = 76hours = 3.2 days	(half life 53 hours or 2.2 days)

Here is the general solution for Samarium-149 build-up:

$$Sm^{149}(t) = \frac{\gamma_{Pm}\Sigma_f}{\sigma^{Sm}}\left\{\left(1 - e^{-\sigma^{Sm}\phi t}\right) - \frac{\left(\frac{\sigma^{Sm}\phi}{\lambda^{Pm}}\right)}{\left(1 - \frac{\sigma^{Sm}\phi}{\lambda^{Pm}}\right)}\left(e^{-\sigma^{Sm}\phi t} - e^{-\lambda^{Pm}t}\right)\right\} \quad \text{(S3)}$$

When $\phi << \lambda/\sigma$ (very low flux) the build-up depends on the slow Sm burnout rate.

$$Sm^{149}(t) = \frac{\gamma_{Pm}\Sigma_f}{\sigma^{Sm}}\left(1 - e^{-\sigma^{Sm}\phi t}\right)$$

When $\phi >> \lambda/\sigma$ (very high flux) the build-up depends on the Pm decay rate

$$Sm^{149}(t) = \frac{\gamma_{Pm}\Sigma_f}{\sigma^{Sm}}\left(1 - e^{-\lambda^{Pm}t}\right)$$

Neither of these flux conditions is correct for a CANDU at high power.

For $\lambda_{Pm}/\sigma_a^{Sm} = \phi_c = 3.63\times10^{-6}/4.2\times10^{-20} = 8.6 \times 10^{13}$ cm^{-2} s^{-1} (a value approaching full power flux for a typical CANDU) both the numerator and denominator of the second term in S3 are zero. In other words, a typical CANDU at full power has conditions such that
the Sm burn up constant ($4.2\times10^{-20} \times 9.1\times10^{13} = 3.82\times10^{-6}$ s^{-1}). is about the same as
the Pm decay constant (3.63×10^{-6} s^{-1})

When $\phi = \lambda/\sigma$, the equation requires L'Hôpital's Rule to evaluate it, giving

$$Sm^{149}(t) = \frac{\gamma_{Pm}\Sigma_f}{\sigma^{Sm}}\left\{1 - \left(1 + \lambda^{Pm}t\right)e^{-\lambda^{Pm}t}\right\}$$

In this equation, by trial and error, Sm builds up to 97% of its equilibrium value for $\lambda t = 5.307$, giving a buildup time of 282 hours or about 11.75 days, approaching the 11 day buildup time in very high flux and about half the time required for buildup in 10% F.P. flux.

As long as the flux is near ϕ_c, i.e. in situations where roundoff errors in (S3) give poor numerical results, the equation with $\lambda = \sigma\phi$ can be used.

EQUILIBRIUM SAMARIUM

Setting equilibrium samarium-149 production, 100% from promethium decay, equal to equilibrium Sm-149 removal, 100% by burnup, yields:

$$\lambda_{Pm} N_{Pm(eq)} = \gamma_{Pm}\Sigma_f\phi = N_{Sm(eq)}\sigma_a^{Sm}\phi \qquad \text{so} \qquad N_{Sm(eq)} = \left(\gamma_{Pm}\right)\frac{\Sigma_f}{\sigma_a^{Sm}} \qquad \text{(S4)}$$

Equilibrium samarium is the same for any flux, but the time to get there changes

The ratio of absorption rates for Xenon and Samarium are $N_{Sm}\sigma_a^{Sm}\phi/N_{Xe}\sigma_a^{Xe}\phi$
$= (\sigma_a^{Sm} N_{Sm}/\sigma_a^{Xe} N_{Xe}) = \gamma_{Pm} / (\gamma_I + \gamma_{Xe}) \times [1+\lambda_{Xe}/\sigma_a^{Xe}\phi]$
$= (1.13/6.6) \times [1 + 2.11\times10^{-5}/3.185\times10^{-4}] = 0.171 \times 1.066 = 0.1825$

Full power equilibrium absorption in Xe-135 is worth 28 mk, so Sm is worth 0.1825 × 28 = 5.1 mk, independent of power level. This estimate depends mainly on how accurately the yields are known, and is nearly insensitive to the estimate of the full power flux.

SAMARIUM BUILDUP AFTER A TRIP

The differential equations are simple to solve for flux dropping to zero. In fact the solution is obvious, since there is no decay of samarium, the promethium equilibrium bank decays to add to the samarium already present:

$$N_{Sme}(t) = N_{Sm(eq)}\left[1 + \frac{\sigma_a^{Sm}\phi}{\lambda_{Pm}}\left(1 - e^{-\lambda_{Pm}t}\right)\right] \qquad \text{(S5)}$$

With $\lambda \approx \sigma\phi$ this becomes, numerically, $Sm(t) = 5.1[2-e^{-\lambda t}]$

The time for most of the increase to show up, about 5 half lives, is 265 hours = 11 days. The final value is,

$$N_{Sm}^{peak} = N_{Sm(eq)}\left[1 + \frac{\sigma_a^{Sm}\phi}{\lambda_{Pm}}\right] \qquad \text{(S6)}$$

= 5.1 mk [1 + 1.05] for $\phi = 9.1 \times 10^{13}$. Thus the shutdown load for a CANDU after a trip is about double the normal load

OTHER FISSION PRODUCTS (& Plutonium)

Several other fission products, mainly Sm-151 and Rh-105, together with Pu-239, contribute small but measurable transient reactivity effects after shutdown and on restart. The effects on operation are mostly negligible. They can affect the required fuelling rate in the first few weeks after a reactor restart following a long maintenance shutdown.