

Mathematics - Course 221

APPENDIX 4: ANSWERS TO ASSIGNMENTS AND REVIEW EXERCISES

421.10-4 Assignment

1. (a) 10^7 (b) 10 (c) 10^6 (d) 10^{12}
 (e) 10^{-8} (f) 10^{-9} (g) 10^{-4} (h) 10^{-4}
2. (a) 10^{-2} (b) 10^{-8} (c) 10^2 (d) 1
 (e) -10^{-7} (f) 10^{13} (g) 10^{10} (h) 10^{-13}
 (i) 10^9 (j) 10^{-11} (k) 10^{13} (l) -10
3. (a) 100 (b) 0.001 (c) 100,000 (d) 0.000001
 (e) 1,000,000 (f) 0.0001
4. (a) 1.65×10^5 (b) 6.93×10^{-3} (c) 3.75×10
 (d) 2.5×10^{-2} (e) 2.934×10^3 (f) 1.01×10^{-3}
 (g) 1×10^4 (h) 2.0×10^{-4} (i) -2.49×10^2
 (j) 9.7×10^{-1} (k) 1.76×10^{-1} (l) 2.7
 (m) 9.57×10^4 (n) 1.75×10^{-14} (o) 2.4×10^7
 (p) 3.2×10^{12}
5. (a) 2.4×10^3 (b) 5.6×10^{12} (c) -1.1×10^{14}
 (d) -4.3×10^3 (e) 4.5×10^{12}
6. (a) 9.3×10^5 (b) 6.9×10^5 (c) 3.4×10^7
 (d) 5.5×10^{-12} (e) 5.5×10^2
7. (a) 2.3×10^{-1} (b) 9.4×10^{-5}

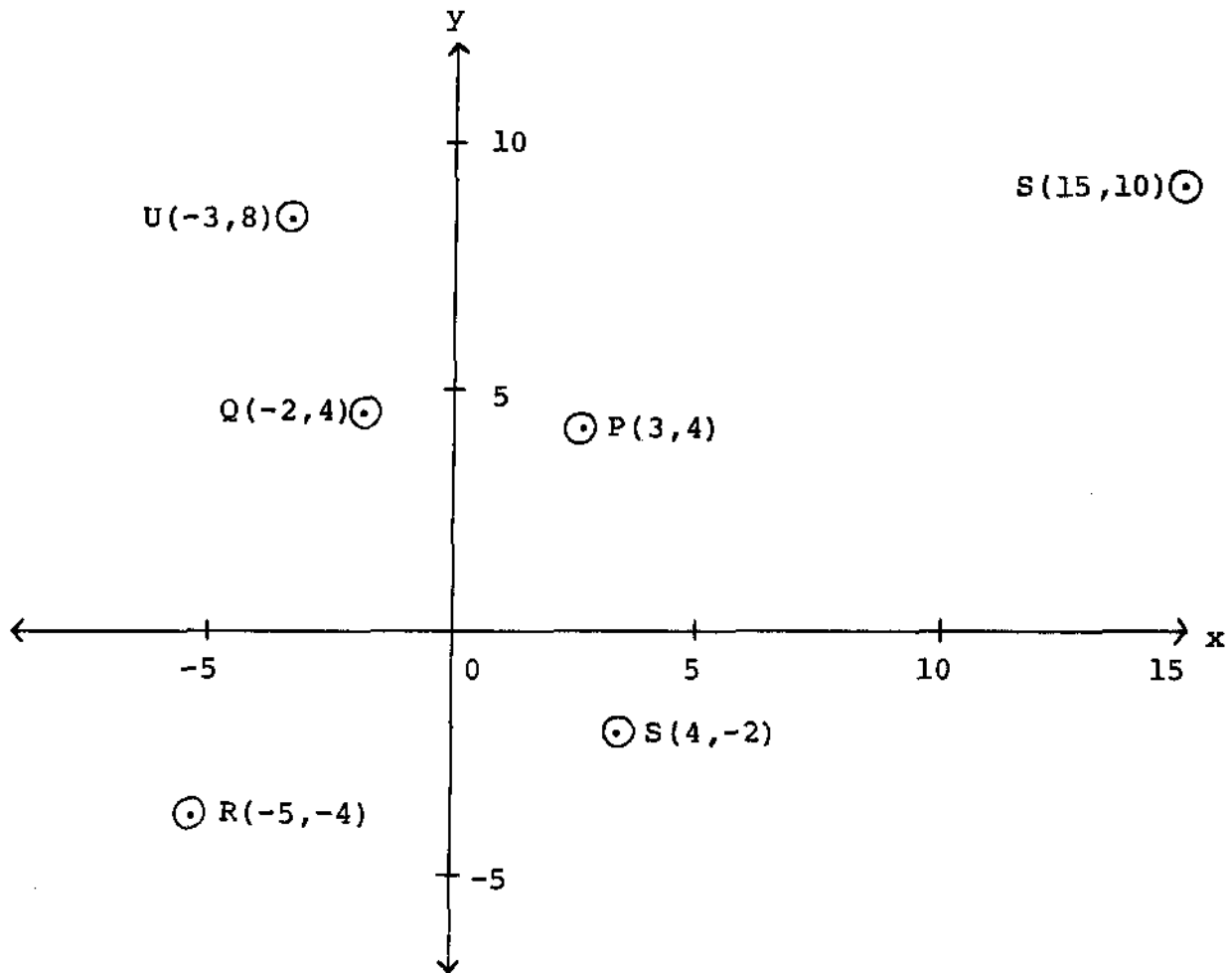
421.20-1 Assignment

1. (a) $-10\frac{5}{6}$ (b) 9 (c) 5
2. (a) a^{10} (b) $\frac{3}{32} a^{10}$ (c) b^{12} (d) 3^7
 (e) m^6 (f) a^{19} (g) $\frac{1}{a}$ (h) b^7
 (i) a^{14} (j) $-27a^6$ (k) $\frac{1}{32} x^{20}$ (l) $a^{\frac{5}{6}}$
 (m) $a^{\frac{1}{6}}$ (n) $9x^2y$ (o) 1 (p) $x y^{-4}$
3. (a) 27 (b) $\frac{1}{32}$ (c) $\frac{1}{2}$ (d) $6\frac{3}{4}$
 (e) $-\frac{1}{27}$ (f) 6 (g) -2 (h) -32
 (i) $1\frac{7}{9}$
4. (a) 1 (b) $4x^8$ (c) $3^5 a^2 b^3$ (d) $\frac{18y^2 z^2}{x^3}$
5. $9.1 \times 10^{-28} \text{g}$
6. $6.2 \times 10^{18} \text{ fissions/s}$
7. (a) $11a$ (b) $6x^2 + 6x + 5xy + 5$
 (c) $25x + 20y$ (d) $8a + 5b + 12c$
 (e) $5k - 2j$ (f) $10a$
 (g) $4xy^2 + y$ (h) $4x^2y^3 + 1 - x$
 (i) $-5x - y + 4z$
8. (a) $-2y$ (b) $\frac{1}{3y}$ (c) $5b$ (d) -2
 (e) x (f) $-3abc$ (g) $\frac{2y}{x}$ (h) $\frac{1}{2} x^2$
 (i) $-24x^2y^4p$ (j) $22p^2qs^2t$

9. (a) $x^2 - 4xy - 32y^2$ (b) $15x^2 + 22x + 8$
(c) $12a^2 - 23ac + 10c^2$ (d) $x^4 - y^2$
(e) 0 (f) $2x + 5y$
(g) $a^2 - 1$ (h) $4x + 5$
(i) $3x - 8$ (j) $-(x^2 + x - 3)$
10. (a) $32m^2nx$ (b) $-36ab^2c^2d$
(c) $-18my + 15ty$ (d) $20h - 30k$
(e) $2x - 23y$ (f) $x^2 + xy$
(g) $3c + k$ (h) $-b + 7c$
(i) $-3a - 6x$ (j) $2ab$
(k) $2xy$

421.40-1 Assignment

1.

3. (a) 12 A (b) 2.4 Ω

4. (a) 893 kPa (b) 12.5 cm

421.40-2 Assignment

1. (a) $C(r) = 2\pi r$
(b) $d(t,v) = vt$
(c) $A(r,h) = 2\pi r^2 + 2\pi rh$
2. $f(6) = 9$; $f(0) = -3$; $f(-2) = -7$
3. $H(0) = 9$; $H(1) = 0$; $H(a) = 0$
4. $d(p) = \frac{\sqrt{2p}}{4}$
5. (a) ± 2
(b) no real roots since curve does not cross x-axis
(c) $-2.9, -0.2, 3.1$
(d) $3, -2$
(e) $-1.53, -0.33, 1.87$

321.10-3 Assignment

1. (a) (i) $R = 1.67\Omega$ (ii) $I = 1.2 \text{ A.}$
 (b) (i) $R = 2.17\Omega$ (ii) $I = 4.6 \times 10^{-2} \text{ A.}$
2. (a) $7.4 \times 10^{-5} \text{ s}^{-1}$ (b) 2.6 h
3. (a) $\frac{1}{2}$ (b) 0.6 (c) 9 (d) 4
 (e) -0.2 (f) -10 (g) 10 (h) 8
4. $0.84 \text{ k}\Omega$
5. $1.1 \times 10^2 \text{ hr.}$
6. (a) -3.1 (b) -9.8 (c) 2.6 (d) 1.7×10^3
 (e) 1.1 (f) -11 (g) $x = 3^{2 \cdot 7} = 19$
 (h) $x = 7^{4 \cdot 8} = 1.1 \times 10^4$ (i) $x = 9^{2 \cdot 1} = 1.0 \times 10^2$
 (j) $x = 4^{5 \cdot 3} = 1.6 \times 10^3$ (k) $x = 17^{16 \cdot 8} = 4.7 \times 10^{20}$
 (l) $x = 6^{7 \cdot 5} = 6.9 \times 10^5$
7. (a) $\frac{3}{4} \log X + \frac{1}{6} \log Y + \frac{5}{4} \log Z$
 (b) $\log X + \frac{15}{2} \log Z - 9 \log Y$
 (c) $\frac{7}{6} \log X - \frac{5}{12} \log Y + \frac{1}{2} \log Z$

321.10-3 Assignment

8. (a) 1339.43 (b) 3.02×10^{-2} (c) 25.31
(d) 50.12 (e) 5.01×10^{-5} (f) 0.76
9. (a) 9.1×10^2 (b) 0.46 (c) 1.2 (d) 3.2×10^3

321.10-4 Assignment

4. 3.3 minutes
5. 1.55×10^3 MPC
6. 19.5 s

221.10-1 Assignment

1. 3×10^{-3}
2. 2×10^{-4}
3. (a) 1.4×10^{-3} (b) 1.4×10^{-3}
4. 1.7×10^{-2}
5. 4×10^{-4}
6. 3×10^{-3}
7. every 4 weeks

221.20-1 Assignment

1.

	Slope	Angle of Inclination	Equation
(a)	$\frac{4}{3}$	53.1°	$4x - 3y = 0$
(b)	$-\frac{2}{3}$	146.3°	$2x + 3y - 6 = 0$
(c)	-1	135°	$x + y = 0$
(d)	0	0°	$y - 2 = 0$
(e)	∞	90°	$x + 3 = 0$

2. Slope PQ = slope QR = $\frac{1}{2}$; Q is common to segments PQ, QR
 \therefore P, Q, R are collinear.

3.

	Slope	x - intercept	y - intercept
(a)	-1	4	4
(b)	$\frac{5}{4}$	4	-5
(c)	0	none	$\frac{6}{5}$
(d)	undefined	$-\frac{4}{15}$	none

221.20-2 Assignment

1.

	Tangent slope at $(x, f(x))$	Tangent slope at $x = 2$
(a)	$10x - 2$	18
(b)	$-\frac{2}{x^2}$	$-\frac{1}{2}$

2. (a) $8x^3 - 12x^2$ (b) $\frac{2x}{a^2} - \frac{2a^2}{x^3}$

(c) $-\frac{3}{2}x^{-3/2}$ or $-\frac{3}{2\sqrt{x^3}}$

3. (a) $2x - 6$ (b) $10x^4 - 3x^2$

(c) $2ax + b$ (c) $\frac{2}{3}x^{-1/3} - x^{-2/3}$

4. (a) 11 (b) -3

(c) $-\frac{1}{3}$ and 1

221.20-3 Assignment

1. (a) -4 (b) -2 (c) $\frac{1}{2}$

2. (-1, -3)

3.

	Tangent Equation	Normal Equation
(a)	$x + y - 2 = 0$	$x - y = 0$
(b)	$y = 4$	$x = 1$

4.

	$v(t)$	$v(2)$	$a(t)$	$a(2)$
(a)	$16t - 3$	29	16	16
(b)	$-8t - 4t^3$	-48	$-8 - 12t^2$	-56
(c)	$20t - 80/t^2$	20	$20 + 160/t^3$	40

5. 25m

6. Roots of $f'(x) = 0$ are $x = 2.73, -0.73$; $y = f(x)$ has a local maximum at $x = -0.73$, and a local minimum at $x = 2.73$.

221.20-4 Assignment

1. (a) $2x e^{x^2-4}$ (b) e^{-x}
 (c) $-x^{-2}e^{-x^{-1}}$ (d) $x^{-3/2}e^{-x^{-1/2}}$ or $x^{-3/2}e^{-1/\sqrt{x}}$
 (e) $\frac{5}{2}(5t^{3/2} - t^{-1/2})et^{5/2} - t^{1/2}$
 (f) $x^{-4}e^{-1/x^3}$

2. (a) (i) $v(t) = e^t - 3t^2$ (ii) $a(t) = e^t - 6t$
 (iii) $v(2) = e^2 - 12$
 (b) (i) $v(t) = -e^{-t} + 2$ (ii) $a(t) = e^{-t}$
 (iii) $v(2) = 2 - e^{-2}$

3.

t	0	0.25	0.5	1	1.5	2.0	2.5	2.75	3
s	1	1.27	1.52	1.72	1.10	-0.61	-3.44	-5.15	-6.91
v	1	1.10	0.90	-0.28	-2.27	-4.61	-6.57	-7.04	-6.91
a	1	-0.22	-1.35	-3.28	-4.52	-4.61	-2.82	-0.86	2.09

4. $9.3 \times 10^{-12} \text{ s}^{-1}$

5. (a) 0.45 Ci
 (b) (i) 5.2×10^{13} (ii) 1.1×10^{12}
 (c) (i) 0.34 Ci (ii) 7.1 mCi
 (d) 48 minutes
 (e) 4.4 hours

6. see text

7. By definition of $t_{1/2}$, $0.5A_0 = A_0 e^{-\lambda t_{1/2}}$

$$\therefore \ln 0.5 = -\lambda t_{1/2}$$

$$\therefore t_{1/2} = \frac{-\ln 0.5}{-\lambda}$$

$$\text{But } \ln 0.5 = \ln \frac{1}{2} = \ln 2^{-1} = -\ln 2$$

$$\therefore t_{1/2} = \frac{\ln 2}{\lambda}$$

9.

t	0	1	2	3
N(t)	10^{20}	7.94×10^{19}	6.31×10^{19}	5.01×10^{19}

t	5	10	15	18
N(t)	3.16×10^{19}	9.99×10^{18}	3.16×10^{18}	1.58×10^{18}

10.

t(s)	P(W)	P(%F.P.)	p^1 (%F.P./s)
0	100	1×10^{-4}	5×10^{-6}
20	2.7×10^2	2.7×10^{-4}	1.4×10^{-5}
40	7.4×10^2	7.4×10^{-4}	3.7×10^{-5}
60	2.0×10^3	2.0×10^{-3}	1.0×10^{-4}
80	5.5×10^3	5.5×10^{-3}	2.7×10^{-4}
100	1.5×10^4	1.5×10^{-2}	7.4×10^{-4}
120	4.0×10^4	4.0×10^{-2}	2.0×10^{-3}
140	1.1×10^5	1.1×10^{-1}	5.5×10^{-3}
160	3.0×10^5	3.0×10^{-1}	1.5×10^{-2}
180	8.1×10^5	8.1×10^{-1}	4.1×10^{-2}
200	2.2×10^6	2.2	0.11
220	6.0×10^6	6.0	0.30
240	1.6×10^7	16	0.81
260	4.4×10^7	44	2.2
280	1.2×10^8	1.2×10^2	6.0
300	3.3×10^8	3.3×10^2	16.3

- (c) The rate at which the needle moves across the linear scale (linear rate) increases exponentially with time--the needle moves imperceptibly for about 3 minutes, then moves ever more rapidly across the scale, covering the final half of the range in just 14 seconds. This is just as one would expect from the mathematical expression

$$P'(t) = \frac{\Delta k}{L} P(t) = \frac{\Delta k}{L} P_0 e^{\frac{\Delta k}{L} t}$$

The rate at which the needle moves across the log scale (rate $\log P$) is constant, ie, the needle advances by the same amount each 20 seconds. This is in agreement with the mathematical expression

$$\frac{d}{dt} \log P(t) = \frac{\Delta k}{L} \log e \quad (\text{cf. question 12})$$

- (d) linear scale more sensitive at high power; log scale at low power
- (e) (i) signal output proportional to $\log P$ is more sensitive to changes in P at low power ($\leq 10\%$ full power, say)
- (ii) signal output proportional to P is more sensitive to changes in P at high power

11. see text

221.20-5 Assignment

1. Force F equals mass m times R/C velocity v wrt time t .

Angular velocity ω equals R/C angular displacement θ wrt time t .

Angular acceleration α equals R/C angular velocity ω wrt time t .

Torque τ equals moment of inertia I times R/C angular velocity ω wrt time t .

Force F equals rate of decrease of potential energy E_p wrt distance r from force center.

Power P equals time rate of energy output (or conversion).

Electric current i equals rate of flow of charge q .

Capacitor current i_c equals capacitance C times R/C capacitor voltage V_c wrt time t .

Inductor voltage V_L equals inductance L times R/C inductor current i_L wrt time t .

Rate of decrease of number N of radioactive nuclei remaining at time t equals decay constant λ times N .

Rate of decrease in radioactive source activity A equals decay constant λ times A .

Rate of decrease in number N of nuclear projectiles equals macroscopic cross section Σ times penetration depth x .

'Linear rate' power P equals reactivity Δk times power P divided by mean neutron lifetime L .

'Rate log power' equals reactivity Δk divided by mean neutron lifetime L .

Specific heat capacity C of a substance equals R/C quantity Q of heat stored in substance wrt temperature T of substance, divided by the mass m of the substance.

Heat Q flow rate (in a fluid) equals specific heat capacity C times temperature difference ΔT across system times mass m flow rate.

Heat H flow rate equals minus thermal conductivity k times cross sectional area A times R/C (rate of increase) temperature T wrt depth x in conducting medium. (Minus sign indicates directions of heat flow and increasing temperature are opposite.)

R/C of gas volume V wrt temperature T equals number n of moles of gas times gas constant R divided by gas pressure P .

Voltage V induced across a coil equals number N of turns in coil times rate of decrease in magnetic flux ϕ linking the coils.

2. See Table 1, lesson 221.20-5.

3. $6.0 \times 10^{-3}N$

4. (a) $V_2 = -M \frac{di_1}{dt}$

(b) (i) $V_2(t) = 6t^2 - 12t$ (ii) $V_2(2) = 0$

221.30-1 Assignment

1. see text

2. see text

3. (a) $-\frac{3}{2}x^2 + C$ (b) $e^t + \frac{1}{4}t^4 + C$

(c) $\frac{2}{3}x^3 + \frac{3}{2}x^2 - 5x + C$ (d) $2e^{\sqrt{x}} + C$

(e) $x^4 - \frac{3}{4}x^{4/3} + C$ (f) $-e^{-t^2} + \frac{2}{5}t^{5/2} + C$

4.

	$v(t)$	$s(t)$
(a)	0	0
(b)	$2t + 10$	$t^2 + 10t + 14$
(c)	$t^2 + v_0$	$\frac{1}{3}t^3 + v_0t$
(d)	$11 - e^{-t}$	$e^{-t} + 11t - 11$

5. $s(t) = v_0t - 4.9t^2$

6. (a) 99 (b) $37\frac{1}{3}$
(c) $72\frac{1}{3}$ (d) $\frac{1}{2}(1 - e^{-4})$

7. (a) $79\frac{7}{15}$ (b) 77.5
(c) $23 - e^2$

221.30-2 Assignment

1. (a) 8 square units (b) $21\frac{1}{3}$ square units
(c) 9 square units
2. (b) (i) 7.2×10^2 mrem (ii) 1.8×10^2 mrem/h
3. (a) $\frac{V_0^2 C}{2T} (1 - e^{-2T/RC})$ (b) 1 W
4. (a) $v(t) = -6 - 9.8t$; $s(t) = 1000 - 6t - 4.9t^2$
(b) 14 seconds
(c) -73 m/s
(d) 6.5×10^2 m
5. 12 J
6. (a) 1.6×10^6 N (b) 9.8×10^4 Pa
(c) at (0,11)

Review Exercise #3

1. (b) 4 m/s (c) $2t + 2$ m/s (d) 6 m/s
2. (a) $v(t) = -1/\sqrt{t}$; $v(4) = -\frac{1}{2}$; $a(t) = \frac{1}{2}t^{-3/2}$
(b) $v(t) = 3t^2 - 4$; $v(5) = 71$; $a(t) = 6t$
3. (a) $15x^2 - 2$ (b) $\frac{8}{3\sqrt{x}}$
(c) $-2x - 4x^{-3}$ (d) $\frac{3x - 1}{2\sqrt{x}}$
4. (a) tangent $4x + y - 2 = 0$
normal $x - 4y + 25 = 0$
(b) tangent $y = 2$
normal $x = 1$
5. (a) 3.0 Ci (b) 1.1×10^{13}
6. (a) 4.2 Ci (b) 16 h (c) 53 h (d) 3.1×10^{16}

Review Exercise #4

1. $(-5, 58\frac{1}{3})$ and $(3, -11)$
2. $t = -\frac{5}{2}$ and $t = \frac{2}{3}$
3. (a) $x = 1$ and $x = -\frac{1}{3}$
(b) $x = -\frac{5}{2}$ and $\frac{3}{5}$
4. $t = \frac{3}{2}$ and $t = -1$

4. (a) $3x + 4y - 14 = 0$
 (b) $x - y - 3 = 0$
5. slope = $-\frac{4}{5}$; y-intercept = $\frac{13}{5}$
6. 8, -2
7. (a) 7.4 kW (b) 1.1×10^2 s or 1.8 minutes
8. (a) $-t^2$ (b) \sqrt{t}
9. (a) $1.2 \times 10^{-9} \text{ s}^{-1}$ (b) 2.6 h
10. (a) $1.3 \times 10^{12} \text{ dps}$ (b) 35 Ci

Review Exercise #7

1. (a) 30 (b) $12 \frac{2}{3}$ (c) 1162.1
2. (a) $v = -2t + 6$ $s = -t^2 + 6t$
 (b) $v = \frac{4}{3}t^{3/2}$ $s = \frac{8}{15}t^{5/2} + 100$
 (c) $v = -t^2/2 + 3t + v_0$
 $s = -t^3/6 + 3t^2/2 + v_0t$
3. (a) $y = 2x^2 + 5x + C$
 (b) $s(t) = \frac{2}{5}t^{5/2} + C$ (c) $v(t) = 3t^2 + C$
4. (a) $v(t) = -9.8t + v_0$; $s(t) = -4.9t^2 + v_0t$
 (b) $v(t) = v_0$; $s(t) = v_0t$
 (c) $v(t) = \frac{3}{4}t^{4/3} + v_0$; $s(t) = \frac{9}{28}t^{7/3} + v_0t + 10$

5. (a) $\frac{x^3}{3} - 2x + C$ (b) $\frac{x^4}{2} - 2x^2 + C$
 (c) $\frac{2}{3} x^{3/2} + C$ (d) $-\frac{x^4}{4} + C$
 (e) $-\frac{2}{x} + C$ (f) $10\sqrt{x} + 14x + C$
6. (a) $t^2 - 3t + C$ (b) $2t^{3/2} + 4t + C$
7. (a) $v(t) = -5t + 10$; $s(t) = -\frac{5}{2}t^2 + 10t$
 (b) $v(t) = \frac{2}{3}t^3$; $s(t) = t^4/6$

Review Exercise #8

1. Roots are $x = -5, 1, 3$
2. Roots are $x = -2.73, 2.07$.
 Significance: $f' = 0 \Rightarrow$ tangent slope = 0
 .'. curve $y = f(x)$ has maximum at $x = -2.73$ and minimum
 at $x = 2.07$.
3. tangent: $12x + y - 12 = 0$
 normal: $x - 12y - 1 = 0$
4. (a) 0.1 (b) t^2 (c) $-\frac{2}{t}$ (d) y
 (e) 0.4 (f) 100 (g) λt
5. (a) $\lambda = 8.7 \times 10^{-5} \text{ s}^{-1}$ (b) 2.2 h
 (c) 39 h
6. (a) $1.4 \times 10^{-3} \text{ s}^{-1}$ (b) 1.4 h

7. (a) $7x^6 - 18x^2 + \frac{1}{3}x^{-2/3}$ (b) $\frac{5}{2}x^{3/2} + \frac{1}{2}x^{-3/2}$
 (c) $\frac{2}{3}x^{-1/3}$ (d) $\frac{2}{5}x^{-3/5} - \frac{a}{x^2}$
 (e) $\frac{4}{t^3}e^{-2/t^2}$ (f) $2xe^{x^2} - 4$
 (g) $\frac{7}{2}x^{5/2} + \frac{1}{2}x^{-3/2}$

8. 0.0054

10. (a) $2x - 5y = 0$ (b) $25x + 10y + 6 = 0$
 (c) $2x - 2y - 3 = 0$

11. (a) $-x^2 + 0.4x - 7$
 (b) $v(t) = 4\sqrt{t} + 1$; $s(t) = \frac{8}{3}t^{3/2} + t + 4$
 (c) $\frac{a}{x} - 10x + C$
 (d) $y = 3x - 5$

12. (a) 155 (correct to 3 S.F.)
 (b) $e^3 - 1$
 (c) $11\frac{1}{4}$

Review Exercise #9

3. (b) $3.8 \times 10^2 \text{ J}$

4. (b) $GM_e M_s / R_e$

5. (a) $T = I \frac{d\omega}{dt}$ (b) $V = L \frac{di}{dt}$

6. (a) 9.2 h (b) $-7.0 e^{-0.35t}$
 (c) 19 mgGd/kgD₂O (d) 1.9 mgGd/kgD₂O per hour
 (e) 1.2 mgGd/kgD₂O, different from (d) since C'(t) is exponential, not linear in time.
 (f) 1.7 mgGd/kgD₂O.
7. (a) 5.3×10^8 N
 (b) 0.13 MPa
 (c) 39 m vertically above "V" bottom

221.40-3 Assignment

1. ± 1.73 2. -2, 4
 3. $\frac{3}{4}$, 3 4. $\frac{4}{3}$
 5. -0.37, 5.37 6. $\frac{-1 \pm \sqrt{-23}}{4}$ (no real roots)

L.C. Haacke