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APPENDIX – A

THE POWER TRANSFER EQUATION

The following is a proof of the power transfer equation for your reference only. **You are not expected to be able to prove or remember this derivation.**

Figure 8.20 shows the vector diagram from Figure 8.4(b) in greater detail. It will be used to show how the power transfer equation can be derived.

Remember that the resistance of the generator and the line are **neglected** in this example, thus, there is **no active power** loss between the generator terminals and the load.

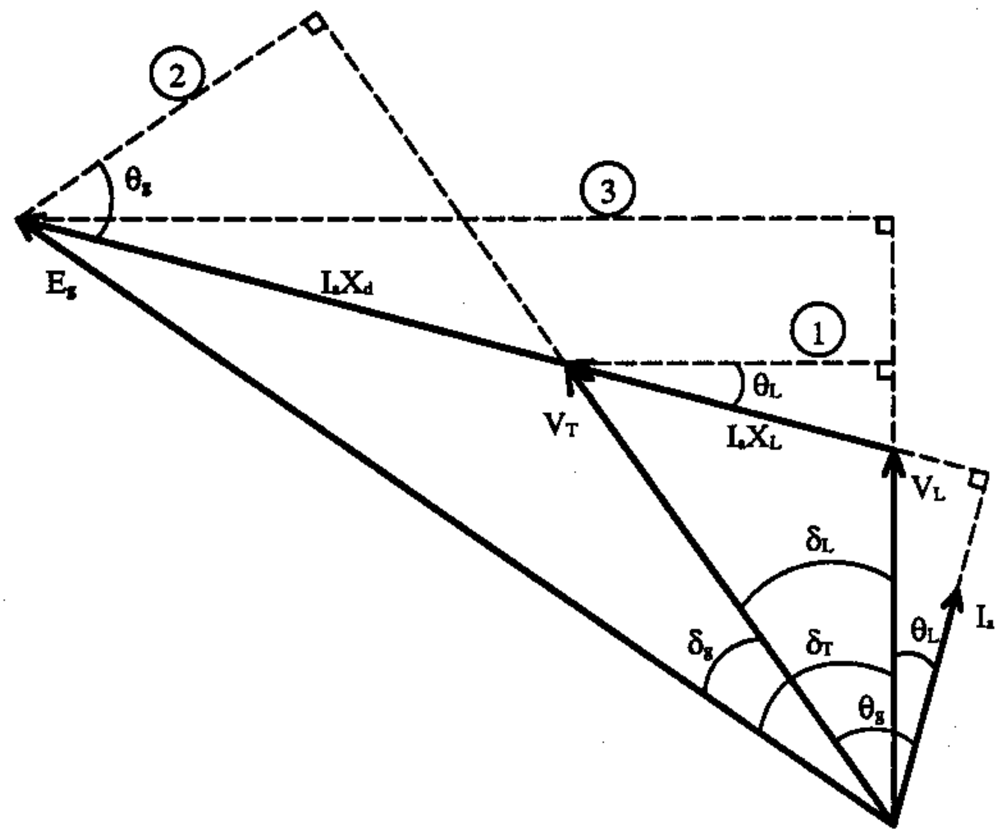

Figure 8.20: Vector Diagram for Derivation of Power Transfer Equation

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The active power P_{gen} at the generator terminals, on a per phase basis, will be:

$$P_{gen} = V_T I_a \cos\theta_g \qquad (1)$$

Similarly, the active power P_{load} at the load terminals will be (note that $I_{load}=I_a$):

$$P_{load} = V_L I_a \cos\theta_L \qquad (2)$$

As losses are neglected,

$$P_{gen} = P_{load} \qquad (3)$$

Thus, it can be stated that the active power at both ends of the circuit is the same. Let this power be called P, which gives,

$$P = V_T I_a \cos\theta_g = V_L I_a \cos\theta_L \qquad (4)$$

Equations (1) and (2) can be used to develop power transfer equations for the line and the generator. This development follows.

In Figure 8.20, the dashed line labelled as 1 (number in a circle) can be expressed in two equivalent trigonometric forms (using sine and cosine rules):

$$I_a X_L \cos\theta_L = V_T \sin\delta_L$$

which can be re–arranged as,

$$I_a \cos\theta_L = \frac{V_T \sin\delta_L}{X_L} \qquad (5)$$

substituting equation 5 into equation 2 gives,

$$P = \frac{V_T V_L \sin\delta_L}{X_L} \qquad (6)$$

where δ_L is the line load angle and X_L is the reactance of the line.

This is the **power transfer equation for the line.**

Also in Figure 8.20, the dashed line labelled as 2 can be expressed in two equivalent trigonometric forms:

$$I_a X_d \cos\theta_g = E_g \sin\delta_g$$

which can be re–arranged as,

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$$I_s\cos\theta_s = \frac{E_s\sin\delta_s}{X_d} \quad (7)$$

substituting equation (7) into equation (1) gives,

$$P = \frac{E_s V_T \sin\delta_s}{X_d} \quad (8)$$

where δ_s is the generator load angle and X_d is the reactance of the generator.

This is the **power transfer equation for the generator.**

Finally, in Figure 8.20, the dashed line labelled as 3 can also be expressed in two equivalent trigonometric forms:

$$I_s(X_d+X_L)\cos\theta_L = E_s\sin(\delta_s+\delta_L)$$

which can be re–arranged as,

$$I_s\cos\theta_L = \frac{E_s\sin(\delta_s+\delta_L)}{(X_d+X_L)} \quad (9)$$

substituting equation 9 into equation 2 gives,

$$P = \frac{E_s V_L \sin(\delta_s+\delta_L)}{(X_d+X_L)} \quad (10a)$$

or,

$$P = \frac{E_s V_L \sin\delta_T}{(X_d+X_L)} \quad (10b)$$

where δ_T is the total load angle.

Equations (10a) and (10b) are the **power transfer equations for the generator and the line together.**