5. Approximate Solutions for Reactivity Ramps

- Reactivity Mechanisms

  - characterized by:

    1. reactivity worth. (mk) $\Delta\rho = \lambda_0 - \lambda_1$.

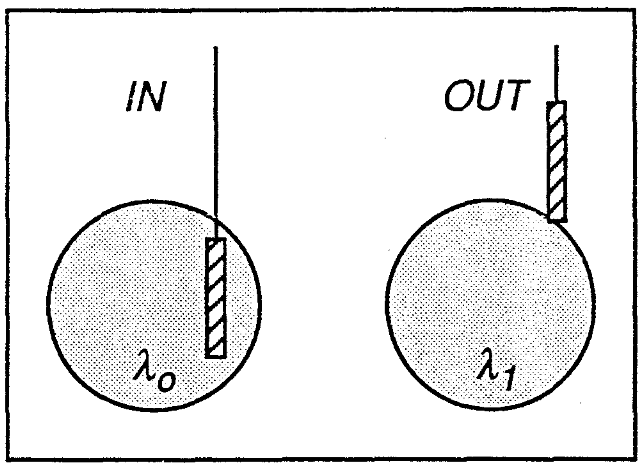
       Must be greater than the reactivity effect to be compensated.

    2. insertion rate (mk/s) $\Delta\rho/\Delta t$

       Must be as rapid as the effect to be controlled.

- The reactivity worth is obtained from the difference of eigenvalues between 2 diffusion calculations:

$$
\begin{aligned}
\delta\rho_{rod} &= \rho_1 - \rho_0 \\
&= (1-\lambda_1) - (1-\lambda_0) \\
&= \lambda_0 - \lambda_1 \\
&= -\Delta\lambda
\end{aligned}
$$



- It is extremely difficult to obtain an analytical solution to the P.K. equations when reactivity varies with time. A numerical solution is usually obtained. For an analytical solution, an approximation is required.

- We will consider a Reactivity Ramp: $\rho(t) = \mu t$

CANDU Reactor Control

Inherent Reactivity Effects in CANDU 600

CAUSE	*REACTIVITY WORTH*	*INSERTION RATE*
Fuel and coolant warm-up: (25 °C to 290 °C)	fresh fuel: - 7 mk equilibrium: +3 mk	seconds, minutes
Reactor power: 0 to 100% FP	fresh fuel: - 9.4 mk equil.: -4.5 mk	seconds, minutes
Moderator temperature: 40 °C to 70 °C	fresh fuel: - 2.1 mk equil.: +2.7 mk	hours (seasons)
Fuel burnup: - initial excess reactivity - reactivity decline - refueling one channel	 ≈ 20 mk -0.4 mk/day 0.35 mk	 2-3 months (continuous) hour
Coolant density: 0.8 → 0 g/cc (LOCA)	fresh fuel: + 15 mk equil.: + 10 mk	0.1 - 2 s

Reactivity Mechanisms in CANDU 600

TYPE	*REACTIVITY WORTH*	*INSERTION RATE*
Reactor Regulating System - primary : 14 Liquid Zone Controllers - secondary: 4 Solid Control Rods	 6 mk 10 mk	 ±0.14 mk/s ±0,09 mk/s
Long Term Reactivity Control - 21 Adjuster Rods (7 banks) - Soluble Poison (boron + gadolinium)	 15 mk large	 ±0,2 mk/s hours, days
Emergency Shutdown Systems - 28 Shut Off Rods - Gadolinium Injection	 -60 mk very large	 1.3 s 2.5 s

Constant Delayed Source (CDS) Approximation

- Delayed source does not vary rapidly: we assume it does NOT vary at all over a limited period of time (t is small)

$$\begin{aligned} s_d^{CDS}(t) &= s_{do} \\ &= \sum_{k=1}^{K} \lambda_k \underbrace{\frac{\beta_k}{\lambda_k} p_o}_{\zeta_{ko}} \\ &= \beta\, p_o \end{aligned}$$

- We obtain:

$$\Lambda \frac{dp}{dt} = [\rho(t)-\beta]p(t) + s_{do} + s(t)$$

- Considerably more simple than original equations, even with only 1 delayed neutron group (1DG).

- The CDS equation can be integrated for an *arbitrary variation* of *ρ(t)* :

$$p(t) = e^{\int_0^t \left(\frac{\rho(\tau)-\beta}{\Lambda}\right) d\tau} \left\{ p_o + \int_0^t \left[\frac{s_{do}+s(t')}{\Lambda}\right] \cdot e^{-\int_0^{t'} \left(\frac{\rho(\tau)-\beta}{\Lambda}\right) d\tau} dt' \right\}$$

- When reactivity is *constant* (step insertion at $t = 0$)

$$p(t) = e^{\left(\frac{\rho-\beta}{\Lambda}\right)t} \left\{ p_o + \int_0^t \left[\frac{s_{do}+s(t')}{\Lambda}\right] \cdot e^{-\left(\frac{\rho-\beta}{\Lambda}\right)t'} dt' \right\}$$

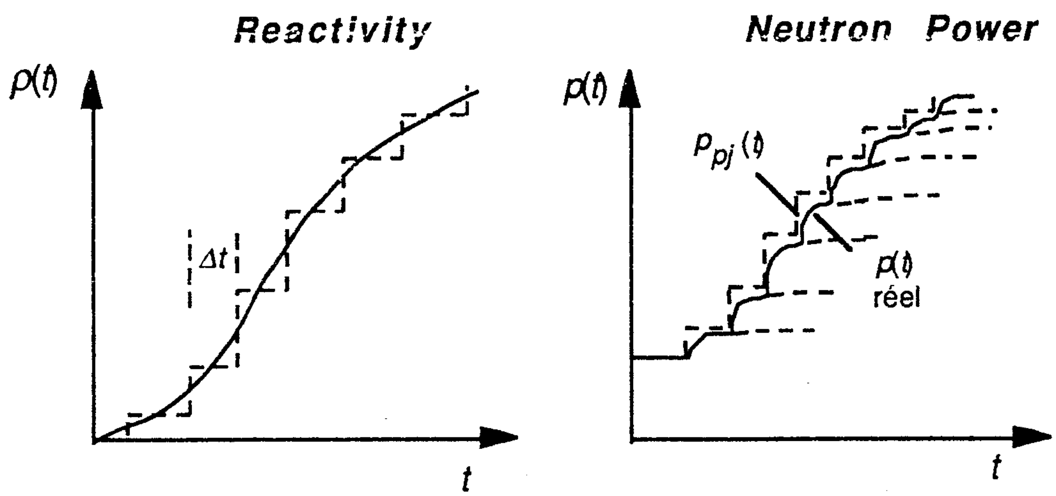
Prompt Jump Approximation (PJA)

- Response to step-change in reactivity showed *one* component responding very rapidly, while all other components were related to the delayed neutron source.

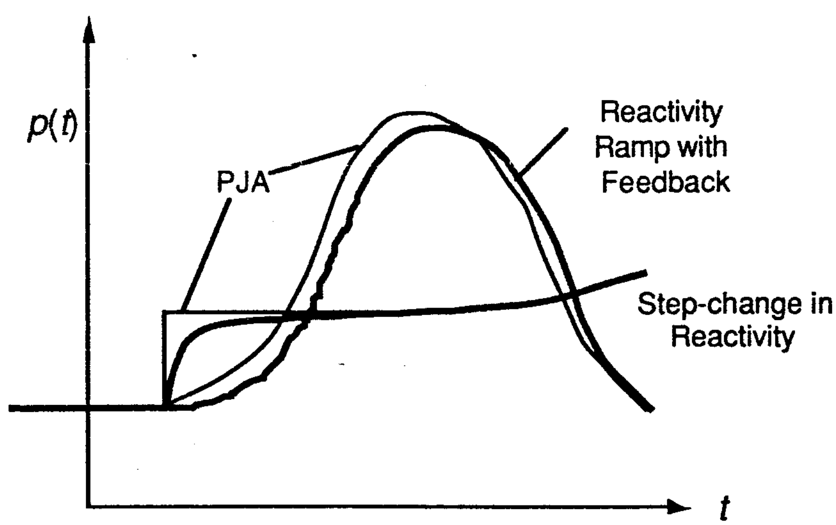
- Assume the prompt response to be essentially instantaneous:

$$T_p = \frac{\Lambda}{\beta - \rho} \rightarrow 0 \quad \text{with PJA}$$

$$\Lambda \rightarrow 0$$



- PJA response is always ahead of the real solution:



Prompt Jump Approximation (cont'd)

- The amplitude equation can be written:

$$\frac{dp}{dt} = \frac{\rho(t)-\beta}{\Lambda}p(t) + \frac{s_d(t)}{\Lambda} + \frac{s(t)}{\Lambda}$$

- Making $\Lambda \rightarrow 0$:

$$p_{pj}(t) = \frac{s_d(t)+s(t)}{\beta-\rho(t)}$$

(see source multiplication formula)

- One Delayed Group:

$$s_d(t) = \lambda\,\zeta(t)$$
$$\frac{d\zeta}{dt} = \beta\,p(t)-\lambda\,\zeta(t)$$

1. from above:

$$[\rho(t)-\beta]p(t) + \lambda\,\zeta(t) + s(t) = 0$$

2. derive with t :

$$[\rho(t)-\beta]\frac{dp}{dt} + \frac{d\rho}{dt}p(t) + \lambda\frac{d\zeta}{dt} + \frac{ds}{dt} = 0$$

3. eliminate $d\zeta/dt$:

$$\frac{dp}{dt} = \frac{[\lambda\,\rho(t)+d\rho/dt]}{\beta-\rho(t)}p(t) + \left[\lambda\,s(t)+\frac{ds}{dt}\right]$$

4. initial condition:

$$p^o = p(0^+) = \left(\frac{\beta-\rho_o}{\beta-\rho^+}\right)p_o$$

- general solution (PJA):

$$p_{pj}(t) = \exp\left[\int_{0^+}^{t}\left(\frac{\lambda\rho(t')+d\rho/dt'}{\beta-\rho(t')}\right)dt'\right]\cdot \left\{p^o + \int_{0^+}^{t}\left(\frac{\lambda s(t')+ds/dt'}{\beta-\rho(t')}\right)\exp\left[-\int_{0^+}^{t}\left[\frac{\lambda\rho(\tau)+d\rho/d\tau}{\beta-\rho(\tau)}\right]d\tau\right]dt'\right\}$$

Log-Rate

- The Log-Rate (or logarithmic derivative) can be measured.

- For an initially critical reactor (no source *s*):

$$\tau(t) \equiv \frac{1}{p}\frac{dp}{dt} = \frac{\lambda\rho(t)+d\rho/dt}{\beta-\rho(t)}$$

- important observations:

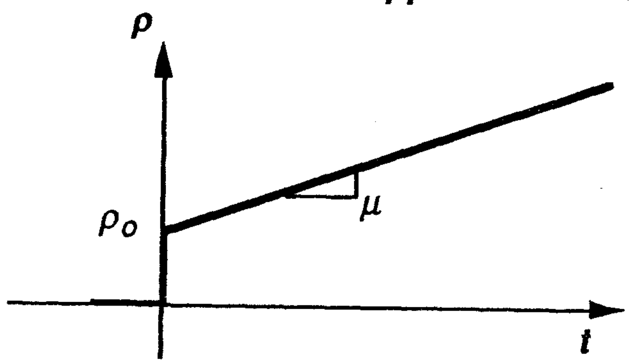
    1. rate of change of power in a nuclear reactor is not only function of the dynamic reactivity ρ(t). It is also a function of the *rate of change* of reactivity dρ/dt. ⇒ denotes the importance insertion rate (speed) of reactivity mechanisms.

    2. power can *decrease* even if reactor is supercritical (ρ(t)>0), if:
    $$\frac{d\rho}{dt} < -\lambda\rho$$
    3. power can *increase* even if reactor is subcritical (ρ(t)<0), if:
    $$\frac{d\rho}{dt} > -\lambda\rho$$

- Concl.:

When reactivity varies continuously, reactor response is sensitive to the rate of change of reactivity.

Application of PJA to a Reactivity Ramp



$$\rho(t) = \begin{cases} 0 & t \le 0 \\ \rho_o + \mu t & t > 0^+ \end{cases}$$

- The solution can be expressed as: $\quad P_{pj}(t) = p^o \; e^{I_1(t)} \cdot e^{I_2(t)}$
  where:

$$I_1(t) = \int_{0^+}^{t} \frac{d\rho/dt}{\beta - \rho(t)} dt$$

$$= \int_{\rho_o}^{\rho(t)} \frac{d\rho}{\beta - \rho}$$

$$= -\ln\left(\frac{\beta - \rho(t)}{\beta - \rho_o}\right)$$

$$I_2(t) = \int_{0^+}^{t} \frac{\lambda \rho(t')}{\beta - \rho(t')} dt'$$

$$= \int_0^t \frac{\lambda \rho_o}{\beta - \rho_o - \mu t'} dt' + \int_0^t \frac{\lambda \mu t'}{\beta - \rho_o - \mu t'} dt'$$

$$= \frac{\lambda \rho_o}{\mu} \ln\left(\frac{\beta - \rho_o}{\beta - \rho(t)}\right) + \lambda \left[\frac{\beta - \rho_o}{\mu} \ln\left(\frac{\beta - \rho_o}{\beta - \rho(t)}\right) - t\right]$$

$$= \frac{\lambda \beta}{\mu} \ln\left(\frac{\beta - \rho_o}{\beta - \rho(t)}\right) - \lambda t$$

Application of PJA to a Reactivity Ramp (cont'd)

- we find:

$$P_{pj}(t) = P_o \left[\frac{\beta}{\beta - \rho(t)}\right] e^{-\lambda t} \left(\frac{\beta - \rho_o}{\beta - \rho(t)}\right)^{\frac{\lambda\beta}{\mu}}$$

- in the absence of an initial step ρ_o :

$$P_{pj}(t) = P_o \; e^{-\lambda t} \left(\frac{\beta}{\beta - \mu t}\right)^{1+\frac{\lambda\beta}{\mu}}$$

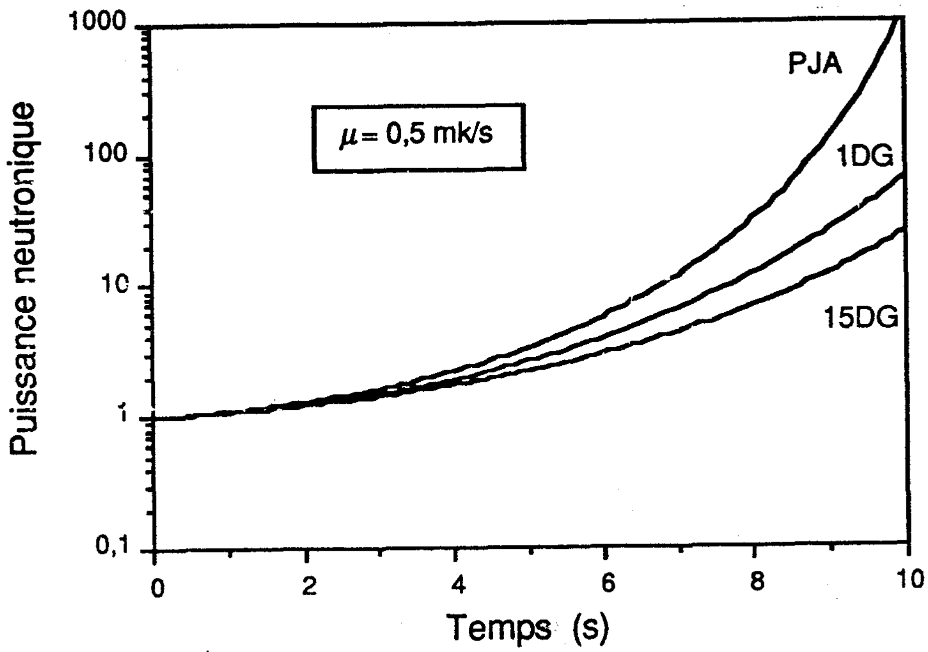
- if μ is zero (no ramp), with an initial step ρ_o, we find:

$$P_{pj}(t) = P_o \left(\frac{\beta}{\beta - \rho_o}\right) e^{\frac{\lambda\rho_o}{\beta - \rho_o} t}$$

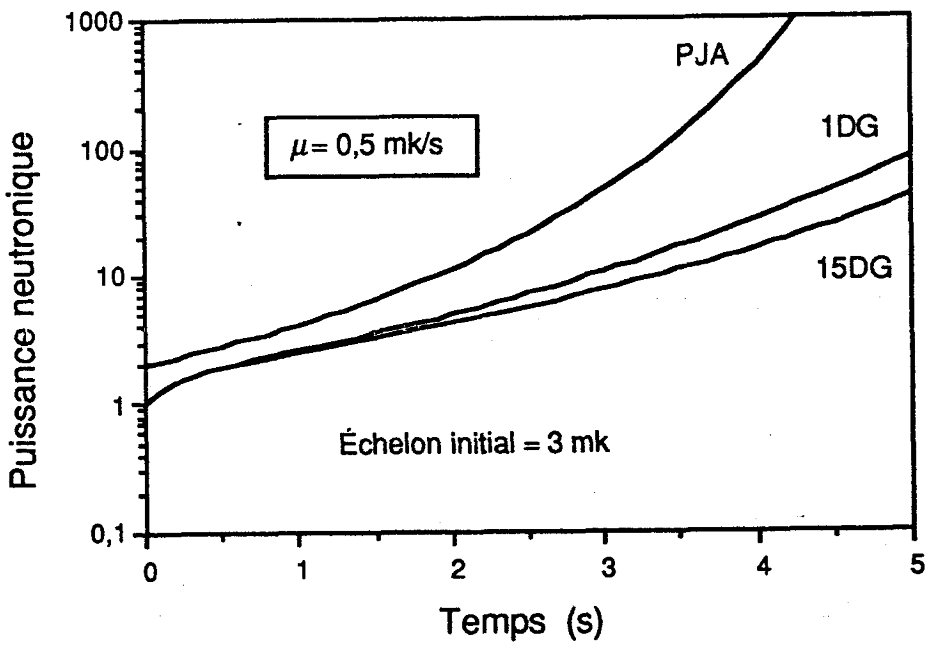
- Conclusion:

  For constant reactivity (no ramp), the PJA reduces to the first term of the equation obtained with 1DG.

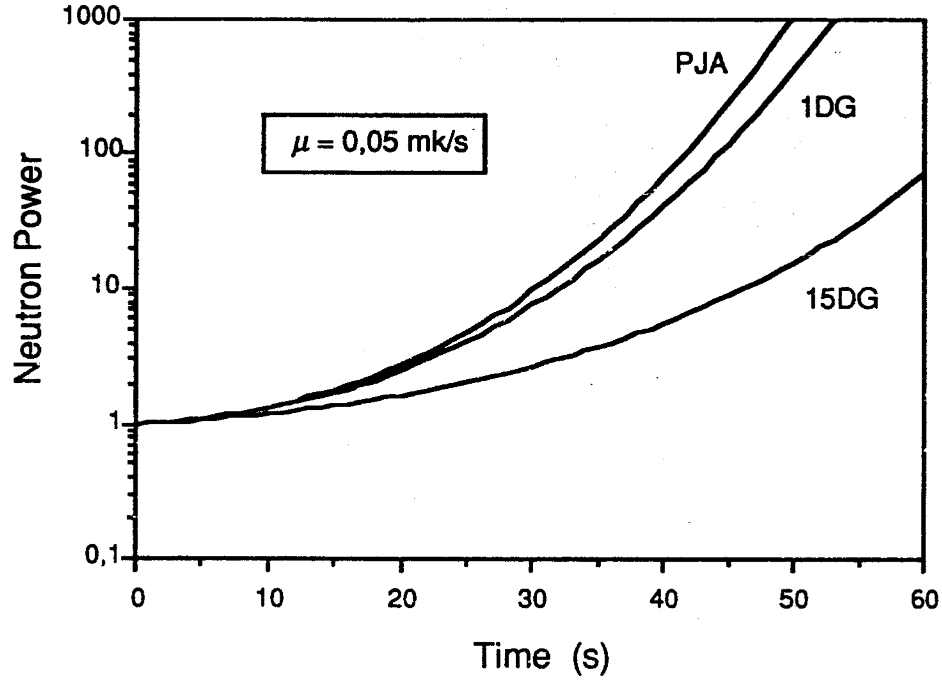
Reactivity Ramp in a CANDU Reactor (0.5 mk/s)

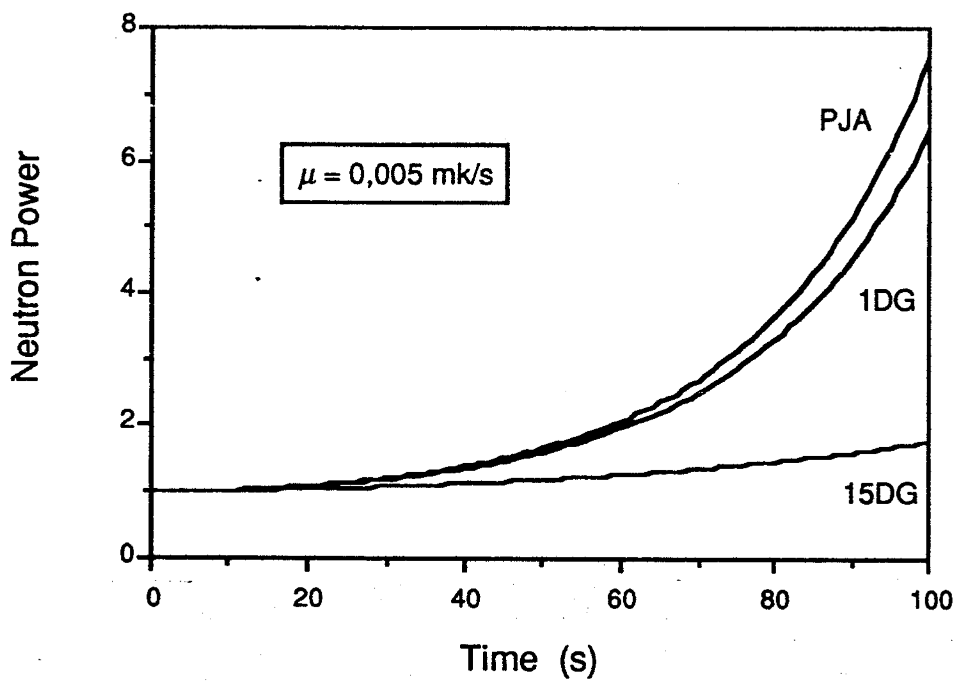


- with an initial step of 3 mk:

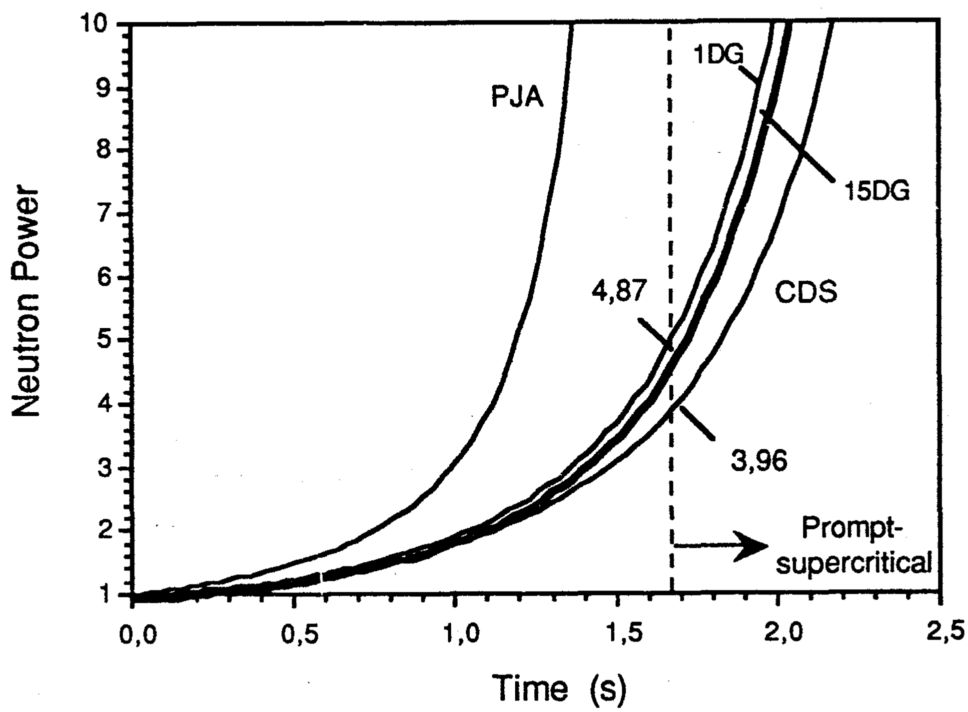


Small Reactivity Ramps in a CANDU Reactor





Large Reactivity Ramp in a CANDU Reactor
(LOCA, 3.5 mk/s)



- Observations:

1. PJA is <u>not</u> applicable (solution diverges as $\rho \Rightarrow \beta$)

2. CDS gives good results as long as $\rho < \beta$

3. For fast ramps, number of delayed groups is not significant (although the presence of the delayed source is important)

Assignment no 6

From the definition of the Point Kinetics Parameters, show that, for the extremely simplified case where:

1. the nuclear properties are uniform;
2. the reactor contains only one fissile isotope;
3. the neutrons are monoenergetic.

a) the effective delayed neutron fraction becomes:

$$\beta = v_d/v$$

b) the mean generation time can be written:

$$\Lambda = 1/(vv\Sigma_f)$$
$$= \Delta t_f/v$$

where Δt_f is the mean time between fission events.

Assignment no 7

A CANDU reactor under equilibrium refueling is operating at full power when the Safety Shutdown System is activated. A negative reactivity of -30 mk is suddenly inserted, and the power drops very rapidly. The neutron power, as indicated by the control flux detectors, does not fall to zero;

a) the power rapidly drops to a particular value, and afterwards, goes down at a much slower pace. Why does the power stop falling rapidly? At what power level (in % of full power, %FP) does the power tend to stabilize?

b) what is the neutron power in the reactor after 1 minute?

Assignment no 8

The previous reactor has been shutdown for some time. The ionization chambers indicate a neutron power of $1.5\text{x}10^{-3}$ %FP. The operator demand a 50% power increase to the Reactor Regulating System (RRS). After a few minutes, power is stable at the new level, and the operators notices that the Liquid Zone Controler level has dropped by 15% and is now indicating 55% fill.
What will be the final Liquid Zone Control level when the reactor becomes critical?

(Assume that 100% displacement of LZC is worth 6 mk in reactivity)