Flux and Power Mapping in RFSP

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Flux and Power Mapping in RFSP

- Fuel Management Program RFSP has FLUX MAPPING and POWER MAPPING capability
- Alternative to unusual RFSP method of solving the finite-difference diffusion equation in 3 dimensions
- Mapping used to monitor core power distribution under nominal equilibrium core conditions at CANDU 6 sites (Gentilly 2 and Point Lepreau)
- Advantage over diffusion-type calculation is direct inclusion of in-core data in the power calculation

*Calculations of the Harmonics (*MONIC)*

Steady State Diffusion Equation: $(R-P)\phi=0$

where R is the Removal matrix $\begin{bmatrix} \nabla.D_1\nabla-(\Sigma_{a,1}+\Sigma m) & \nabla.D_2\nabla-\Sigma_{a,2} \\ \Sigma_m & 0 \end{bmatrix}$

P is the Production matrix $\begin{bmatrix} 0 & \nu\Sigma_{f,2} \\ 0 & 0 \end{bmatrix}$

is the Flux vector $\begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}$

Objective: To find eigenfunctions of $R=\frac{1}{\lambda_i}P\phi_i$

At n[th] iteration $\phi^n=R^{-1}P\phi^{n-1}=R^{-1}P\Sigma_i A_i \phi_i=\Sigma_i \lambda_i A_i \phi_i$

The eigenfunction with the largest λ_i will emerge and dominates.
Solution converges to this predominate mode.

Calculations of the Harmonics (con't)

The Adjoint Flux Vector $\phi^* = (\phi_1^* \ \phi_2^*)$ **satisfy the** $\phi^*(R-P)=0$

Bi-Orthogonality of the natural modes:

$$\int (\phi_{1M}^*(\vec{r})\,\phi_{2M}^*(\vec{r}))\,P \begin{bmatrix} \phi_{1N}(\vec{r}) \\ \phi_{2N}(\vec{r}) \end{bmatrix} d\vec{r} = 0$$

for any two different harmonics M .NE. N

For a pseudo-one-group flux $\phi_T = \phi_1 + \phi_2$ **, it is self-adjoint:**

$$\int \phi_{TM}(r)\,\nu\Sigma_f(r)\,\phi_{TN}(r)\,dr = 0 \qquad M \neq N$$

Calculations of the Harmonics (con't)

Calculation of the Nth harmonic mode involves subtracting off from the unconverged flux the components of the previous harmonics:

$$\phi'_{uc} = \phi_{uc} - \sum_{I=1}^{N-1} A_I \phi_I$$

The Amplitude A_I of the component of the Ith harmonic determined using the approximate orthogonal property of the total flux:

$$A_I = \frac{\int \phi_{TI}(\vec{r})\nu\Sigma_f(\vec{r})\phi_{Tuc}(d\vec{r})}{\int \phi_{TI}(\vec{r})\nu\Sigma_f(\vec{r})\phi_{TI}(\vec{r})d\vec{r}} \qquad I = 1,\ldots,N-1$$

Calculations of the Harmonics (con't)

- **A repetitive "Iterate - Subtraction" procedure forces convergence to the next higher harmonic. Harmonics generated are orthogonal (Gram-Scmidt Orthogonalization Procedure).**

Selection of Mode Set

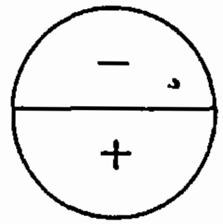
During normal full-power operation the set of modes consists of a fundamental mode (based on a recent core diffusion calculation) and the first 10-14 harmonic modes.

For the normal simulation e.g., a derating during which adjuster banks are withdrawn, a set of 22 flux modes is used:

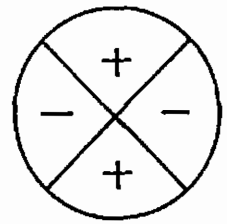
a. fundamental based on diffusion calculation of core state before derating

b. 14 harmonic modes

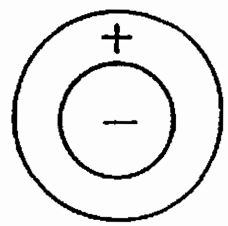
c. 7 power recovery modes with 1 through 7 adjuster banks removed from core



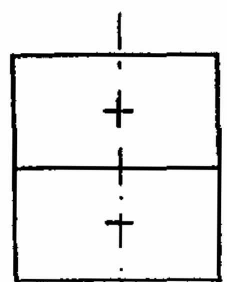
FIRST AZIMUTHAL



SECOND AZIMUTHAL



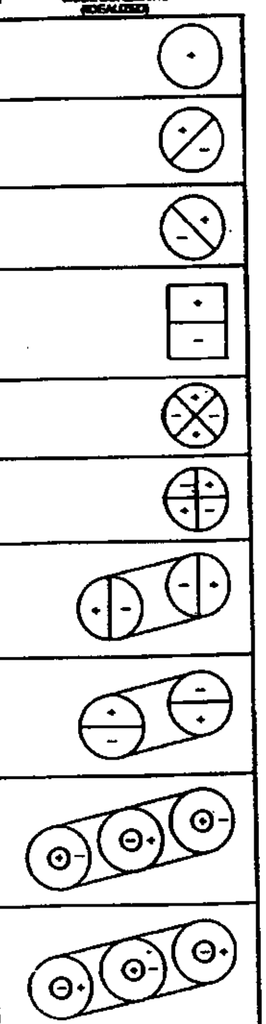
FIRST RADIAL



FIRST AXIAL

FLUX HARMONICS

MODE NUMBER	DESIGNATION	SUBCRITICALITY	MODE SCHEMATIC (IDEALIZED)
0	FUNDAMENTAL	0	
1	FIRST AZIMUTHAL-A	[illegible]	
2	FIRST AZIMUTHAL-B	[illegible]	
3	FIRST AXIAL	27.1	
4	SECOND AZIMUTHAL-A	[illegible]	
5	SECOND AZIMUTHAL-B	[illegible]	
6	FIRST AZIMUTHAL-A X FIRST AXIAL	[illegible]	
7	FIRST AZIMUTHAL-B X FIRST AXIAL	47.7	
8	FIRST RADIAL X SECOND AXIAL-A	[illegible]	
9	FIRST RADIAL X SECOND AXIAL-B	[illegible]	



Harmonics for CANDU (Cylindrical Reactor)

- **For homogeneous bare cylindrical reactor, flux shape given by (in r, θ , z co-ordinate):**

$\phi(r,\theta,z=J_M(\alpha_{ML}.r/R_0)*\cos(M*\theta)*\sin(N*\pi*z/H)$ M even

$\phi(r,\theta,z=J_M(\alpha_{ML}.r/R_0)*\sin(M*\theta)*\sin(N*\pi*z/H)$ M odd

where J_M **is the M**[th] **order Bessel function,**

α_{ML} **is the L**[th] **zero of** J_M

R_0 **is the radius of the reactor**

H is the height of the reactor

- **Various combinations of M and N give the Harmonics. Flux shape used as initial guess in *MONIC**

Harmonics - Natural Modes (Example)

1-D Problem Slab reactor, thickness from $x = -a/2 \text{ to} + a/2$

$$\frac{d^2\phi}{dx^2}+B^2\phi=0$$

Ignore flux extrapolation beyond slab surface,

i.e. assume $\phi = 0 \text{ at } x = a/2 \text{ and } -a/2$

Note also symmetry $\phi=(x)=\phi(-x) \text{ and } \frac{d\phi}{dx}=0 \text{ at } x=0$

Solution: $\phi(x)=A\cos Bx+C\sin Bx$

$\frac{d\phi}{dx}=0 \text{ at } x=0$ **forces** $C=0$

$\phi(\frac{a}{2})=0$ **forces** $\cos(\frac{Ba}{2})=0$

Therefore, $\phi(x) = A\cos(B_n x) = A\cos(\frac{n\pi}{a}x) \quad n = 1,3,5...$

B_n **are the eigenvalues, cos** $(B_n x)$ **are the eigenfunctions (harmonics)** $B_1 2$ **is the buckling of the fundamental mode** $(\pi/a)^2$

Examples of Higher Harmonics - Natural Modes of a Slab Reactor

Steady State One-Group Diffusion Equation

$$D\nabla^2\phi - \Sigma_a\phi + s = 0$$

Define $L^2 = \frac{D}{\Sigma_a}$ (Unit cm^2)

Since $s = \eta\Sigma_{aF}\phi$ and $f = \Sigma_{aF}/\Sigma_a$

then $s = \eta f \Sigma_a\phi = k\infty\Sigma_a\phi$

$$\nabla^2\phi + \frac{k\infty - 1}{L^2}\phi = 0$$

Define $B^2 = \frac{k\infty - 1}{L^2}$

then $\nabla^2\phi + B^2\phi = 0$

Auxiliary Calculation Modules (Con't)

- *RIPPLE
  Creates a new fundamental mode from current fluxes, and stores data in "FLUX MODES" "LATESTFUND" for subsequent use by *FLUXMAP

- *MAPMATRIX
  Creates flux-mapping matrices for a specified flux mode N, and stores data in "FLUX MODES" "MODE N" for subsequent use by *FLUXMAP; If N=1, data stored in "FLUX MODES" "LATESTFUND"

Auxiliary Calculation Modules (con't)

- ***READAMODE**
  Reads specific mode previously created, and stores data in "FLUX MODES" "MODE N" for subsequent use by *FLUXMAP; If N=1, data stored in "FLUX MODES" "LASTESTFUND"

- ***ONLINEMAT**
  Calculates and copies to a specified file all mapping matrices for subsequent use by the on-line flux-mapping program at Point Lepreau

Sample *MONIC Input

```
*START    ALAN GRAY
480    SEU    0.9    %    U235    FUEL       MONIC    THIRD    HARMONIC    -    1ST    AZIMUTHAL    B

*MODEL          480   SEU   0.9   %   U235   FUEL     MONIC   THIRD   HARMONIC   -   1ST   AZIMUTHAL   B

*READ TAPESEUMONIC02
*MONIC                         3
A                  10    1      0.05        20  100   30
E    2  15 3243600.0 0.95470    0.00001   1.5       0.99    0.05       1  10 600
GUESS        1    1                  1.0    422.9    3.832  270.0
LABEL      MONIC THIRD HARMONIC - 1ST AZIMUTHAL B
*RITE TAPESEUMONIC03*
TITLE                480   SEU   0.9   %   U235   FUEL   MONIC   THIRD   HARMONIC   -   1ST   AZIMUTHAL   B

*STORE
FROM       HARMONICS HARMONIC 3SLOW FLUX CELL PHI
TO         FLUX/POWERSLOW FLUX CELL PHI
*PRINT     CELL PHI
*RITE CARD
BLOCK      FLUX/POWERSLOW FLUX CELL PHI
FORMAT     (12E12.5)
WRITE      CELLPHI03          1      5760
*PRNT MASS
*CLOSE     NORMAL TERMINATION
```

Flux Mapping

The aim of flux mapping is the determine the amplitudes A_n to obtain the best fit of the mapped fluxes to the measured fluxes $F_{d.}$

There are many more detectors than modes, i.e., D>N. In the CANDU 6 there are 102 in-core detectors, i.e., D=102, and the number of modes used in the flux-mapping expansion, N, ranges between 15 and 28.

Since it is impossible to obtain a perfect fit to D detector fluxes using a smaller number N of unknowns A_n , the flux-mapping method obtains a least-squares fit of the mapped fluxes to the measured fluxes $F_{d.}$

Solving for the Mode Amplitudes

Working through the algebra in Matrix Notation:

$$A = \begin{pmatrix} A_1 \\ \vdots \\ A_N \end{pmatrix} \quad (7)$$

$$F = \begin{pmatrix} F_1 \\ F_2 \\ \vdots \\ F_D \end{pmatrix} \quad (8)$$

$$M = \begin{pmatrix} M_{11} & M_{12} & \cdots & M_{1N} \\ M_{21} & & & \\ \vdots & & & \\ M_{D1} & M_{D2} & \cdots & M_{DN} \end{pmatrix} \text{ where } M_{dn} = \psi_n(\vec{r}_d) \quad (9)$$

$$W = \begin{pmatrix} W_1 \\ \vdots \\ W_D \end{pmatrix} \quad (10)$$

Solving for the Mode Amplitudes (con't)

The detector measurements (readings) are electric currents generated by the in-core detectors and the lead cables. The readings converted to effective fluxes by dividing by sensitivity factors:

$$Fd = \frac{E_d}{S_d} \equiv K_d E_d \qquad \textbf{d = 1, ..., D} \qquad \textbf{(3)}$$

where E_d is the reading detector d.

$K_d \equiv \frac{1}{S_d}$ **is the inverse sensitivity of detector d and F_d is the derived "measured flux" (also sometimes called the "calibrated flux") for detector d.**

Solving for the Mode Amplitudes (con't)

Define a sum of squares of differences between the mapped and measured fluxes:

$$\in = \sum_{d=1}^{D} W_d^2 \left\{ \phi_d - F_d \right\}^2 \qquad \textbf{(4)}$$

where W_d is the weighting assigned to the Detector d.

Using Equation (1) for the mapped fluxes,

$$\in = \sum_{d=1}^{D} W_d^2 \left\{ \sum_{n=1}^{N} \psi_n(\vec{r}_d)\, A_n - F_d \right\}^2 \qquad \textbf{(5)}$$

$\in$ is minimized by imposing the condition for an extremum:

$$\frac{\partial \in}{\partial A_n} = 0 \qquad \textbf{n = 1,..., N} \qquad \textbf{(6)}$$

Solving for the Mode Amplitudes (con't)

Condition imposed by Eq. 6 leads to:

$$M^T \cdot (W \cdot W^T) \cdot M \cdot A = M^T \cdot (W \cdot W^T) \cdot F \quad \textbf{(11)}$$

Inverting this equation, the amplitude vector is obtained as

$$A = H \cdot F \quad \textbf{(12)}$$

where the NxD "pseudo-inverse" matrix H is given by:

$$H = \{(M^T \cdot (W \cdot W^T)) \cdot M\}^{-1} \cdot (M^T \cdot (W \cdot W^T)) \quad \textbf{(13)}$$

Once the modes-at-detectors matrix M has been computed and the weight vector W has been chosen, the matrix H can be calculated by inversion (Equation 13), and the amplitudes A_n can be determined by a simple matrix multiplication, Equation (12).

If w = 1, then Equation 13 reduces to : $H = (M^T \cdot M)^{-1} \cdot M^T$ **(14)**

Choice of Weighting

- Choice arbitrary in principle
- Uniform Weighting - W=I
  - Equal absolute errors
  - Matrix H can be pre-calculated
  - Computation of A by a single matrix multiplication
  - Used in on-line flux mapping
- Relative Weighting - $W_d = 1 / F_d$
  - Sum of relative errors (percentage errors) minimized
  - High flux reading carries relatively smaller percentage error
  - Matrix H re-computed every time W changes
  - Used in off-line mapping

Inclusion of Lead-Cable Effects

Detector current is the sum of the currents generated by the detector proper and its lead cable. The Ψ in Eq. (1) should then be replaced by the effective quantities.

$$\psi_n^{\text{total}}(\vec{r}_d) = \psi_n^{\text{detector}}(\vec{r}_d) + \alpha_d \cdot \psi_n^{(\text{lead-cable } d)}$$

where $\psi_n^{\text{detector}}(\vec{r}_d)$ is the average flux in mode n at detector d

$\psi_n^{(\text{lead-cable } d)}$ is the lead-cable flux, summed over lengths of cable equal to the modelled length of the detector, α_d is the sensitivity of unit length lead cable relative to the detector d sensitivity.

The remainder of the flux-mapping methodology is unchanged.

Three Dimensional Flux Distribution

Using the Modal Amplitudes A, 3-D flux distributions can be obtained:

$$\phi(\vec{r}) = \sum_{n=1}^{N} A_n \Psi_n(\vec{r})$$

With Lattice-Cell Modes (Bundle-Flux Modes Matrix B,

$$B_{kn} = \Psi n(\vec{r_k})$$

$k = 1, \ldots, N_B$

$n = 1, \ldots, N$

Mapped Lattice-Cell Flux is given by:

$$\phi_k \equiv \phi(\vec{r_k}) = \sum_{n=1}^{N} B_{kn} A_n$$

$$= \{B \cdot A\}_k$$

Fuel Flux is deduced from Mapped Cell Flux by:

$$\phi_{k,\,fuel} = \phi_k \cdot F_k(\omega_k)$$

Three Dimensional Flux Distribution (con't)

With Channel-Flux-Modes Matrix C,

$$C_{jn} = \Sigma_{\text{k in channel j}} B_{kn}$$

j = 1,..., N_c
n = 1,..., N

Mapped Channel Flux is given by:

$$\phi_j = \Sigma_{n=1}^{N} C_{jn} A_n$$
$$= \{C \cdot A\}_j$$

j = 1,..., N_c

Zone-Average Thermal Fluxes

With Zone-Average-Modes Matrix Z,

$$Z_{in} = \frac{\Sigma_{\text{k in zone i}} B_{kn}}{\Sigma_{\text{k in zone i}} 1} \qquad i = 1, \ldots, N_Z \quad n = 1, \ldots, N$$

Mapped Zone-Average Flux is given by:

$$\phi i = \frac{1}{N_{BZ_i}} \sum_{n=1}^{N} Z_{in} A_n$$

$$= \frac{1}{N_{BZ_i}} \{Z \cdot A\}_i \qquad i = 1, \ldots, N_Z$$

where N_{BZi} is a number of bundles in Zone i

Mapped Powers

- The mapped bundle power is obtained from the mapped cell flux by:

  $$P_k = \phi_k \cdot H_k(\omega_k)$$

  where $H_k(\omega_k)$ is the H-factor of bundle k at instantaneous irradiation ω_k ϕ_k is the mapped thermal flux for bundle k

  Absolute normalization of the fluxes and powers is then imposed from the assumed the total reactor thermal power P_{th}, i.e., by renormalizing all fluxes such that:

  $$\sum_{k=1}^{N_B} P_k = P_{th}$$

Mapped Powers (con't)

- Note: Only F-Factors and H-Factors are required for deducing the power distribution and for fuel irradiation and burnup increments. Device incremental F and H need to be included.

Failed Detectors

- Detector Reading deemed irrational if less than 0.05 or greater than 1.5 of average value
  - Exclude irrational detectors in the mapping calculation: smaller matrix dimension, re-calculate the pseudo-inverse H Matrix
  - Replace failed detector reading by best-estimate, given by the fundamental mode detector coupling coefficient M_{d1} properly normalized
- (Absolute) difference between mapped and measured detector fluxes larger than an acceptable range (e.g. 3-sigma)
  - Multiple Passes: 1st pass with best-estimate, subsequent passes with mapped value, check for failed detectors after each pass (up to 4 passes)

Comparison to Vanadium Fluxes (1992 Lepreau Restart Power Run-up Transient)

Simulation Method	RMS Difference
Pre-Simulation (Diffusion)	3.0%
Standard Diffusion	2.4%
History-Based Diffusion	2.2%
Mapping with Standard Diffusion Solution as Fundamental	1.7%
Mapping with History-Based Diffusion Solution as Fundamental	1.7%

Auxiliary Calculation Modules

- *DLSENSIT
  - Calculates lead cable relative sensitivity factors for a given FPD
  - Detector sensitivity is a function of accumulated irradiation, same for lead cable
  - Irradiation by time-average flux for FPD full power days assumed
- *MONIC

  Computes higher harmonics of the diffusion equation. Modal Mesh fluxes for Mode N stored in Index "HARMONICS" "FUNDAMENTAL" or "HARMONICS" "HARMONIC N"

Auxiliary Calculation Modules (con't)

- *ORTHOG
  Orthogonalizes a set of flux modes, using *MONIC orthogonalization procedure

Core-Tracking Application

- *FLUXMAP Calculation Requires
  - Updated fundamental Flux Mode (if desired)
  - Harmonic modes specification
  - Raw Vanadium detector readings
  - Detector sensitivity factors
- *SIMULATE (POWERMAP Option) Calculation Requires
  - Updated F-Factors and H-Factors at each bundle position
    - Updated fuel irradiation ( ), fuelling bundle movements, and current lattice cell conditions (*POWDERPUF calculations may be required).

Core-Tracking Application (con't)

- Incremental F-Factor and H-Factor due to devices (current Zone fills required)
- Mapped Channel Powers and Bundle Powers used in Power Limit Compliance statistics