NUCLEAR ELECTRIC G.S. TECHNICAL TRAINING COURSE

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NUCLEAR ELECTRIC G.S. TECHNICAL TRAINING COURSE

2 - Science Fundamentals - T.T.2

5 - Heat & Thermodynamics

-2 - Energy, and First Law of Thermodynamics

0.0 INTRODUCTION

This lesson will deal with energy, and the First Law of Thermodynamics.

1.0 INFORMATION

What is energy? Energy is inherent in all matter. We live with it every moment and can sense it when we move or get hot or get cold, or fall, etc.

Scientists usually define energy as the ability to do mechanical work.

In addition to this it has been proved that energy is just another form of matter. A good example of this is contained in the Radiation Protection Training Course regarding gamma ray absorption by means of pair production. That is a gamma ray (electromagnetic energy) divides into a position and an electron which have a certain mass. These two particles of mass subsequently collide again and convert to gamma rays. However, this aspect of energy will not concern us in this course.

The types of energy that we will deal with are briefly described below:

Mechanical Potential Energy

This type of energy concerns the relative position of bodies in a system. In figure I the weight "w" stood on the floor originally. By applying a force "F" on the rope we did work on the weight and raised it through "z" ft. For a frictionless pulley and weightless rope, F=w. Work done = wz

In the upper position the weight has potential energy with reference to the floor. Letting the weight fall allows it to do mechanical work equal to that done in raising it. The formula for potential energy is

$$P.E. = wz \text{ ft.lbs}$$

The units are in foot pounds. The graph of figure I shows the increase in potential energy with rise in height. Raising the weight halfway to "a" gives the weight a potential energy "a b" which equals the work done. At full height, potential energy and work done equal "d e".

Mechanical Kinetic Energy

This type of energy concerns the relative movement of bodies. A body moving relative to the earth has kinetic energy. A bullet fired from a gun acquires kinetic energy, figure 2. This energy comes from the work that the expanding gases of the exploded powder do in pushing the bullet out of the gun barrel.

Once free of the barrel, the bullet travels at constantly decreasing speed; its kinetic energy at a given velocity is:

$$K.E.\ \frac{wv_s^2}{2g} = \frac{wv_s^2}{64.4} \text{ft.lbs.}$$

where:

K.E. = kinetic energy ft.lbs.
w = weight of moving body lb.
v_s = velocity of moving body ft/sec
g = acceleration due to gravity, ft/sec/sec.

Example: At the instant a bullet leaves a rifle it has a speed of 1200 f.p.s. If the bullet has a weight of 2 Oz. what is its kinetic energy?

Solution:

$$w = \frac{2}{16} = 0.125 \text{ lb.}$$

$$K.E. = 0.125 \times \frac{(1200)^2}{64.4} = 2{,}797 \text{ ft. lb.}$$

By the time the bullet comes to a standstill, it will do work equal to this amount.

Internal Energy

Matter is composed of an aggregation of molecules which are moving continuously but haphazardly. There is a theory regarding gases called the kinetic theory of gases which states that it is the impact of these moving molecules on a surface which accounts for the pressure exerted by a gas on the surface. Now, since the molecules have weight or mass, we think of them as having kinetic energy similar to that of any more tangible body or mass of substance in motion. Air or steam in a closed tank which to all

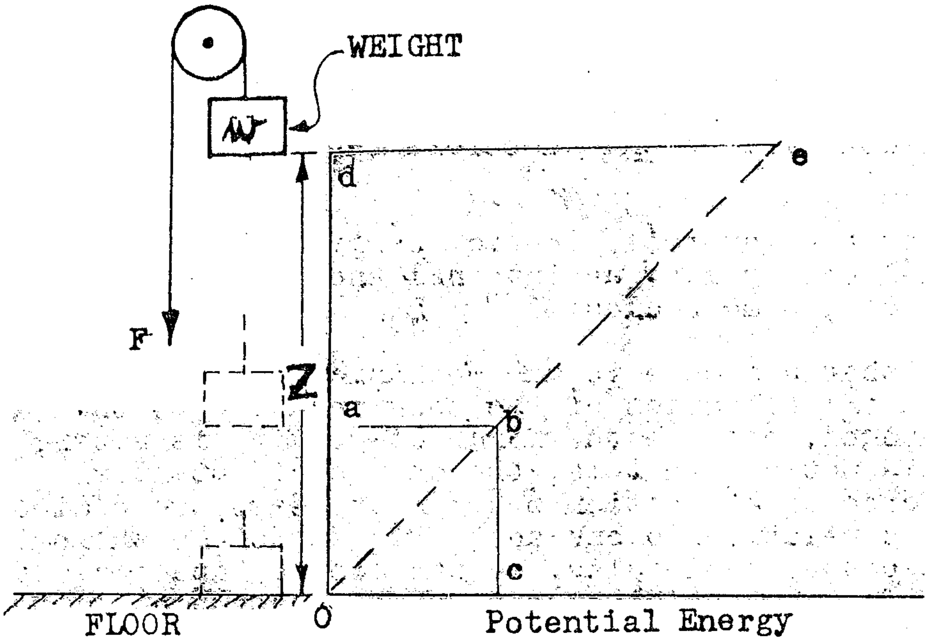
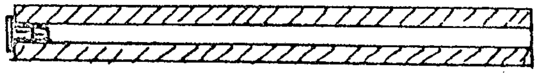
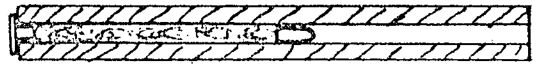


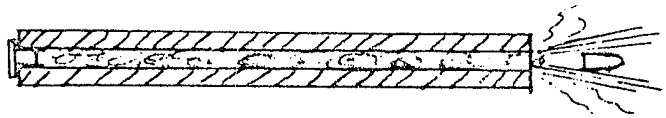
Figure 1: Suspended weight gains potential energy as it rises above floor.



a) Bullet at rest, kinetic energy = 0



b) Expanding gases from exploding powder do work on bullet giving it kinetic energy.



c) Immediately after leaving barrel, bullet has gained **maximum** kinetic energy.

Figure 2: Exploding powder in cartridge produces gas that does mechanical work on bullet in gun, giving it kinetic energy.

appearances is stagnant, actually possesses a considerable amount of this internal kinetic energy due to a motion of the molecules. Of importance is the fact that the higher the temperature, the faster the molecular motion will be and therefore the higher the internal kinetic energy.

In addition to the internal kinetic energy, substances have an internal potential energy i.e. an internal energy due to the relative positions of its molecules.

We have said that a change in the mechanicalpotential energy of a body occurs when the elevation of the body relative to the earth as a datum is changed, because the force of gravity is acting to aid or hinder this motion; that is, the change of potential energy results from a force of attraction between at least two bodies which change position in relation to one another. Now since there is an attractive force between molecules, we can see, by analogy with the concept of mechanical potential energy, that if anything happens to increase or decrease the average distance between molecules there will be a change in the internal potential energy, that portion of internal energy due to the relative position of molecules.

As an illustration let us consider again, as in the lesson on changes of State of Water T.T.3, one pound of water being boiled. As water it occupies about 0.016 cu. ft. If the pound of water is entirely evaporated at atmosperic pressure and the steam collected, it will be found that the one pound of steam occupies about 27 cu. ft., a volume about 1700 times that occupied by the water. Since the number of molecules in the pound of water is virtually the same as the number in the pound of steam, it follows that the molecules must be much more widely separated in the steam than in the water. To move these molecules apart against their attractive forces requires a large amount of energy (latent heat of vaporization) which is stored in the steam as part of the steam's internal energy. When and if the steam is condensed, this same amount of internal potential energy will be given up.

Summarizing then we can say that internal kinetic energy and internal potential energy added together comprise what we generally refer to as internal energy which we can now define as follows:

Internal Energy of a substance is the energy stored within the substance due to the rapid motion and relative position of its molecules.

This internal energy, if applied correctly, can be made to do work, in the same way as mechanical potential energy and mechanical kinetic energy can be made to do work. Referring back again to figure 2, the gas formed during the explosion has internal energy which is being used to impart velocity to the bullet. Similarly we apply intermal energy in steam to drive a steam turbine.

It is generally true that the higher the temperature and pressure that a substance is subjected to, the greater its stored internal energy.

Changing Energy Form

Most modern machines are able to perform the functions they were intended for because of the fact that energy can be changed from one form to another.

Take for example the illustration in figure I. The arrangement of weight and floor can be called a system. The weight lifted to height "z" has potential energy. When we cut the rope holding the weight, what happens to the potential energy? It decreases in proportion to the loss in height. But energy cannot just disappear. What happens to the lost potential energy? As soon as the weight is loose, it starts falling with increasing velocity. Increasing motion means that the weight gains kinetic energy. That, then is the answer. Disappearing potential energy reappears as kinetic energy. At any level of the weight, the sum of the potential and kinetic energies is equal to its total energy.

When the weight is at "d" and stationary, kinetic energy = 0 while potential energy = total energy. The instant the weight hits the floor, it comes to a dead stop. Potential energy is now = 0, and, at the instant of impact kinetic energy has reached its maximum value and is = total energy. But if the speed drops to zero the kinetic energy must drop to zero too, according to formula for K.E. since $v_s = 0$.

Where did the kinetic energy go? Touching the bottom of the weight and the floor at point of contact, we will find that they are both warmer than before impact. The kinetic energy has changed to intermal energy loosely called heat.

Summarizing, then, we have changed potential energy, to kinetic energy and it in turn has been changed to internal energy. Similarly we could use the example of the steam turbine. The internal energy of steam is converted to kinetic energy which exerts a force on the blades attached to the shaft. This causes the shaft to rotate. The rotation of the shaft can be used to do mechanical work, or it has potential energy to do mechanical work.

First Law of Thermodynamics

We have found that energy exists in many forms and that it can change from one form to another quite easily. Continuing our discussion on energy we can say that by definition:

The First Law of Thermodynamics states that energy cannot be created or destroyed.

This is also called the Law of Conservation of Energy. It says in effect that the quantity of energy in the universe is constant. (This law is based on physical observations and is not subject to mathematical proof.)

Since energy can be changed from one form into another it is useful to be able to compare the quantities of the various energies. To be able to do this we must have suitable units to which we can apply conversion factors and thereby determine how much work a certain quantity of energy can do.

We have said that work, mechanical potential energy and mechanical kinetic energy are measured in foot pounds, where as intermal energy is measured in Btu. A man named Joule did a lot of work on establishing a conversion factor and the following constant was established:

$$1 \text{ Btu} = 778 \text{ ft. lbs.}$$

This is known as Joule's constant, and is generally represented by the symbol J.

D. Dweck

NUCLEAR ELECTRIC G.S. TECHNICAL TRAINING COURSE

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5 - Heat & Thermodynamics
-2 - Energy and First Law of Thermodynamics
A - Assignment

1. What is energy?

2. A weight of 1 ton is held 50 ft. above the ground. Calculate the potential energy of the weight in ft. lbs.

3. A car weighing 3,220 lbs, is travelling at a speed of 60 mph. a) What is the kinetic energy of the car in ft. lbs. b) If the car travelling at this speed slammed into a solid brick wall, and assuming all the kinetic energy is converted to internal energy how much internal energy would be developed? Give your answer in Btu's.

4. Define

   a) Internal energy
   b) First Law of Thermodynamics
   c) The symbol J as it is understood in thermodynamics

NUCLEAR ELECTRIC G.S. TECHNICAL TRAINING COURSE

2 - Science Fundamentals - T.T.2

5 - Heat & Thermodynamics

-1 - Work, Power & Efficiency.

0.0 INTRODUCTION

In this lesson we shall discuss the scientific definitions of work, power and efficiency.

1.0 INFORMATION

WORK

In everyday conversation the word "work" is usually associated with the means of earning a living. However, when we speak of work in these lessons we will be referring to it in its scientific sense, which is defined in the following paragraphs.

In scientific terms, when a force acting through a distance overcomes a resistance we say that work has been done. For example when a man exerts a force to lift a weight against the force of gravity he has done work. It is important to understand that whenever work is being done, movement has to take place. It doesn't matter how fast or how slowly this movement takes place, but without movement no work is being done. For example lets say a bulldozer is pushing against a brick wall. If the wall doesn't move, even if the bulldozer has been at it for hours, we say that no work has been done.

We can now define work as follows:

Work = the force in pounds times the distance in feet through which it is applied. It is expressed in foot-pounds.

The amount of work can be expressed and calculated by the following simple formula:

$$\begin{aligned} \text{Work done} &= \text{Force x distance} \\ W &= F \times \ell \\ &= (P \times A) \times \ell \end{aligned}$$

where:

W = Work in ft. lbs.
F = Force in lbs
ℓ = Distance in feet
P = Pressure in psi
A = Area in square inches.

Sample Problem #1.

How much work is done when an object weighing 10 pounds is raised 3 feet?

Solution: $W = F \times \ell$
$= 10 \times 3 = 30$ ft. lbs.

Sample Problem # 2

The area of a power piston is 50 sq. in. and the uniform effective pressure on the piston is 40 psig. Find the amount of work done per stroke when the piston stroke is 6 in. long.

Solution: Force on piston = area x pressure
$F = 50 \times 40 = 2000$ pounds
$\therefore$ Work done $W = F \times \ell$
$= 2000 \times \frac{6}{12} = 1000$ ft. lbs. per stroke.

POWER

You will notice that work is independent of time - i.e. the number of ft. lbs of work performed is not altered whether the movement is slow or at lightning speed.

However when we speak of power we refer to the rate at which work is being done, and therefore involves time. The rate at which we perform mechanical work is measured in horsepower. The rate at which electrical work is performed is measured in kilowatts. Similarly the rate at which heat performs work is measured in Btu's per hour.

Some of the units of power and how they are inter-related are listed below:

1 horsepower (H.P.) = 33,000 ft.lbs. per minute
= 746 watts
1 kilowatt = 1000 watts
1 kilowatt = = 3412 Btu/hr
1 horsepower = 2545 Btu/hr. (1 Btu = 778 ft. lbs,)

Sample Problem #3

A locomotive travelling at 33 mph. is pulling a flatcar. The tension on the drawbar of the locomotive is 30,000 lbs. Calculate the horsepower required to do the job. Assume efficiency of equipment = 100%.

Solution: $33 \text{ mph} = \frac{33 \times 5280}{60}$ ft/min.

$$\text{Horsepower} = \frac{\text{force in lbs} \times \text{distance in ft/min.}}{33{,}000 \text{ ft. lbs/min.}}$$

$$= \frac{30{,}000 \times 33 \times 5280}{33{,}000 \times 60} = \underline{\underline{2{,}640}} \text{ H.P.}$$

EFFICIENCY

We compare the performance of different devices by comparing their efficiencies. The efficiency of a device is usually expressed in percentage and it is the ratio of output to input times 100.

Thus: $\text{Efficiency} = \frac{\text{output}}{\text{input}} \times 100\%$

In this lesson we shall discuss two different types of efficiencies:

a) mechanical efficiency
b) thermal efficiency

Mechanical efficiency of an engine is expressed by the formula:

$$E_M = \frac{BHP}{IHP} \times 100\%$$

where: BHP = shaft output of engine in brake horsepower
IHP = engine's indicated horse power.
E_M = mechanical efficiency of engine in percent

The thermal efficiency of an engine is expressed by the formula:

$$E_{th} = \frac{\text{brake horsepower output from engine} \times 100\%}{\text{heat supplied to the engine}}$$

$$= \frac{BHP}{W_f \times HHV} \times 100\%$$

where: E_{th} = engine thermal efficiency in percent
BHP = engine's brake horsepower in Btu/hr (1 HP=2545 Btu/hr)
W_f = fuel consumption in lbs/hr.
HHV = higher heating valu of fuel in Btu's/lb.

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5 - Heat & Thermodynamics

-1 - Work, Power & Efficiency

A - Assignment.

1. Define:

   (a) Work
   (b) Power
   (c) Efficiency

2. Calculate the height in feet to which an object weighing 50 lbs has to be raised in order to do 125 foot-pounds of work on it.

3. Calculate the thermal efficiency of an engine which has a fuel consumption of 50 pounds/hr. and an output of 95 BHP. The HHV of the fuel is 18,500 Btu's/lb.

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2 - Science Fundamentals - T.T.2

5 - Heat and Thermodynamics

-3 - General Energy Equation

0.0 INTRODUCTION

This lesson will describe how the various forms of energy can be written as an equation, (referred to as the general energy equation) representing the energy balance for a particular machine.

1.0 INFORMATION

In the previous lesson we have discussed the various forms of energy that occur. We have also established the relation between heat and work units. We are now in a position to write an equation representing the energy balance for a "steady flow" device.

The expression "steady flow" suggests that the fluid going to and from the device is moving across any section, such as 1-1 or 2-2, figure 1, continuously and at a constant rate. Furthermore, we assume in this analysis that thermal equilibrium exists; that is, that there is no accumulation or diminution of energy within the device G of figure 1. This means that all parts of the machine are at constant operating temperatures, so that heat transferred to or from these parts is at a steady unvarying rate. Finally, the quantity of matter entering the device is equal to the quantity leaving.

The general energy equation is set up in accordance with the law of conservation of energy; if no energy is stored in the device, the energy entering it is equal to the energy leaving it. All forms of energy mentioned in the previous lesson are in general pertinent to engines which are manufacturing or consuming power, and hence all these forms will appear in our equation. The following discussion then develops the general energy equation step by step.

Potential Energy

In. The substance entering the device G, figure 1, is at some elevation Z_1 above a datum line, and in reference to this line possesses mechanical potential energy of

P.E. = wz_1 ft. lb. for w lb. of substance, or z_1 ft. lb, per pound of substance; or if we divide by 778 $P.E._1 = z_1/J$ Btu/lb.

Out. At exit point 2-2, figure 1 the potential energy above the assumed datum line is $P.E._2 = z_2/J$ Btu/lb. and wz_2/J for w lbs.

Kinetic Energy

In Approaching the machine, the substance has a velocity v_{s1} and therefore the kinetic energy of $K.E._1 = w\, v_{s1}^2/2g$ ft. lbs for a weight of w lbs. or $v_{s1}^2/2g$ ft. lbs. per lb. of substance which converted to heat units is $K.E._1 = v_{s1}^2/2gJ$ Btu/lb.

Out The substance must leave with some residual velocity v_{s2} in order that it move away from the machine. The corresponding kinetic energy is $K.E. = v_{s2}^2/2gJ$ Btu/lb. and $w\, v_{s2}^2/2gJ$ Btu for w lbs.

Internal Energy

In Since molecular activity is indicative of internal energy and since the molecules of any substance at a finite temperature are active, the internal energy at section 1 will be assumed to a finite value. The symbol we use for internal energy is u Btu/lb. and at section 1-1, it will be u_1 Btu/lb. Further let $wu_1 = U_1$ Btu for w lbs. as measured above a convenient datum state. Since the units of the general energy equation are in Btu, we need not use a conversion factor here because internal energy is measured in Btu.

Out Similarly, the outgoing internal energy is u_2 Btu/lb. or U_2 for w lbs.

Flow Work

Flow work is the type of work involved in moving a mass of fluid, say along a pipe from point 'A' to point 'B'. As we have said previously, work = force x distance through which it acts. In the case involving flow:

force (F) = pressure (p) x area (A) over which pressure is exerted.

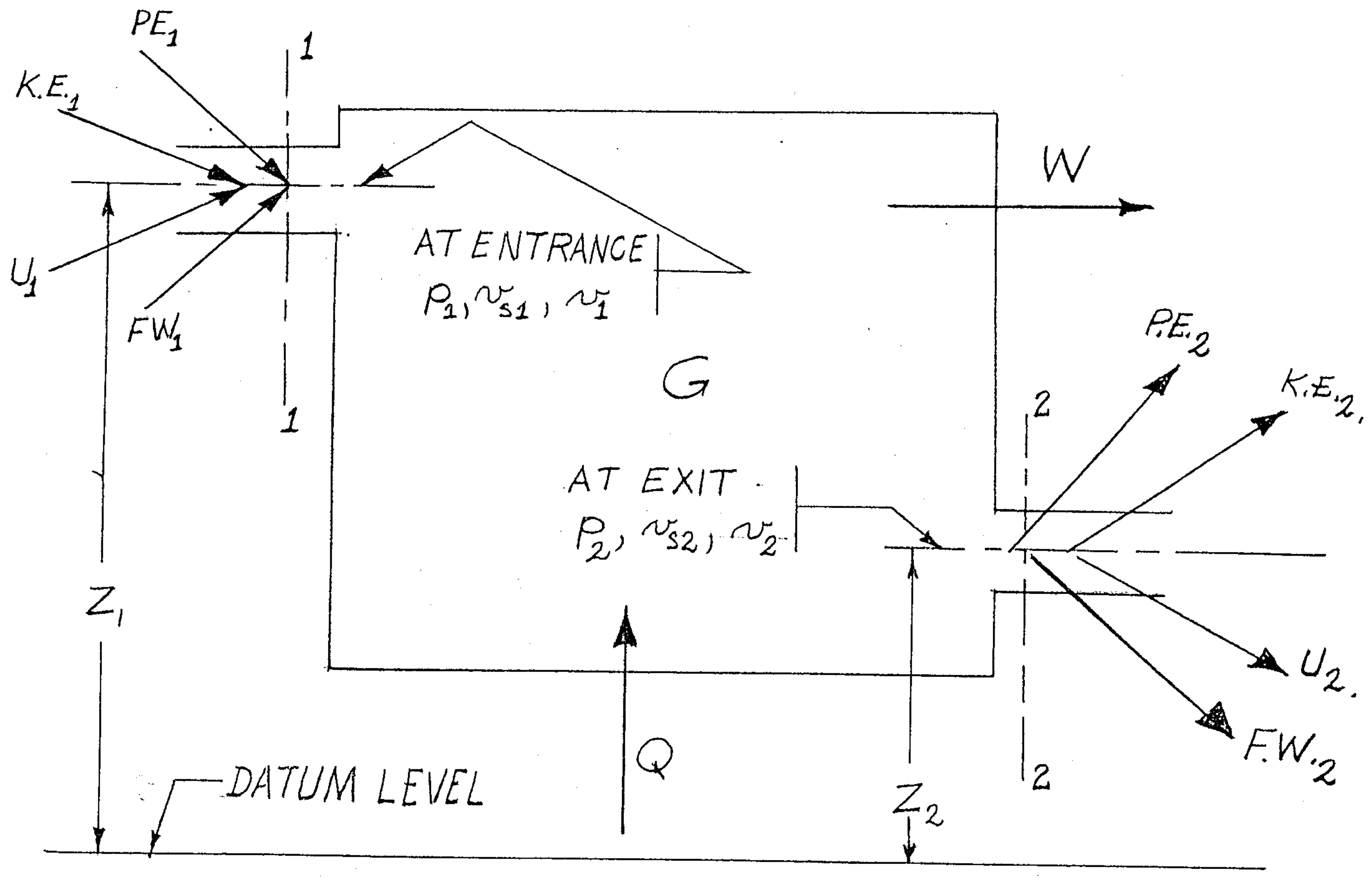

Figure 1

ENERGY DIAGRAM FOR A STEADY-FLOW MACHINE

An energy diagram is a representation of a device with an indication thereon, of all energy quantities passing in or out. This diagram represents the general case and includes all energy quantities, except chemical energy, with which we shall be concerned. In a particular application, one or several of the energy terms may be zero or negligible. Observe that W = the net work and Q = the net transferred heat.

The work done in pushing a certain weight w across a point say section 1-1 is equal to the force exerted times some distance 'ℓ' through which it acts. Thus the flow work done is:

$$F.W. = p \frac{lb}{sq.ft.} \times A \text{ sq.ft.} \times \ell \text{ ft.}$$

But Area x Length = Volume and in this case it would represent the volume of the weight of fluid moved:

$$F.W. = p \times V \text{ ft. lbs.}$$

where F.W. = flow work, ft. lbs.
p = pressure, lbs/sq.ft.
V = volume of fluid, cu.ft. for w lbs.

Note that flow work only occurs when there is motion of fluid along the pipe. If there is no flow, then no flow work is being done even though the volume of fluid concerned is under pressure.

Having briefly dealt with flow work, we can now continue to develop the general energy equation by dealing with flow work going in at 1-1, and flow work coming out at 2-2, figure 1.

<u>In</u> Since the substance is in motion there will be flow energy at section 1 equal to $F.W._1 = p_1v_1$ ft. lb. per lb. of substance if v_1 represents the specific volume in cu.ft. per lb. In heat units we have p_1v_1/J Btu per lb., or $F.W._1 = wp_1v_1/J = p_1V_1/J$ Btu for w lbs.

<u>Out</u> The flow work at section 2-2 is p_2v_2/J Btu per lb. or $F.W._2 = wp_2v_2/J = p_2V_2/J$ for w lbs., where V_2 is the total volume ($V_2 = wv_2$) and where in each case the pressure is in lbs/sq.ft.

The generalized symbols P.E., K.E., and F.W., may represent values for any weight of substance, but they must each be for the same weight in a particular energy equation. Likewise, their units may be any convenient energy unit but each term in an energy equation must have the same unit.

The symbols in the foregoing discussion may be somewhat confusing since a capitalized letter may stand for one thing, whereas the small case letter may stand for another. Therefore these are summarized below for clarity:

w = weight of substance, lbs.
W = mechanical work done, ft. lbs.
v_s = velocity, ft. per second.
v = volume, cu.ft./lb.
V = volume, cu.ft. for w lbs.
u = internal energy, Btu/lb.
U = internal energy, Btu for w lbs.

Mechanical Work

This is the type of work done on a shaft or by a shaft connected to the device and is to be distinguished from flow work which is done by a fluid.

In the general case, the energy corresponding to the work may be leaving the working substance in the device, as in a turbine, or may be entering the working substance in the device, as in a centrifugal air compressor. We have assumed in figure 1 that the mechanical work W represents outgoing energy and we shall place the term accordingly in our general energy equation. The resulting equation will be nevertheless perfectly general. If it happens that the energy in the form of work is entering the substance in the machine, the value of W will be negative.

For convenience, this device may be considered as ideal insofar as mechanical friction of the moving parts is concerned so that the work of the fluid is also the shaft work. However, if the energy equivalent to the mechanical frictional loss is known, this energy may be indicated on the energy diagram and accounted for in the energy equation. The frictional energy is outgoing energy, energy that may come from the store of energy in the working substance or energy that would otherwise be a part of the substance's store of energy. It is worth noting that the work term "W" represents the net work. Since there is both a work input and a work output in many cases, $W = W_{out} - W_{in}$ = net work.

Transferred Heat.

As in the case of work the transferred heat may be either incoming or outgoing. In thermodynamic machines, the working substance within may be at a temperature higher than that on the outside. In this event, heat will naturally flow outward. In certain thermodynamic processes, heat is intentionally added to the working substance as in a boiler. Thus heat may be transferred naturally and unavoidably, or on intentional heating or cooling (for example note the cooling system on an internal combustion engine) may occur. Let us use the symbol Q for transferred heat. If for figure 1 we place the term for transferred heat Q in our general energy equation as energy entering the device, then its numerical value when found for the case of outgoing heat will be negative. Should heat be transferred inward and outward, the value of Q will represent the net amount of heat transferred; $Q = Q_{in} - Q_{out}$.

Now having established a symbol or equation for each different form of energy, it is a simple matter to write the general energy equation, which is simply a mathematical expression for the First Law of Thermodynamics (law of conservation of energy). Since energy cannot be created or destroyed the following situation must exist:

Energy in = Energy out.

Therefore we can add up all the forms of energy going "in" on one side of the equation and all the forms of energy going "out" on the other side of the equation and get:

$$P.E._1 + K.E._1 + U_1 + F.W._1 + Q = P.E._2 + K.E._2 + U_2 + F.W._2 + W.$$

Compare this equation with figure 1. Using the known expressions for P.E., K.E., and F.W. as given in the previous pages, we write:

$$\frac{wZ_1}{J} + \frac{wv_{s1}^2}{2gJ} + U_1 + \frac{p_1V_1}{J} + Q = \frac{wZ_2}{J} + \frac{wv_{s2}^2}{2gJ} + U_2 + \frac{p_2V_2}{J} + W$$

for w pounds, and for one pound we would write:

$$\frac{Z_1}{J} + \frac{v_{s1}^2}{2gJ} + u_1 + \frac{p_1v_1}{J} + Q = \frac{z_2}{J} + \frac{v_{s2}^2}{2gJ} + u_2 + \frac{p_2v_2}{J} + W$$

Each term must of course have the same unit of energy; the unit as given is the Btu.

In most heat engines, the change in potential energy from z_1 to z_2 is negligible; that is there is little difference in the elevation of inlet and exit openings. However, in hydraulic turbines, for example, the change of potential energy from reservoir to tail race represents a most significant change of energy; so the potential energy must be retained in such cases. In all applications of this equation to heat engines, the change in potential energy will be negligible, therefore, the equation for this use may be written for w pounds, in the form:

$$\frac{w\ v_{s1}^2}{2gJ} + U_1 + \frac{p_2V_2}{J} + Q = \frac{w\ v_{s2}^2}{2gJ} + U_2 + \frac{p_2V_2}{J} + W$$

The general equation in the foregoing forms is explicit in in the matter of computing the kinetic energy and the flow work;

but the internal energy, the transferred heat, and the work are represented by single symbols. The difference in the two cases lies in the fact that the magnitudes of kinetic energy and flow work are independent of the nature of the substance and the process involved, whereas the internal energy change depends on the nature of the substance, and the transferred heat and work depend as well upon the kind of process which the substance is undergoing.

This discussion of the general energy equation should not be closed without a reminder of the meaning of the mathematical signs of the terms Q and W.

If W as solved for is positive, net work is done by the substance.

If W as solved for is negative, net work is done on the substance.

If Q as solved for is positive, net heat is added to the substance.

If Q as solved for is negative, net heat is abstracted from the substance.

Application of General Energy Equation

The general energy equation is so called because other energy equations applicable to particular machines and devices may be obtained from it. Although the general energy equation strictly applies to such steady-flow machines as the turbine, and the centrifugal compressor, the principle may also be applied to reciprocating engines like the steam engine, and the reciprocating compressor, even though the admission of the working substance is intermittent and not steady.

In a particular machine one or more terms may be dropped - e.g. in a steam turbine the change of K.E. from inlet steam to outlet steam can generally be neglected, and since it is so well insulated the heat transferred Q = 0.

Whenever a problem regarding a heat engine is to be solved, an energy diagram such as in figure 1 should be drawn to help clarify the problem.

Sample Problem - General Energy Equation

An air compressor takes in 1 lb.of air at 15 psia with a specific volume of 2 cu.ft./lb. The air is discharged at 100 psia with a specific volume of 0.5 cu. ft/lb. The increase of internal energy is 40 Btu and the work done is 70 Btu. The changes in potential energy and in kinetic energy are zero. How much heat is transferred?

Solution: Considering the general energy equation we note that:

$$w = 1; \quad P.E._2 - P.E._1 = 0 \quad K.E._2 - K.E._1 = 0$$

$$F.W_1 = \frac{w\,p_1v_1}{J} = \frac{(1)x(15 \times 144)x(2)}{778} = 5.55 \text{ Btu}$$

$$F.W_2 = \frac{w\,p_2v_2}{J} = \frac{(1)x(100 \times 144)x(0.5)}{778} = 9.25 \text{ Btu.}$$

Since the work is done on the air W = -70 Btu. Since the problem states that the internal energy increases, $U_2 > U_1$, and $U_2 - U_1 = +40$ Btu. ($U_2 - U_1$ would be negative for a decrease). Apply the general energy equation in the form,

$$F.W._1 + Q = U_2 - U_1 + FW_2 + W$$
$$5.55 + Q = +40 + 9.25 + (-70)$$
$$\therefore \quad Q = +40 + (9.25 - 5.55) + (-70)$$
$$Q = -26.3 \text{ Btu for one lb. of air.}$$

The negative sign for Q indicates that the transferred heat is abstracted from the substance.

The above solution may also be made directly from an energy diagram. Figure 2 shows the known flow of energy as given in the problem. At this stage it may not be known whether Q is heat added or heat rejected. Therefore, we may show this energy as flowing inward. Then if the answer is positive, the direction shown is correct; if the answer is negative this flow of energy occur opposite to the manner pictured. Setting up the energy equation to accord with figure 2 we have:

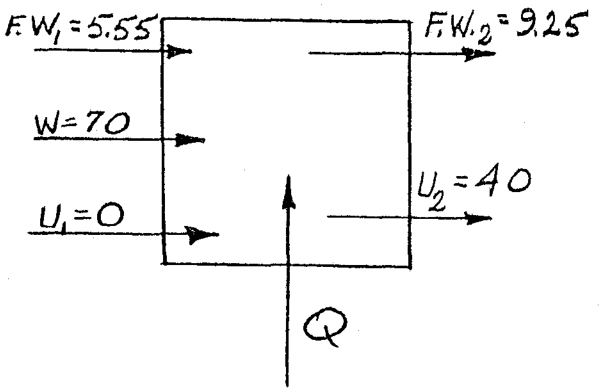

Fig. 2

$$\text{Energy In} = \text{Energy Out}$$
$$5.55 + 70 + 0 + Q = 9.25 + 40$$

from which Q = -26.3 Btu, heat abstracted as before.

We have written the general energy equation in two different forms thus far. But there is another form which will be made use of in the T.T.1 level. This form will be developed as follows:

We have defined enthalpy as heat energy added to a substance as it is changed from ground state or zero state to its present state. We can expand this definition now by saying that:

Enthalpy = internal energy + flow work
or $h = u + p\ v/J$ for one pound.
and $H = U + p\ V/J$ for w pounds.
Thus the general energy equation becomes:

$$\frac{w\ v_{s1}^2}{2gJ} + H_1 + Q = \frac{w\ v_{s2}^2}{2gJ} + H_2 + W$$

The last sheet of this lesson summarizes the various forms of energy that one may expect to come across in Heat and Thermodynamics. We have dealt with several of these in this and previous lessons. The summary sheet gives an indication of where the various forms of energy, we have dealt with, fit into the general scheme of things.

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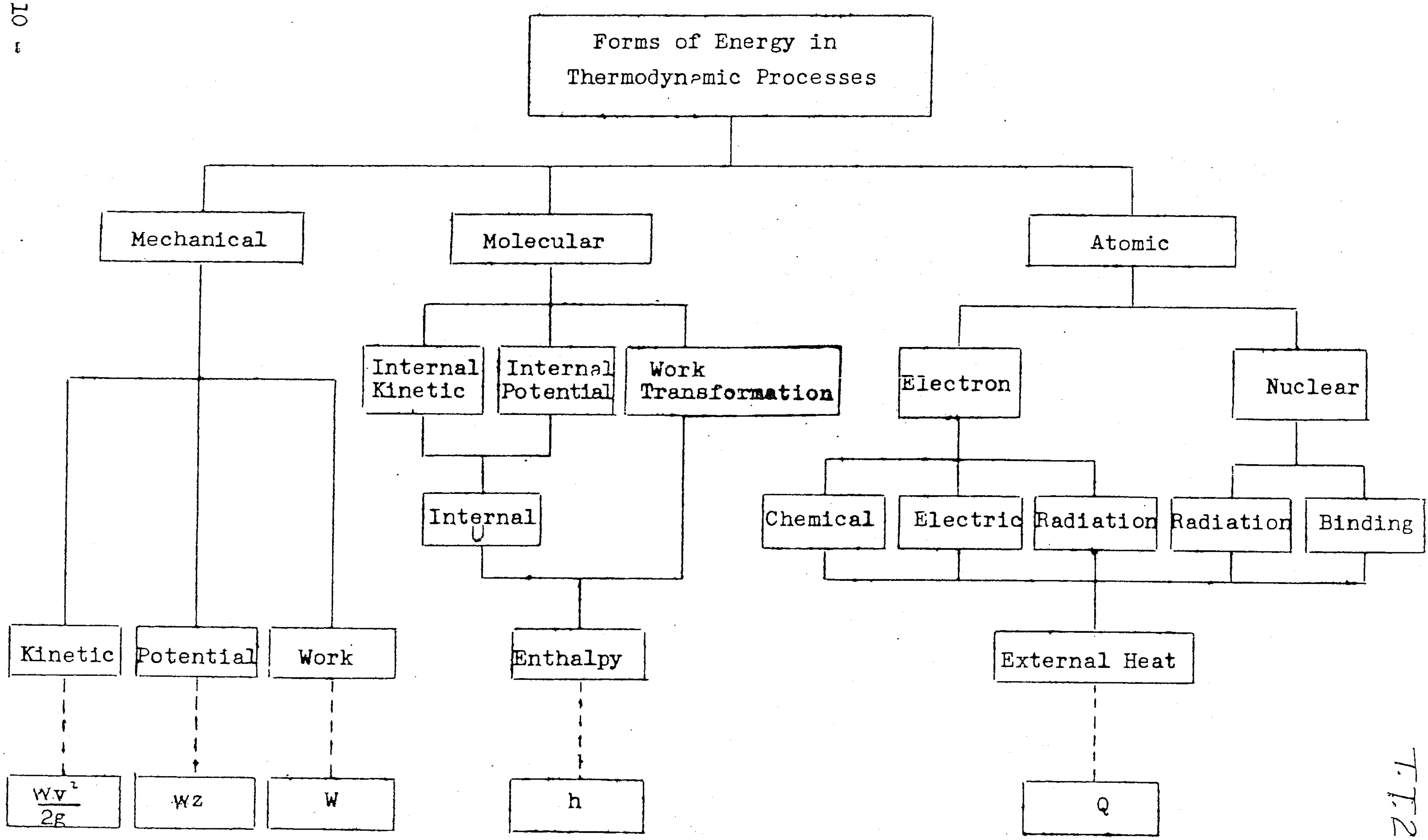
T.T.2-2.5-3



$$\frac{wz_1}{J} + \frac{wv_{s_1}^2}{2gJ} + U_1 + \frac{P_1V_1}{J} + Q = \frac{wz_2}{J} + \frac{wv_{s_2}^2}{2gJ} + U_2 + \frac{P_2V_2}{J} + W$$

NUCLEAR ELECTRIC G.S. TECHNICAL TRAINING COURSE

2 - Science Fundamentals - T.T.2

5 - Heat and Thermodynamics

-3 - General Energy Equation

A - Assignment

1. What is a "steady-flow" device?

2. In your own words describe what flow work is and give the equation for it.

3. The pressure in a certain pipe containing water is equal to 1440 psi. If 50 lbs of water are made to move past a point A in the pipe, calculate how much flow work has been done. Assume that the specific volume of water is 0.02 cu. ft./lb. Express your answer in Btu's.

4. Is the mechanical work done in rotating the following pieces of equipment negative or positive:

   (a) compressor?

   (b) internal combustion engine?

5. If the heat added to a substance during one stage of a process is 45 Btu, and the heat abstracted during another stage of the process is 15 Btu, is the net Q positive or negative?

NUCLEAR ELECTRIC G.S. TECHNICAL TRAINING COURSE

2 - Science Fundamentals - T.T.2

5 - Heat & Thermodynamics

-4 - Adiabatic & Isothermal Expansion & Compression

0.0 INTRODUCTION

This lesson will explain adiabatic expansion and compression processes and isothermal expansion and compression processes.

In the lesson on the "General Energy Equation we have dealt with a steady-flow process. All the information in this lesson deals with a non-flow process.

1.0 INFORMATION

Adiabatic Expansion or Compression

An adiabatic process is defined as process in which no heat is transferred to or removed from the working fluid.

Let us consider an example of adiabatic compression and subsequent expansion. Let's say figure 1 shows a perfectly insulated cylinder of an air compressor, with the piston in readiness to compress the air. (By perfectly insulated we mean one through which no heat whatsoever can pass, either inward or outward.)

Now compress the air and then allow it to expand again till the piston is in its original position.

Notice that there is no flow into or out of the piston and therefore is termed a non-flow process. When compressing the air volume will obviously decrease and the pressure increase. If we plot the pressure against cylinder volume at any time during the process we would plot the curve A to B.

During compression the temperature of the air will rise. The work done to move the piston is absorbed by the air and stored as internal energy, resulting in a higher temperature. (You will recall that since this is an adiabatic process no heat is added to, or removed from the working fluid through the cylinder walls.)

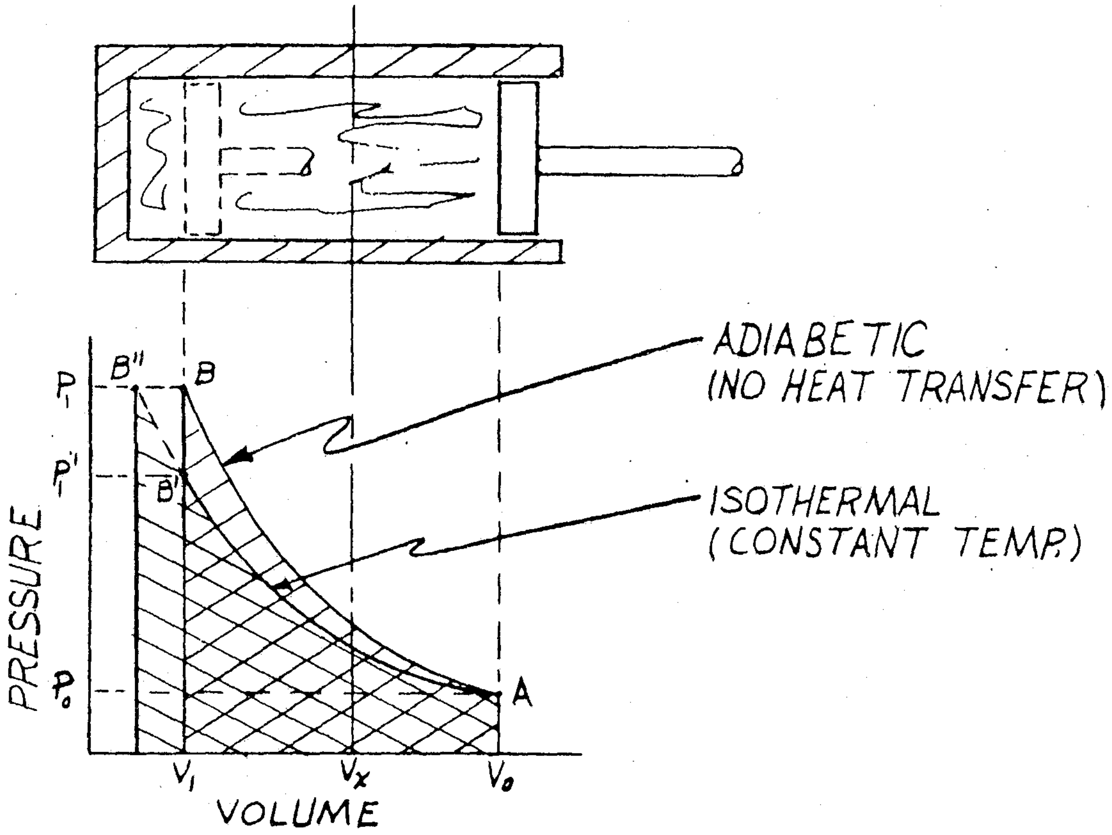


Fig. 1

During expansion, the air will push the cylinder back to its original position changing all the internal energy, formed during compression to mechanical work done on the piston. The result is that on the pressure-volume graph we are following curve AB exactly again only in the reverse direction-- i.e. from B to A. (The above discussion describes an ideal situation because in practice there are losses such as friction and eddy currents that would prevent the piston from arriving exactly at its original position.)

Reversible & Irreversible Processes

In thermodynamic usage, a process is designated as reversible if it can be performed in both directions, the path taken during the initial change of condition being exactly retraced during the return to the original state of the system. True reversibility will involve no change in the availability of energy at any point during either the direct or the reversed process.

The frictionless adiabetic example given on the preceding pages is one of a reversible process. A more familiar example of a process that approaches reversibility as a limit is found in a ball thrown upward. If we remove the retarding influence of air resistance and if we say we have a ball with perfect eleasticity then we can attain a truly reversible process --i.e. the ball would keep on bouncing up and down the same distance once it had received energy for its initial upward motion.

However, the foregoing discussion is theoretical only. The presence of friction will always make a process irreversible since it will have the effect of changing work, a form in which energy is completely available, into thermal energy, of which only a part can be available.

Briefly then an irreversible process is one which cannot repeat itself, since energy must be supplied from an external system to return the thermal energy to its original location.

A familiar example of an irreversible process is fruit falling from a tree.

The concept of reversible and irreversible processes plays an important role in thermodynamics and these words will be used frequently in this course.

Equations

In engineering it is desirable to be able to express processes in the form of equations so that we can predict the performance of a particular machine.

You will recall that in the lesson of Definitions T.T.3 we had defined specific heat as the amount of heat required to be added or removed in order to change the temperature of one pound of a substance one degree fahrenheit. The symbol used was the letter "c" and the units in Btu/lb./°F. If we multiply specific heat times temperature we get the internal energy of a substance at a particular temperature. Thus:

$$u = c \frac{\text{Btu}}{\text{lb.} \times {}^\circ\text{F}} \times T\ {}^\circ\text{F} = cT \text{ Btu/lb.}$$

where: u = internal energy, Btu/lb.
T = temperature °F

In the example in figure I the work done to compress air from A to B or to expand from B to A can be written as:

$$W = c\,(T_B - T_A)\ J \text{ ft. lbs./lb.}$$

where: W = work done ft. lbs.

T_B = temperature at B, °F

T_A = " at A, °F

J = 778 ft. lbs. = 1 Btu.

Experiments have shown that the specific heat for a process taking place at constant pressure is different than the specific heat for a process taking place at constant volume. We therefore let the following symbols stand for these two specific heats:

c_p = specific heat at constant pressure Btu/lb./°F.
c_p for air = 0.24 Btu/lb/°F

c_v = specific heat at constant volume Btu/lb/°F.
c_v for air = 0.17 Btu/lb/°F.

Many calculations involve c_p & c_v as a ratio for which we use the letter k thus:

$$k = c_p/c_v$$

k is known as the <u>specific heat ratio</u>
For air $k = \frac{0.24}{0.17} = 1.4$ for conditions up to 900°F

Having introduced c_v, c_p and k we can now introduce another equation for work when we're dealing with a reversible adiabatic process. In figure I the work "W" is equal to the area under the the curve "AB". This type of reversible adiabatic P-V curve for a perfect gas can be expressed by the relation:

$$PV^k = C$$

where "C" is a constant.

The relation between temperature pressure, and volume for a reversible adiabatic process can be expressed as follows:

$$\frac{T_A}{T_B} = \left(\frac{V_B}{V_A}\right)^{k-1} = \left(\frac{P_A}{P_B}\right)^{\frac{k-1}{k}} \text{ or } \frac{P_A}{P_B} = \left(\frac{V_B}{V_A}\right)^{k}$$

The letters T,V, and P stand for temperature, volume and pressure respectively whereas the subscripts A and B refer to the respective points on figure I.

Isothermal Expansion or Compression

An isothermal process is defined as a process that takes place at constant temperature. (Iso = one) therm = temperature) Therefore isothermal = one temperature. The addition of the latent heat of vaporization to water in a boiler or the rejection of this heat in a condenser are examples of isothermal processes.

Let us go back to figure I and imagine that the insulation has become a "heat sink" that will receive or give up heat so easily that it will soak up the heat of compression and return it during expansion so that compression and expansion takes place at constant temperature,--i.e. the temperature at point A. If we again plot the pressure for any cylinder volume we would plot the line A to B' and back to A. (Note that again we are talking about a non-flow process.)

Since the temperature remains constant during this process, the pressure will not rise as rapidly. Pressure is therefore inversely proportional to volume thus (Refer to lesson Expansion of Gases--Gas Laws T.T.3):

$$P_o V_o = P_1 V_1$$

Therefore we can say that for any point during compression, say V_x (see figure 1) the pressure is higher during a diabatic compression than during isothermal compression. This means that the piston is always working against a higher pressure, i.e. doing more work, when compression is adiabatic than if it was isothermal.

This is one reason that cylinder cooling is used on reciprocating compressors. That is there is an attempt to approach isothermal compression and therefore, require less work to compress the air.

Referring to figure I again, when this cylinder is the cylinder of an air compressor and pressure P_1 is the discharge pressure the piston will have to travel past volume V_1 (to the left) in order to discharge the air. The isothermal compression line A to B' has been extended to B" to show at what point discharge pressure will be reached when we compress isothermally.

Again as in the caseof adiabatic processes, we can speak in terms of an isothermal process as being reversible (or approaching reversibility) or irreversible.

Equations

In the lesson on Expansion of Gases in the T.T.3 level we derived the equation:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

This relationship also applies to an isothermal process, except that $T_1 = T_2$ i.e. temperature remains constant. Applying this equation to the illustration in figure I and using the proper subscripts, we get:

$$P_A V_A = P_{B''} V_{B''}$$

The work done is again equal to the area underneath the curve A to B". Without going into the proof of the equation, the quantity of work done can be expressed as follows:

$$W = (P_{B''} V_{B''})\ 2.3\ \log_{10}\left(\frac{V_A}{V_{B''}}\right) \text{ ft. lbs.}$$

where: W = work done ft. lbs.
$P_{B''}$ = pressure at point B", lbs./sq.ft.
V_A, V_B = volume at points A, & B respectively ft.^{3}

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NUCLEAR ELECTRIC G.S. TECHNICAL TRAINING COURSE

2 - Science Fundamentals - T.T.2
5 - Heat & Thermodynamics
- 4 - Adiabatic & Isothermal Expansion & Compression
A - Assignment

1. Define an adiabatic process.

2. Explain what is meant by:

   Reversible process
   Irreversible "
   Give an example of each.

3. A compressor takes atmospheric air at a temperature of 80°F and after compression passes it into an air receiver. During this process the temperature rises to 100°F. If the total quantity of air involved is 10 lbs., how much work has to be done to complete the process. Assume a reversible, frictionless adiabatic process. Use c = 0.17.
   Express your answer in ft. lbs.

4. Define an isothermal process.

5. A compressor takes 2 lbs. of air at atmospheric pressure (14.7 psi.) and a volume of 14 cu. ft/lb and compresses it isothermally to a pressure of 84 psi. What is the final volume of air?

NUCLEAR ELECTRIC G.S. TECHNICAL TRAINING COURSE

2 - Science Fundamentals - T.T.2

5 - Heat and Thermodynamics

-5 - Second Law of Thermodynamics

0.0 INTRODUCTION

In the previous two lessons we have dealt with the "general energy equation" and adiabetic compression and expansion as well as isothermal compression and expansion. The next step in developing the subject of Heat and Thermodynamics is to deal with how this knowledge can be applied to a device called a heat engine which will be able to convert heat into work. All heat engines operate by passing the thermodynamic system or working substances through a number of thermodynamic processes, returning it periodically to its original condition. These processes, which are necessary to return the system to its original condition, taken as a whole and in order, constitute the heat engine cycle.

However, in discussing heat engine cycles we will need to be familiar with the term "entropy". Before we discuss the term "entropy" it will be to our advantage to be familiar with what is known as the "Second Law of Thermodynamics".

Therefore, this lesson will discuss the "Second Law of Thermodynamics". The next lesson will deal with "entropy" after which we can discuss heat engine cycles.

1.0 INFORMATION

The second law of thermodynamics may be called the law of degradation of energy. We have learned in the lesson on "Energy and the First Law of Thermodynamics", that energy cannot be created or destroyed, but that all forms of energy are mutually convertible. Some changes of energy take place more readily than others. For example, all of a given quantity of work may be easily converted into heat, but, if the attempt is made to reconvert it to the form of work, the process is much more difficult and only a part of the original energy can be reconverted. Work or any form of energy which is theoretically capable of complete conversion into other forms is called high-grade energy; heat, not being capable of complete conversion, is classed as low-grade energy. A general tendency exists for all high-grade forms of energy to become low-grade, and it is with the principles that govern this tendency that the second law deals.

No two authors on this subject seem to have the same wording for the second law of thermodynamics. However, for our purposes the meaning of this law can be given in two statements as follows:

<u>The Second Law of Thermodynamics</u> states that: -

1. No heat engine can convert all the heat which is supplied to it into work. A portion must be rejected in the form of unused heat.

2. Heat cannot flow from a body at a lower temperature to another body at a higher temperature.

The first statement establishes definite conditions and limits for the performance of heat engines. A portion of the heat transferred to a working substance is inherently unavailable to do work. We can say then that that portion of heat which is converted into mechanical work is <u>available energy</u>; that portion of heat which is rejected by a heat engine is called <u>unavailable energy.</u>

Regarding the second statement, we can say that thermal energy can be continuously "lifted" to a body at higher temperature, only if energy is continuously supplied by some external system to bring about the change in location. This, in fact defines the requirement for the operation of a refrigerating machine, which will continuously remove thermal energy from a cold body and store it in a body at higher temperature. The external source of energy required in this case is supplied by the motor driving the compressor.

The efficiency of a heat engine may be defined by:

$$e = \frac{Q_{in} - Q_r}{Q} \text{ X } 100$$

where e = efficiency %

Q_{in} = heat in, supplied from a high temperature source, Btu.

Q_r = rejected heat which is unavailable to do work, Btu.

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NUCLEAR ELECTRIC G.S. TECHNICAL TRAINING COURSE

2 - Science Fundamentals - T.T.2
5 - Heat and Thermodynamics
-5 - Second Law of Thermodynamics
A - Assignment.

1. Define the Second Law of Thermodynamics.

2. A heat engine receives 500 Btu from a high temperature source and converts all but 150 Btu into useful mechanical work. What is the efficiency of the heat engine?

NUCLEAR ELECTRIC G.S. TECHNICAL TRAINING COURSE

2 - Science Fundamentals - T.T.2

5 - Heat and Thermodynamics

-6 - Entropy

0.0 INTRODUCTION

In the previous lesson on the Second Law of Thermodynamics we have said that in all heat engines a portion of heat energy is always rejected and is not available to do work. Further we have said that there is a tendency for all high-grade forms of energy to become low-grade energy, and eventually unavailable energy. Thus heat added to a fluid or system may be divided into two parts, namely available energy (Q_A) and unavailable energy (QR). This fact involves us with the term called entropy, for which we use the symbol 's'.

This lesson will deal with entropy and discuss its significance.

1.0 INFORMATION

Every substance has entropy in the same sense that it has pressure or volume or internal energy. It is a property of a thermodynamic substance of which great use is made. Entropy is not something that one can think of in a physical sense. It is best to consider it simply as a defined property (as is enthalpy, for example). Entropy changes always accompany actual transfers of heat. We would like to emphasize at this point that entropy does not have an absolute value, but generally when we refer to it we are interested in changes of entropy rather than absolute values.

Suppose we have a change in entropy from s_1 to s_2; the difference can be expressed as ΔS. (Δ is the Greek letter delta and the expression is therefore read as "delta S"). Thus we can write any entropy change as follows:

$$\Delta S = S_2 - S_1$$

There are two processes that cause the so-called degradation of available energy to unavailable energy:

1. The heat transfer process.
2. The friction process.

When a fluid absorbs energy as heat, its entropy increases; when it gives up energy as heat its entropy decreases. When a fluid experiences friction, its kinetic energy changes to heat energy; A good example of this is the friction losses experienced by steam as it flows through a turbine; it results in an increase in entropy.

Temperature Entropy Diagram

We have said previously that for a pressure volume diagram the work done is equal to the area underneath the curve. It would be very useful if we could use a similar method for evaluating the amount of heat transferred, Q. We have said that both temperature and entropy are properties of a substance, (or a system). If changes of pressure and volume can be expressed in equation form to calculate the amount of work done, then it follows that it should also be possible to develop a similar equation representing the absolute temperature in terms of entropy of the system during that same change of conditions.

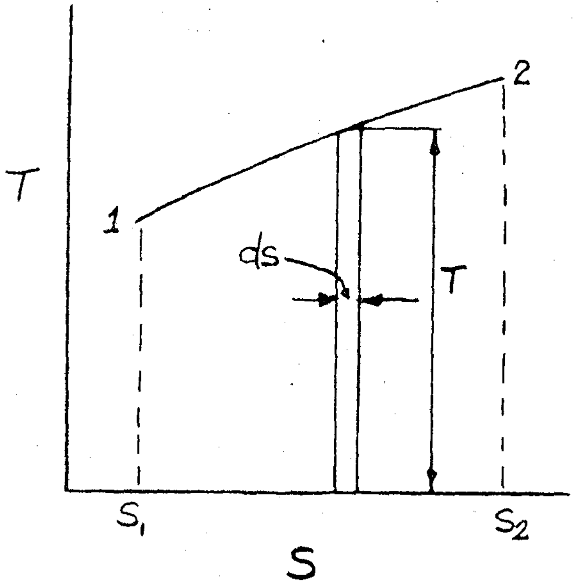


Fig. 1

Let us say we have a change of conditions in temperature and entropy from point 1 to point 2. This change of conditions can be plotted on a graph with coordinates of absolute temperature and entropy as is shown in figure 1. We know that a change in temperature means that heat transfer Q must have taken place. Similarly, we said that changes in entropy occurred as a result of heat transfer. Thus the area under the curve 1-2 represents the total amount of heat transferred Q, as the process took place from point 1 to point 2. If we cut the area s_1-1-2-s_2 into an infinite number of slices, and let each slice be ds wide then the area of each slice is T x ds. Now if we add all the small slices together this should give us the total area represented by s_1-1-2-s_2 which can be expressed as $T(s_2-s_1)$ or $T \times \Delta s$. But the total area s_1-1-2-s_2 is equal to the total heat transferred Q.

$$\therefore \quad Q = T \times \Delta s$$

$$\text{or} \qquad \Delta s = \frac{Q}{T}$$

where: Δs = change in entropy
Q = amount of heat transferred Btu/lb.
T = absolute temperature.

We said in the introduction of this lesson that entropy was somehow associated with unavailable energy. In Heat and Thermodynamics the expression $T\Delta s$ is in fact equivalent to the unavailable energy which in this case is also given by the quantity Q. Thus Δs is directly proportional to the unavailable energy and we say that entropy is a measure of the unavailability of energy.

From the above discussion we can now give a definition of entropy as follows:

Entropy - is a measure of how much of a system's thermal energy (or heat energy) is unavailable for conversion to mechanical work. In mathematical terms we can say that entropy is the amount of heat transferred to or from the substance divided by the absolute temperature at which the heat was added or abstracted. ($\Delta s = \frac{Q}{T.}$)

You will recall from the lesson on definitions, T.T.4 level that absolute temperature was defined as follows:
absolute T. = 460 + °F which is measured in °R (Rankine)
" T. = 273 + °C which is measured in °K (Kelvin)

Entropy itself cannot be measured directly but must be calculated from the above equation which means that the amount of heat transferred and the temperature (expressed as absolute temperature) at which this transfer of heat took place must be known.

Quite extensive experiments have been done by specialists, with water and steam and values of entropy at various temperatures and pressures have been compiled in steam tables, which have been given to you in tables 1 to 3 in the lesson on "Steam Tables" at the T.T.3 level. The temperature in these tables is given in °F and not in °R. In the case of water entropy is considered to be zero at 32°F.

The units of entropy are, as you can observe from the above definition, Btu/lb/°temperature; notice that the units are the same as for specific heat. Nevertheless, this unit of entropy has no important significance; so we speak of so many units of entropy, ordinarily without referring to the dimensional units.

Examples

We said in the beginning of this lesson that when a fluid absorbs energy as heat its entropy increases; when it gives up energy as heat, its entropy decreases. However, we have also said that entropy is a measure of the degradation or unavailability of energy. If entropy is a measure of the unavailability of energy how can it decrease when you would actually expect it to increase all the time? This will be explained in the following two examples.

Example No. 1

Two pounds of water at 25 psig are divided into two equal parts having temperature of 140°F and 240°F respectively. Determine the decrease in available energy caused by heat transfer if these two parts are mixed so that they arrive at a common temperature. Assume the reference or datum temperature as 40°F and a constant specific heat of C = 1 Btu/lb/°F. Figure 1 is a T-s diagram portraying this situation.

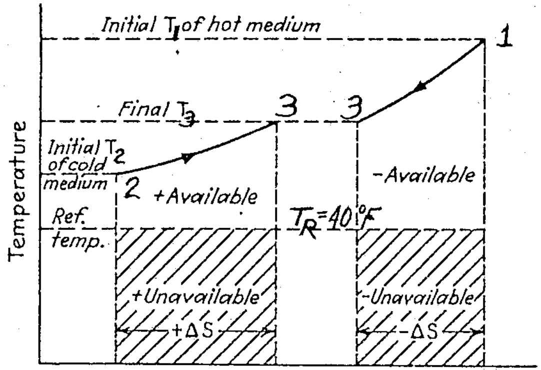


Fig.2 - T-S diagram of heat transfer process.

$w_1 = 1$ lb. water; $w_2 = 1$ lb. water
$T_1 = (240 + 460) = 700°R$;
$T_2 = 140 + 460 = 600°F$

Reference Temp. $T_R = 40+460 = 500°R$

During temperature equalization the heat rejected by the hotter part is equal to the heat received by the colder part. Thus:

$$w_1C(T_1-T_3) = w_2C(T_3-T_2)$$

Since the weights and specific heats are the same on either side of the equation:

$$T_1 - T_3 = T_3 - T_2.$$

$$\therefore \text{ Mixture temperature } T_3 = \frac{T_1 + T_2}{2} = \frac{700 + 600}{2} = 650°R$$

Or 190°F

If we let Q_{in} = the heat transferred to the cold part in Btu/lb. and Q_{out} = the heat transferred from the hot part in Btu/lb. then

$$Q_{in} = Q_{out} = Q = w_1C\ (T_1 - T_3)$$

$$= 1 \times 1\ (700 - 650) = 50.00 \text{ Btu.}$$

$\therefore$ $Q_{in} = +50.00$ Btu (positive meaning that it is added)
$Q_{out} = -50.00$ Btu (negative meaning that it is abstracted)

From steam tables for 1 lb. water at 240°F: $s_1 = 0.3531$
at 190°F: $s_3 = 0.2785$
at 140°F: $s_2 = 0.1984$

The unavailable energy change for the hotter

medium $= w_1T_R\Delta s = w_1T_R\ (s_3 - s_1)$

$= 1 \times 500\ (.2785 - .3531) = -\ 37.30$

The unavailable energy change for the colder medium
$= w_2\ T_R \Delta S = w_2\ T_R\ (S_3 - S_2)$
$= 1 \times 500\ (0.2785 - .1984) = +\ 40.05$ Btu.

We know that:
Total heat energy change Q = available energy change + unavailable energy change
or available energy change = Q - unavailable energy change

∴ Available energy given by the hot part to the cold part

$$= -\ 50.00 - (-37.30)$$

$$= -\ 12.70 \text{ Btu.}$$

and available energy absorbed by the cold part

$$= +\ 50.00 - (+\ 40.05) = +\ 9.95 \text{ Btu.}$$

∴ The decrease in available energy = - 12.70 + 9.95 = - 2.75 Btu.

The increase in unavailable energy = -37.30 + 40.05 = + 2.75 Btu.

Entropy decrease for the hot medium $= w_1(s_3 - s_1)$

$$= 1\ (.2785 - .3531) = -\ 0.0746$$

Entropy increase for the cold medium $= w_2\ (s_3 - s_2)$

$$= 1\ (.2785 - .1984) = +\ 0.0801$$

The net change of entropy for the 2 lb. is:

$$\Delta s = \frac{\text{unavailable energy change, QR}}{\text{T.}} = \frac{2.75}{500} = 0.0055 \text{ Btu/lb/°R}$$

The energy and entropy values are summarized as follows:

Medium	Total Energy Change Q	Unavailable Energy change Q_R	Available Energy change Q_A	ΔS
Hot	- 50.00	- 37.30	- 12.70	- 0.0746
cold	+ 50.00	+ 40.05	+ 9.95	+ 0.0801
mixture	0	+ 2.75	- 2.75	+ 0.0055

In this example we have shown that when a fluid absorbs energy as heat its entropy increases and when a fluid gives up energy as heat, its entropy decreases. The cold water absorbs heat energy raising its temperature and increasing its entropy by 0.0801 units. The hot water gave up energy as heat, lowering its temperature and decreasing its emtropy by 0.0746 units.

However, the net overall result is that entropy for the two parts, has increased by 0.0055 units, unavailable energy has increased by 2.75 Btu, and available energy has decreased by 2.75 Btu. So, the net result is that we have transferred heat which has meant a degradation of energy.

Example No. 2

In the previous example one of the things we have shown is that when transfer of heat takes place there is degradation of energy. In this example we will show that the greater the temperature difference between the heat source and the temperature of the working medium in a heat engine the greater the increase in entropy; hence the greater the unavailable portion of energy and the smaller the available portion of energy to do useful work.

We have said in this lesson that $Q = T \cdot \Delta S$. The energy Q can be shown graphically on T-S coordinates similarly to the temperature-enthalpy graphs we developed in T.T.3

In figure 3 we have a heat engine taking heat from a source at 3000°R. There is a solid wall between the furnace and the piston, so heat has to be transferred by conduction or radiation. The piston draws in air from the surroundings. The air absorbs energy Q = 300 Btu/lb, does work, and then is discharged into a receiver. If the energy is absorbed reversibly by the air so that $T_g = 3000°R$ then the entropy change is:

$$\Delta S = {}^{300}/_{3000} = 0.10$$

We can plot this on the T-S graph in figure 3 as a bar. The horizontal axis of the graph is not graduated in entropy units, but the width of the bars is measured in entropy units; you can see that the area of the bar is the same as the area of a rectangle, namely $\Delta S \times T = Q = 300$ Btu/lb. The heat engine discharges into a receiver and any heat contained in the discharge is not available to do any more work in this system. If the lowest natural available receiver temperature is 500°R, then energy at this temperature with a ΔS of 0.10 will equal

$$\begin{aligned} Q &= T \cdot \Delta S \\ &= 500 \times 0.10 \\ &= 50 \text{ Btu/lb.} \end{aligned}$$

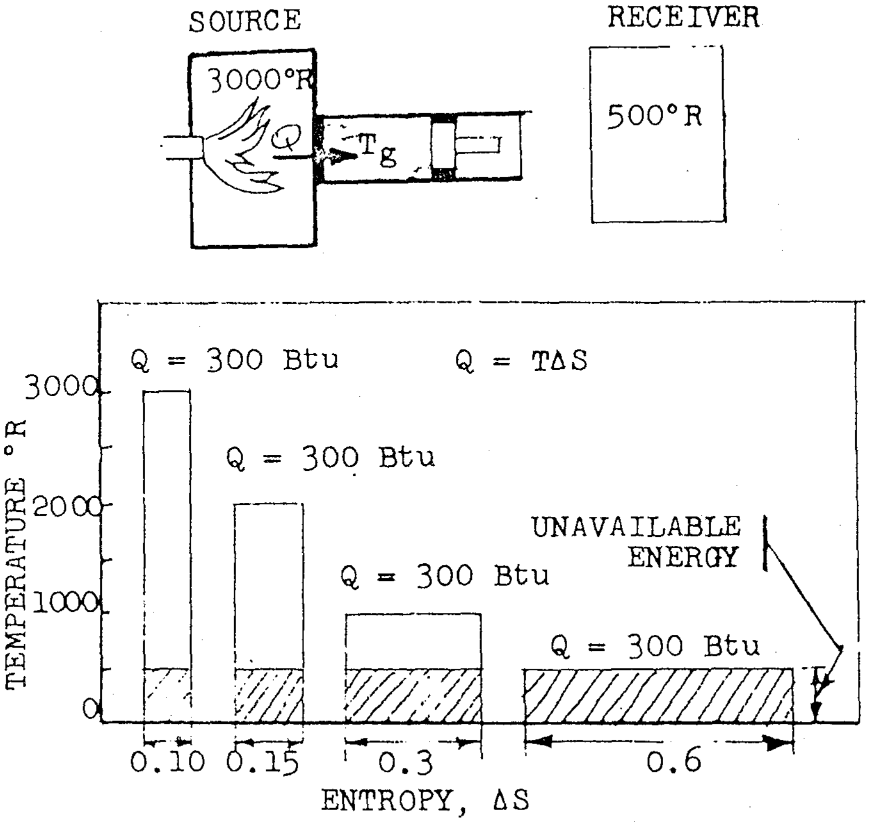


Figure 3

We can superimpose this as a crosshatched section on the first bar. The meaning comes easily: Of the total 300 Btu at 3000 °R, 50 Btu is unavailable energy which can't be used in this system, and 250 Btu is available for conversion to mechanical shaft work. The upper part of the bar is a T-S cycle diagram of a heat engine working between temperature limits of 3000°R and 500°R. The thermal efficiency of the cycle, as expressed in the lesson on the Second Law of Thermodynamics is:

$$e = \frac{Q_{in} - Q_r}{Q_{in}} \times 100$$

$$= \frac{300 - 50}{300} \times 100$$

$$= 83.3\%$$

We get the same efficiency in this case when using absolute temperatures - i.e. of we let T_{in} = temperature at inlet (°R) and T_r = temperature of rejected gas (°R) we can write the equation:

$$e = \frac{T_{in} - T_r}{T_{in}} \times 100 = \frac{3000 - 500}{3000} \times 100 = 83.3\%$$

This is true because this particular example happens to be one of a heat engine operating on a cycle called the Carnot Cycle. However we are getting ahead of ourselves here, and we will learn more of the Carnot Cycle at the T.T.1 level.

Next let us assume that because of external thermal irreversibility, T_g must be at 2000°R to accept the 300 Btu/lb of heat from the 3000°R source. We find that the energy entropy change in the engine gas is now 300/2000 = 0.15 unit. It has gained 0.05 unit of entropy in dropping 1000°R of temperature. The unavailable energy now has grown to 500 x .15 = 75 Btu/lb., and the available energy has shrunk to:

300 - 75 = 225 Btu/lb.

The corresponding thermal efficiency has decreased 225/300 = 0.75 or 75%.

In the third case, we let the gas attain only 1000°R, before it does useful work. The entropy change rises to Q/T = 300/1000 = 0.30 and unavailable energy to 500 x 0.3 = 150 Btu/lb. making the thermal efficiency drop to (150/300) x 100 = 50%. In the last case we let the 300 Btu transfer directly to the receiver at 500°R, raising the entropy change to 0.60 unit. (This can be compared to the case in a Nuclear Power Station where the steam from the boiler is discharged directly to the reject condenser.) All the energy of course becomes unavailable.

You will notice of course that if there was another heat engine, with its own separate system, whose inlet temperature could be 500°R and discharge temperature of say 460°R, then some of the heat energy which we have called unavailable energy would be available for doing work. In this sense we could say that unavailable energy had decreased. Thus what is unavailable energy for one system, may be available energy for another system.

In all of the preceding steps we have found that (if the source temperature remains constant) as we drop the temperature of a given quantity of energy (that is if the heat engine is operating at a temperature below the temperature of the source) its entropy increasees as well as the unavailable energy.

The above example combined with figure 3 gives us a graphic illustration of the penalty of running heat engines at temperatures lower than the heat source temperature. For example, furnace flame temperatures in a modern conventional steam boiler run at about 3000°R, but the energy picked up by the steam is only at 1050°F (1510°R). The entropy of the energy more than doubles in this case, and the actual thermal efficiency, is less than half the theoretical potential efficiency, figured from furnace temperature. At present, we are forced into this position because of lack of materials that can withstand temperatures above 1050°F continuously. We do somewhat better with our internal combustion engines, which use hot gases to work directly on the pistons, but cooling them with water jackets forces the degrading of some high temperature energy.

In discussing the above example on entropy you will notice that we have used an irreversible process. Invariably, there will be an increase in the entropy of a thermally isolated system of bodies (such as a source, heat engine and receiver) for any irreversible transfer of heat or energy.

Since all actual processes are irreversible in some respect, the entropy of the universe is steadily increasing. Moreover, in as much as entropy is a measure of the unavailability of energy, unavailable energy is steadily increasing while the total quantity of energy is considered constant - i.e. available energy + unavailable energy = total energy. Energy is thus being degraded into a form in which it cannot be used for the purpose of doing work. It is because of this that the Second Law of Thermodynamics is often referred to as the Law of Degradation of Energy.

The second part of the 2nd. Law of Thermodynamics states that heat cannot flow from a body at a lower temperature to another body at a higher temperature. This can be revised to state that no heat will flow between two bodies which are at the same temperature. Hence no work can be done by the one body on the other. Thus, since there is a constant tendency towards the degradation of energy one can imagine that one day all forms of energy will have been degraded to unavailable energy and there will be a uniform temperature of all matter in the universe and heat transfer cannot take place anymore. The conclusion derived from this is that when the entropy of the universe reaches the maximum, all forms of motion and life will have ceased to exist.

Summary

In our discussion on entropy and the degradation of energy we have mentioned several points to be noted. If there is an increase in entropy it is in some way related to the heat transfer process or the friction process. The points are summarized below:

1. To achieve a change in entropy a transfer of heat to or from the substance involved must take place.
2. In a reversible adiabatic process there is no transfer of heat and therefore entropy must remain constant during the process.
3. In an irreversible adiabatic process entropy is increased. A good example of this is the case where steam expands adiabatically as it passes through a turbine. It is adiabatic because a negligible amount of heat escapes through the casing. However, there is turbulence and fluid friction, as steam passes through the turbine, which is converted to heat. This heat is absorbed by the steam. Kinetic energy losses due to friction and turbulence are irreversible losses. But there has been a transfer of heat. An increase of heat energy means an increase in entropy.
4. In an irreversible non-adiabatic process the entropy decreases if heat is given off; entropy increases if heat is added

either from an outside source or within the substance due to friction.

5. If no outside heat is supplied during a process, and there is a decrease in enthalpy which is completely converted to work (reversible process,) then there is no change in entropy i.e. entropy remains constant.

6. The converse of point No.5 is also true - i.e. if the process involves work done on a substance, and all the work is converted to heat (or enthalpy) then the entropy also remains constant.

   For example when a piston compresses steam, work is done by the piston on the steam. This work appears as heat in the steam raising its temperature. Heat imparted this way does not change the entropy of steam.

7. An increase in entropy is a measure of irreversibility, since the heat lost in the form of unavailable energy is directly proportional to the increase in entropy.

We have mentioned "constant entropy" several times and this brings us to a term called isentropic process which we can now define as follows:

An Isentropic Process is a reversible adiabatic process during which entropy remains constant.

Entropy and Quality of Steam

In the lesson on "Steam Tables", T.T.3 level we gave the following equations regarding the quality of wet steam:

$$h = h_f + \frac{x}{100} h_{fg}$$

$$h = h_g - \frac{y}{100} h_{fg}$$

and

$$v = v_g - \frac{y}{100}(v_g - v_f) = v_g - \frac{y}{100}(v_{fg})$$

Since we have now covered the subject of entropy we can introduce another equation which will help us in finding the quality of steam. It can be derived in a manner similar to the above equations and is expressed as follows:

When % quality is known:

$$s = s_f + \frac{x}{100} s_{fg}$$

When % moisture is known:

$$s = s_g - \frac{y}{100} \; s_{f_g}$$

where:

h = total enthalpy of wet steam Btu/lb.
h_f = enthalpy of saturated water Btu/lb.
h_g = enthalpy of saturated steam Btu/lb.
h_{f_g} = latent heat of vaporization Btu/lb.

v = total specific volume of wet steam cu.ft./lb.
v_f = specific volume of saturated water cu. ft./lb.
v_{f_g} = specific volume of water & steam mixture cu.ft/lb.

v_g = specific volume of saturated steam cu.ft/lb.
s = total entropy of wet steam.
s_f = entropy of saturated water,
s_{f_g} = entropy of water and steam mixture.

s_g = entropy of saturated steam.
x = % steam quality in mixture
y = % moisture of mixture.

Sample problem No. 1

Steam enters a turbine at a pressure and temperature of 100 psia and 400°F and expands isentropically to a pressure of 5 psig at the exhaust. What is the quality of steam at the turbine exhaust?

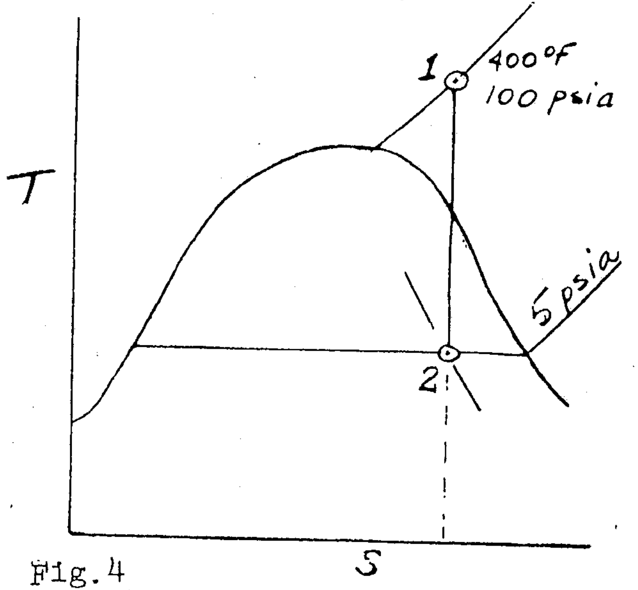


Fig. 4

Solution:

$$s = s_f + \frac{x}{100} \; s_{f_g}$$

$$x = \frac{(s - s_f)}{s_{f_g}} \times 100$$

The problem is to find the quality 'x' at point 2, Figure 4, which is the turbine exhaust. Examining the above equation we find we have two unknowns. We can look up the values of s_f and s_{f_g} for 5 psia, but we do not know the total entropy 's' at point 2. However, since this is an isentropic process the total entropy S at point 1 is the same as at point 2. We can therefore obtain 'S' by looking in superheat steam tables at 400°F and 100 psia. Thus:

NUCLEAR ELECTRIC G.S. TECHNICAL TRAINING COURSE

2 - Science Fundamentals - T.T.2

5 - Heat and Thermodynamics

-6 - Entropy

A - Assignment

1. Define entropy.
2. The heat transferred in a certain process is 1320 Btu/lb. The process takes place at 200°F. Calculate the change of entropy involved.
3. In what way does an irreversible non-adiabatic process influence entropy changes?
4. Define an isentropic process.
5. Why is it desirable to operate a heat engine at or near the same temperature as the temperature of the source of energy.
6. Steam enters a turbine at a pressure and temperature of 100 psia and 400°F and expands isentropically to a pressure of 1 psia (2.05 Hg.) at the exhaust. What is the % moisture of steam at the turbine exhaust?

NUCLEAR ELECTRIC G.S. TECHNICAL TRAINING COURSE

2 - Science Fundamentals - T.T.2

5 - Heat and Thermodynamics

-7 - Heat Engine Cycles

0.0 INTRODUCTION

This lesson will deal with the many different cycles that can be employed in heat engines to develop useful work.

1.0 INFORMATION

We have mentioned heat engines and cycles in previous lessons but as a review they will be defined as follows:

A heat engine is the name given in thermodynamics to any device used for converting heat into work. All heat engines operate by passing the thermodynamic system or working substances through a series of events or processes, returning it periodically to the original condition. These processes, which are necessary to return the system to its original condition taken as a whole and in order constitute a heat engine cycle.

One cycle, by itself, would, of course, be of little use but when a large number of cycles occur in rapid succession then useful work can be developed.

The internal combustion engine provides a familiar example of a heat engine cycle where we have the following processes taking place: intake, compression, combustion, expansion, exhaust and intake again.

The Cycle

The essential elements of any thermodynamic cycle, (as shown in Figure 1) which involves a heat engine are:

1. A working substance, - i.e a medium for receiving and rejecting heat, a substance to carry energy to and from a heat engine.

2. A source of heat or hot body wherewith heat may be added to the working substance.

3. A receiver or cold body, a body to which heat may be rejected by the working substance.

4. A heat engine or heat pump, wherein the working substance may do work or have work done on it.

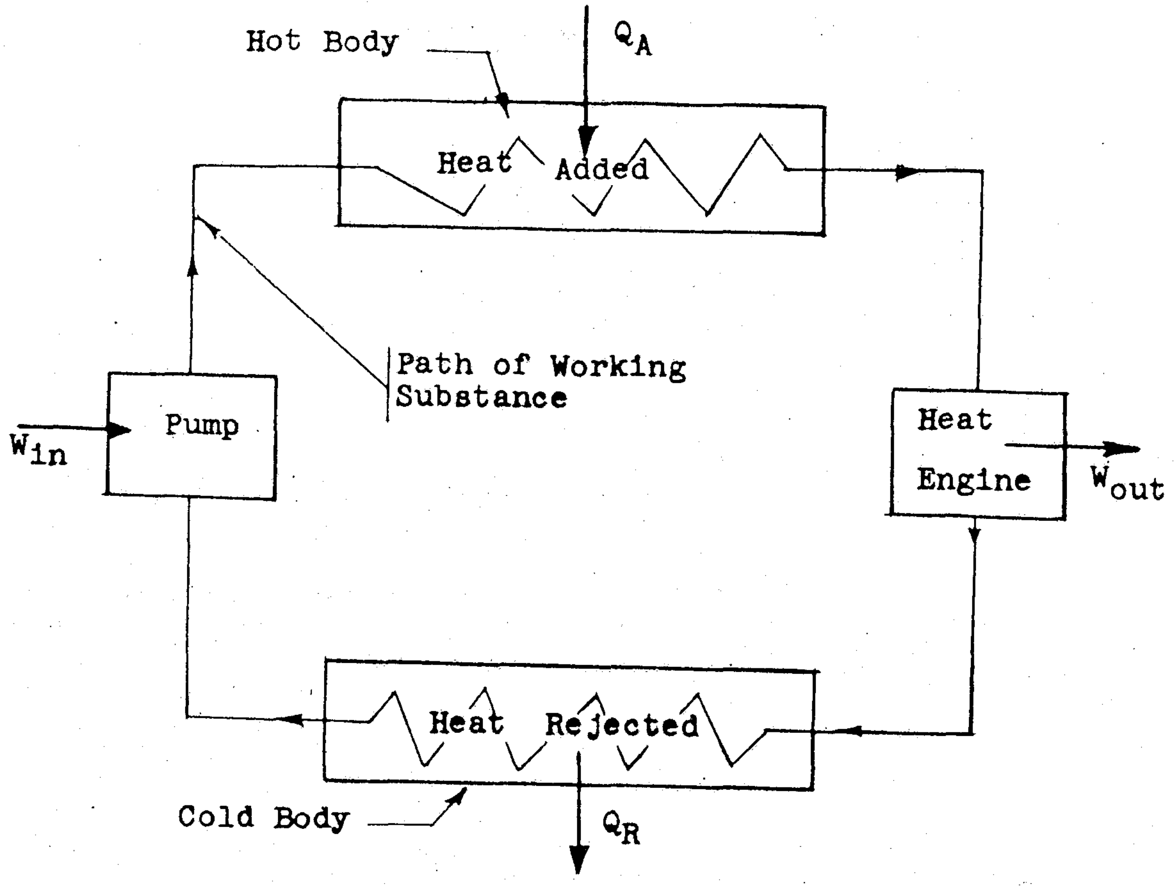


Fig. 1 ELEMENTS OF A THERMODYNAMIC CYCLE

An energy diagram. If W_{out} is greater than W_{in}, the cycle is one designed to generate power. A supply of heat, Q_A, is added to the substance at the source or hot body, part of which is converted into work in the engine, the remainder of which, Q_R, is rejected to the cold body or receiver. A pump is generally necessary to move the working substance from the cold body region which is normally a region of low pressure, to the hot body region, which is ordinarily a region of high pressure. Application of the law of conservation of energy gives:

Energy in = Energy out

$$Q_A + W_{in} = Q_R + W_{out}$$

or $$W_{out} - W_{in} = Q_A - Q_R$$

Man, by chosen devices, may close the cycle, or he may allow nature to close the cycle for him.

An open cycle is one that nature closes for us. The internal combustion engine is the open cycle type where air is drawn in from the atmosphere and the exhaust discharged to the atmosphere.

A closed cycle is one closed by the inventions of man. The modern steam power plant is an example of such a cycle. That is, the same working substance is used over and over again and never leaves the system.

The Work of a Cycle

The net work done during the performance of any cycle may be taken from the general energy equation:

$$\frac{v_{s1}^2}{2gJ} + \mu_1 + \frac{p_1v_1}{J} + Q = \frac{v_{s2}^2}{2gJ} + \mu_2 + \frac{p_2v_2}{J} + W$$

For a substance that passes through a cycle and returns again to its original condition, the kinetic energy, internal energy and flow work terms in the general energy equation are the same on either side of the equation and, therefore, cancel each other out. Hence, neglecting losses, we find that $Q = W$ for a cycle. Therefore, as is shown in Figure 1,

$$W_{out} - W_{in} = Q_A - Q_R \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where W_{out} = work out, Btu

W_{in} = work put in, Btu

Q_A = heat added, Btu

Q_R = heat rejected, Btu

Thermal Efficiency

Efficiency may well be thought of in its simplest form, output divided by input, thus:

$$\text{Efficiency } (e) = \frac{\text{Net Work}(W)}{\text{Heat Added}(QA)} = \frac{Q_A - Q_R}{Q_A} \quad \ldots\ldots(2)$$

This is the same equation as given in the lesson on the Second Law of Thermodynamics.

We are now at the point where we can analyze a few different types of heat cycles and compare them. The following are the cycles we will be concerned with: Otto Cycle, Diesel Cycle, Brayton Cycle, Carnot Cycle, and Rankine Cycle. The first two are known as the internal combustion cycles, since the Otto cycle is used in a gasoline engine, and the Diesel cycle, as the name suggests, in a diesel engine. The Brayton cycle is used in gas turbines. These three all employ gases as working substances and the whole cycle takes place within the engine itself. The Carnot cycle can use either gases or vapors as a working substance. It is the most efficient cycle conceivable and is used as a basis of comparison with other cycles. If the cycle is used in reverse, it can be used as a refrigerator. The Rankine cycle is used for vapors only and is the one employed in steam power plant cycles. In this cycle the working substance passes through several pieces of equipment before returning to its origin.

The first three cycles mentioned in the preceding paragraph will be covered in this lesson. The Carnot and Rankine cycles will be covered on the T.T.1 level.

In the analysis of gas cycles, we shall be particularly interested in:

1. The heat supplied to the cycle.
2. The heat rejected
3. Net work, i.e. Work in - Work out.
4. The efficiency.

Otto Cycle

The Otto cycle is an ideal type of cycle used in most internal combustion engines today. The first engine to operate on this cycle was built by a German, named Nicholas A. Otto in 1876.

The ideal Otto cycle consists of two isentropic and two constant volume processes 1-2-3-4-1 as shown in Figure 2. We use the word ideal, because in practice the processes do not take place exactly isontropically, nor exactly at constant volume. However, if we know what the ideal conditions would be, then we can compare the actual engine with the ideal situation and see how closely it approaches the ideal situation.

Referring to Figure 2, the detailed operations that take place are as follows:

1. Suction stroke 0-1, during which a mixture of air and fuel is drawn into the cylinder at constant pressure.

2. A compression stroke 1-2 during which the charge is compressed isentropically.

3. The ignition and consequent explosion of the charge 2-3, occurring at constant volume (instantaneously) and causing the temperature and pressure to rise owing to the addition of thermal energy developed by combustion.

4. An expansion stroke (or power stroke) 3-4, providing for the isentropic expansion of the burned gases.

5. Rejection of the burned gases from the cylinder at constant volume 4-1, and constant pressure 1-0 and consequent return to the starting point of the cycle.

It is customary to analyze the ideal cycle as though there were no suction and exhaust strokes 0-1 and as through the working substance were air only. Such an analysis is referred to as an air standard analysis.

From the energy equation of a cycle mentioned in the preceding pages, we have

$$W = Q_A - Q_R = \text{net work}$$

Assuming constant specific heats, we find that:

$$Q_A = w\ c_v\ (T_3 - T_2)\ \text{Btu} \quad \ldots\ldots\ldots\ldots(3)$$

$$Q_R = w\ c_v\ (T_4 - T_1)\ \text{Btu} \quad \ldots\ldots\ldots\ldots(4)$$

where:

W = net work

Q_A = heat added, Btu.

Q_R = heat rejected, Btu.

w = weight of air, lbs.

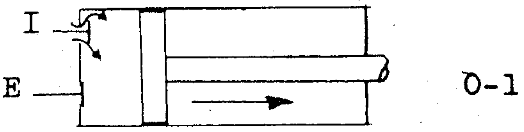
c_v = specific heat at constant volume Btu/lb/°F.

T = ° RANKINE

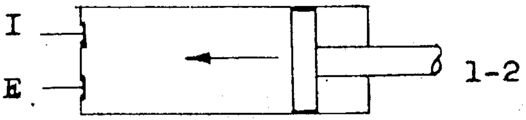
These equations are similar to the one for W in the lesson on adiabatic expansion and compression.

$$\therefore W = wc_v(T_3 - T_2) - wc_v(T_4 - T_1)$$

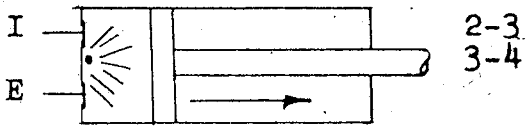
The process 0-1 is a process during which fresh air is drawn in, or burned gases are discharged (1-0). It is a process of flow wherein the weight of substance varies. It is not shown on the T-S plane, because the properties of the substance do not change from 0 to 1. Neglecting the frictional effects, as one would in the ideal case, we notice that the combination 0-1 and 1-0 has no effect whatsoever on the net work or efficiency.



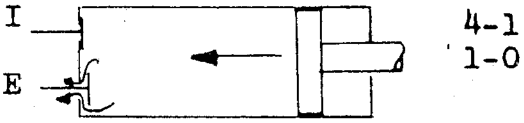
a) Suction Stroke



b) Compression Stroke



c) Expansion Stroke



d) Exhaust Stroke

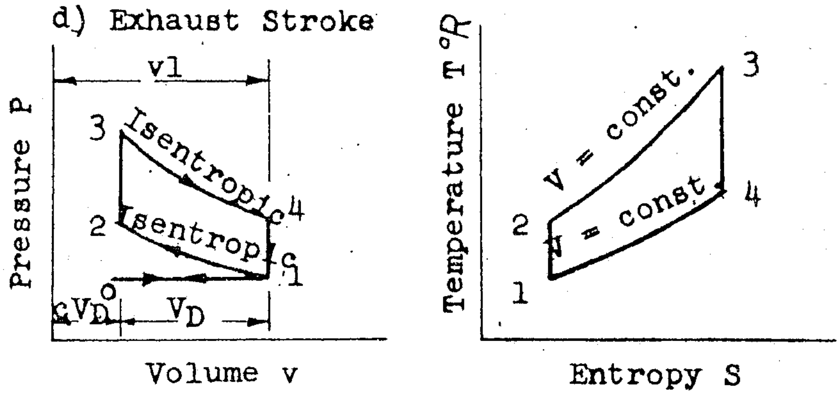


Fig. 2 OTTO CYCLE

The thermal efficiency of the Otto cycle is:

$$e = \frac{W}{Q_A} = \frac{wc_v\,(T_3 - T_2) - wc_v\,(T_4 - T_1)}{wc_v\,(T_3 - T_2)}$$

$$\therefore \quad e = 1 - \frac{T_4 - T_1}{T_3 - T_2} \quad \ldots\ldots\ldots\ldots\ldots (5)$$

Again from the lesson on adiabatic expansion and compression, we learned that $\frac{T_A}{T_B} = \left(\frac{V_B}{V_A}\right)^{k - 1}$.

Applying it to our situation, we get

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{k-1} \quad \text{and} \quad \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{k - 1}$$

and since $V_3 = V_2$ and $V_4 = V_1$,

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{k - 1} = \left(\frac{V_2}{V_1}\right)^{k - 1}$$

$$T_4 = T_3 \left(\frac{V_3}{V_4}\right)^{k - 1} = T_3 \left(\frac{V_2}{V_1}\right)^{k - 1}$$

and

$$T_1 = T_2 \left(\frac{V_2}{V_1}\right)^{k - 1}$$

Substituting into equation (5), we get

$$e = 1 - \left[\frac{T_3\left(\frac{V_2}{V_1}\right)^{k - 1} - T_2\left(\frac{V_2}{V_1}\right)^{k - 1}}{T_3 - T_2}\right] = 1 - \left[\frac{\left(\frac{V_2}{V_1}\right)^{k - 1}(T_3 - T_2)}{T_3 - T_2}\right]$$

$$\therefore \quad e = 1 - \left(\frac{V_2}{V_1}\right)^{k - 1}$$

But we have learned in the T.T.3 course that V_1/V_2 is what is called the compression ratio, and in this case, it is the adiabatic compression ratio.

Let $r_k = V_1/V_2$ = the adiabatic compression ratio.

Then $e = 1 - \frac{1}{(V_1/V_2)^{k - 1}}$

$$e = 1 - \frac{1}{(r_k)^{k-1}} \quad \ldots\ldots\ldots\ldots\ldots(6)$$

You can see thus that the efficiency of an engine operating on the Otto cycle, for a given value of k, depends only on the compression ratio, V_1/V_2. The greater this ratio, the more efficient the engine will be.

Our discussion has been based on a four stroke cycle. However, numerous engines are also designed to operate on a two-stroke Otto cycle. Calculations for a two-stroke cycle are similar to the above.

Diesel Cycle

The cycle under which a diesel engine operates is shown on PV and TS co-ordinates figure 3. It follows the following sequence:

1. Air only is drawn into the cylinder 0-1.
2. The air is compressed isentropically 1-2, until its temperature is high enough to cause diesel fuel to ignite. (Note there is no system of ignition and combustion takes place due to the high temperature of compression).
3. Then diesel fuel is injected at such a rate that its burning will maintain the pressure in the cylinder constant for part of the working stroke 2-3.
4. The expansion stroke (or power stroke) then takes place isentripically 3-4.
5. Then as in the Otto cycle, discharge takes place at constant volume 4-1 and constant pressure 1-0.

The diesel engine may also operate on either the two-stroke or four-stroke cycle. In either event, the corresponding ideal air-standard cycle is the same.

In writing our equations this time, we have to take note of the fact that one process takes place at constant pressure (2-3) and we will have to use c_p for the specific heat here; another process takes place at constant volume and we will have to use c_v for specific heat in this case.

As in the case of the Otto cycle, we find: -

$$Q_A = wc_p\ (T_3 - T_2)\ \text{Btu}$$

$$Q_R = wc_v\ (T_4 - T_1)\ \text{Btu.}$$

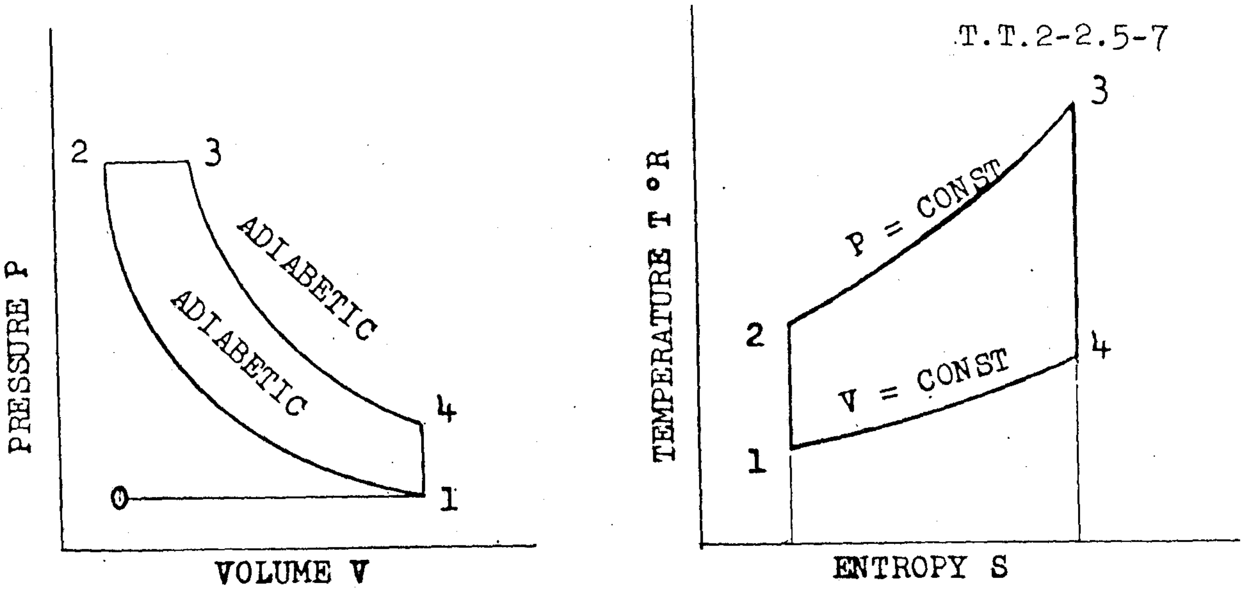


Fig. 3 DIESEL CYCLE

Between the same temperature limits, the constant volume curve in the TS plane is steeper than the constant pressure curve. However, both curves get steeper as the temperature increases. Thus, the temperature at 3 is so high that the constant pressure curve is steeper than the constant volume curve, which is at the lower temperatures between 1 and 4.

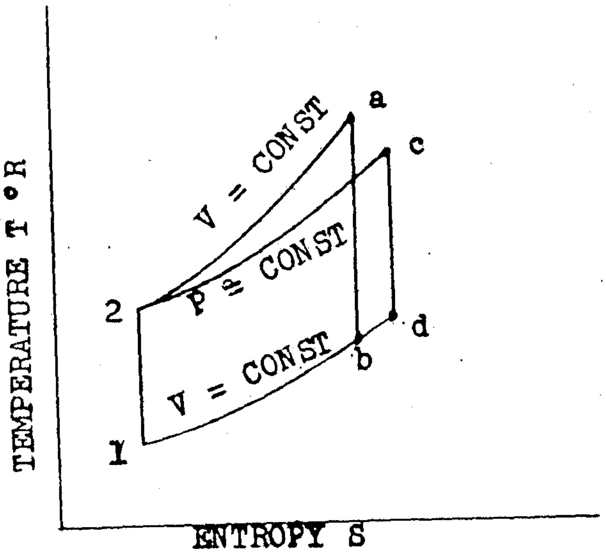


Fig. 4 COMPARISON OF OTTO AND DIESEL CYCLES

This diagram is drawn for the same compression ratio V_1/V_2 for the Otto and Diesel cycles. Moreover, Q_A is the same in each; that is $Q_{2-a} = Q_{2-c}$.

Since obviously the rejected heat is greater for the Diesel than for the Otto, $Q_{d-1} > Q_{b-1}$, the work ($W = Q_A - Q_R$) and therefore, the thermal efficiency for the Diesel is less than for the Otto, when the compression ratio is the same for each. Actual Diesel engines have compression ratios of around 15 as compared to around 8 for the Otto engine in which case the Diesel engine is more efficient.

$$W = Q_A - Q_R = wc_p\,(T_3 - T_2) - wc_v\,(T_4 - T_1) \text{ Btu.}$$

$$e = \frac{W}{Q_A} = \frac{wc_p\,(T_3 - T_2) - wc_v\,(T_4 - T_1)}{wc_p\,(T_3 - T_2)}$$

$$= \frac{wc_p\,(T_3 - T_2)}{wc_p\,(T_3 - T_2)} - \frac{wc_v\,(T_4 - T_1)}{wc_p\,(T_3 - T_2)}$$

$$= 1 - \frac{c_v(T_4 - T_1)}{c_p\,(T_3 - T_2)} = 1 - \frac{1(T_4 - T_1)}{\frac{c_p}{c_v}\,(T_3 - T_2)}$$

But $k = \dfrac{c_p}{c_v}$

$$\therefore e = 1 - \frac{(T_4 - T_1)}{k(T_3 - T_2)} \quad \ldots\ldots\ldots\ldots\ldots(7)$$

This equation is usable, but it may be placed in a much more convenient and revealing form by eliminating the temperatures. Assuming that the working substance is a perfect gas, we may express three of the temperatures in terms of the fourth, say in terms of T_1. Thus along the isentropic 1-2, $T_2/T_1 = (V_1/V_2)^{k-1}$. However, V_1/V_2 is defined as the adiabatic compression ratio r_k.

$$\therefore T_2 = T_1\left(\frac{V_1}{V_2}\right)^{k-1} = T_1\, r_k^{\,k-1} \quad \ldots\ldots\ldots\ldots(8)$$

Along the constant pressure line 2-3, Charles's Law holds and $T_3/T_2 = V_3/V_2$. Let $V_3/V_2 = r_c$. This ratio r_c is called the *fuel cutoff ratio* for a diesel engine.

Then:

$$T_3 = T_2\left(\frac{V_3}{V_2}\right) = T_1 \cdot r_k^{\,k-1} \cdot r_c \quad \ldots\ldots\ldots\ldots\ldots\ldots(9)$$

For the isentropic process 3-4, $T_4/T_3 = (V_3/V_4)^{k-1}$ ....(10)

But from 2-3, we have said $T_3/T_2 = V_3/V_2 = r_c$

Transposing we get $V_3 = \left(\dfrac{T_3}{T_2}\right) V_2 = r_c \cdot V_2$ ....(11)

and transposing equation (10)we get: $T_4 = T_3\left(\dfrac{V_3}{V_4}\right)^{k-1}$

Now using the values of V_3 from equation (11), T_3 from equation (9), we get:

$$T_4 = T_1.r_k^{k-1}.r_c \left(\frac{r_c.V_2}{V_4}\right)^{k-1}$$

By transposing algebraically, this can be changed to:

$$T_4 = T_1.r_k^{k-1}.\ r_c \frac{(r_c)^{k-1}}{\left(\frac{V_4}{V_2}\right)^{k-1}}$$

but from the pV diagram, figure 3, we see that $V_1 = V_4$, and we have said that $V_1/V_2 = r_k$. Therefore, $V_4/V_2 = r_k$. Substituting this into the above equation for T_4, we get:

$$T_4 = T_1\ \cancel{r_k^{k-1}}.r_c. \quad \frac{(r_c)^{k-1}}{\cancel{(r_k)^{k-1}}}$$

Cancelling out equivalent terms in the numerator and denominator, and remembering our rules from algebra that:

$r_c.r_c^{k-1} = r_c^{k-1+1} = r_c^k$, we get

$$T_4 = T_1.r_c^k \quad \ldots\ldots\ \ldots\ldots(12)$$

Now substituting the values of T_2, T_3 and T_4, just found, into equation (7), we get:

$$e = 1 - \frac{T_1 r_c^k - T_1}{k(T_1.r_k^{k-1}.r_c - T_1 r_k^{k-1})} = 1 - \frac{T_1(r_c^k - 1)}{T_1.r_k^{k-1}.k(r_c-1)}$$

Cancelling T_1, we get:

$$e = 1 - \frac{1}{r_k^{k-1}}\left[\frac{r_c^k - 1}{k(r_c - 1)}\right] \quad \ldots\ldots\ldots\ldots(13)$$

Observe that this expression for the efficiency of the Diesel cycle differs from that of the Otto cycle only in the bracketed factor. This factor is always greater than 1, because $r_c = V_3/V_2$ is always greater than 1, since V_3 is always greater than V_2. Thus for a particular compression ratio, r_k, the Otto cycle is more efficient. (See also Figure 4).

However, if the compression ratio is too high in an Otto engine, a knocking occurs due to auto-ignition of the fuel. Since the Diesel engine compresses air only, the compression ratio is much higher than in an Otto engine. Therefore, an actual Diesel engine with say $r_k = 15$ is more efficient than an actual Otto engine with $r_k = 8$.

There is another point of interest to be mentioned here. There are two ratios in the diesel engine that influence its performance. These are the compression ratio r_k and the cutoff ratio r_c. In the Otto cycle it is the compression ratio only.

Brayton Cycle

As we mentioned previously, the Brayton cycle is used in the gas turbine. This type of heat engine has been developed only relatively recently, although the theory of the cycle has been known for quite some time.

Until recently, there were no metals that could withstand very much pressure at temperatures above 800°F. Today, we have available metals that will retain fairly high strength at 1200°F and upwards. Also, only recently axial flow compressors have been developed with a relatively high efficiency of compression. The combination of these two developments has made it possible to make use of the internal combustion turbine on a commercial scale.

Figure 5 shows diagramatically, the plan of operation of one of these turbines. First, it is necessary to start rotation (up to about 25% of operating speed) with the starting motor, since the compressor must operate at a fairly high speed, in order to compress

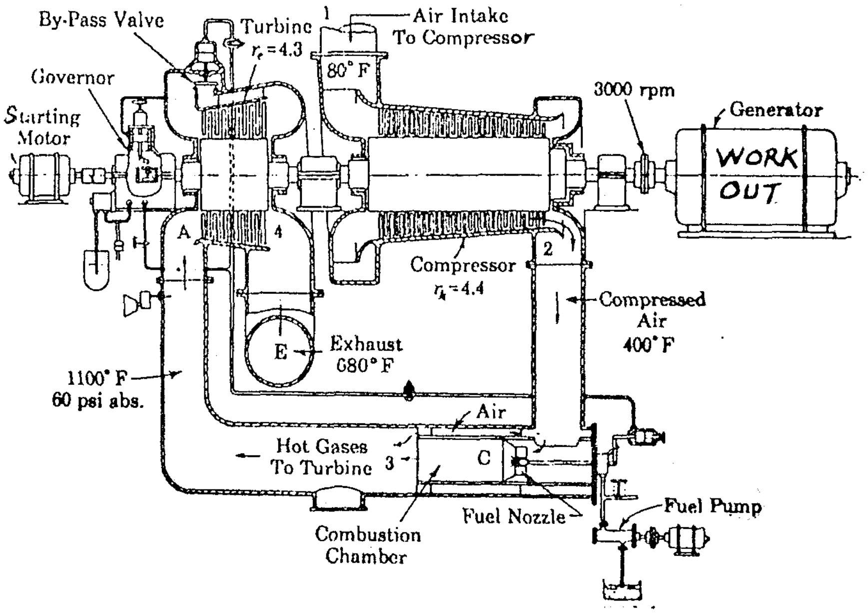
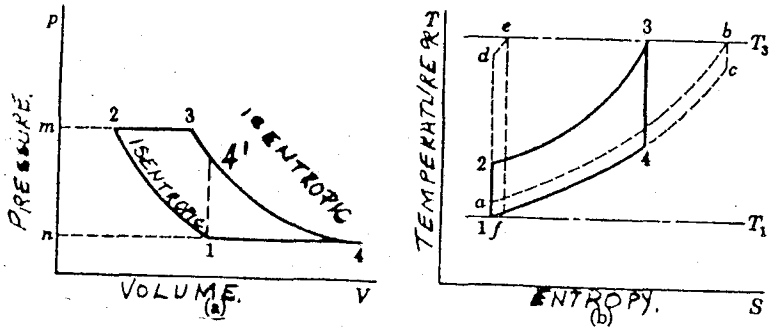


Fig. 5: DIAGRAMATIC LAYOUT OF INTERNAL COMBUSTION TURBINE

The temperatures and pressures given are typical of actual values.



Also known as the Joule cycle. The line n–1 may be taken as the intake to the compressor and the area under it represents the flow work of the intake. The area under 2–m represents the flow work of the discharge of the compressor. The area under m–3 represents the intake flow work as the air enters the turbine. The area under 4–n represents the exhaust flow work from the turbine. The enclosed area n–1–2–m represents the work of the compressor. The enclosed area m–3–4–n represents the work of the turbine. The enclosed areas 1–2–3–4 on each diagram represent the net work output of the combined unit.

The net work may be obtained by an application of the steady flow equation, as well as by the cycle analysis, equation   Thus, assuming adiabatic compression ($Q = 0$) and $\Delta KE = 0$, the work of the compressor from the general energy equation is

$$W_c = h_1 - h_2 = wc_p(T_1 - T_2) = -wc_p(T_2 - T_1)$$

Similarly the work of the turbine is

$$W_t = h_3 - h_4 = wc_p(T_3 - T_4).$$

The net work is the algebraic sum of these work quantities, or

$$W_{\text{net}} = wc_p(T_3 - T_4 - T_2 + T_1),$$

Fig. 6 BRAYTON CYCLE

the air. Air is taken into the compressor at 1 and delivered at 2. At 'C' part of the compressed air mixes with the fuel usually a petroleum oil), which burns in the combustion chamber, the rest passes around the combustion chamber. The mixture of hot gases at 3 passes on and enters the turbine at A. After expanding to 4, the gasses pass out of the exhaust at E.

The corresponding cycle mentioned below describes the ideal Brayton Cycle. It is composed of two isentropic processes and two constant pressure processes. The points in Figure 6 are numbered to correspond with the numbers on Figure 5. The following processes are involved in the cycle:

1. Isentropic compression, 1 - 2.
2. Combustion at constant pressure 2-3, during which heat is added.
3. Isentropic expansion, 3-4, during which power is produced.
4. Discharge at constant pressure 4-1.

A point to notice is that the Brayton Cycle as drawn in Fig. 6

is an extension of the Diesel Cycle. If the cycle were drawn as 1-2-3-4'-1 as shown in Figure 6(a), it would be an exact duplication of the Diesel Cycle. In the Brayton cycle, the gas expansion is prolonged past the point 4' until the pressure p_1 is reached. The amount of work performed is thus increased by the triangular area 4'-4-1 at no additional expenditure of heat added to the cycle.

To develop equations for calculating the work done, we proceed in a manner similar to the other cycles.

$$W = Q_A - Q_R$$

$$Q_A = wc_p(T_3 - T_2) \qquad Q_R = wc_p\,(T_4 - T_1)$$

Notice we use c_p since the temperature changes take place at constant pressure and therefore specific heat to be used is the one for constant pressure.

$$W = wc_p\,(T_3 - T_2) - wc_p\,(T_4 - T_1)$$

$$\therefore W = wc_p\,(T_3 - T_4 - T_2 + T_1) \quad \ldots\ldots\ldots\ldots\ldots (14)$$

The thermal efficiency is:

$$e = \frac{W}{Q_A} = \frac{Q_A - Q_R}{Q_A} = \frac{wc_p(T_3 - T_2) - wc_p\,(T_4 - T_1)}{wc_p\,(T_3 - T_2)}$$

$$e = \frac{T_3 - T_2}{T_3 - T_2} - \frac{T_4 - T_1}{T_3 - T_2}$$

$$e = 1 - \frac{T_4 - T_1}{T_3 - T_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

This equation may be reduced to a simple form by a procedure similar to that used for the Diesel Cycle. Suppose all temperatures are put in terms of T_2 as follows:

The process from 1-2 is an adiabatic process. We have learned in the lesson on adiabatic expansions and compression, T.T.2, that

$$\frac{T_A}{T_B} = \left(\frac{P_A}{P_B}\right)^{\frac{k-1}{k}}$$

Applying the correct numerals to this particular case, we get:

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{k-1}{k}}$$

$$T_1 = T_2\left(\frac{P_1}{P_2}\right)^{\frac{k-1}{k}} = T_2\left(\frac{V_2}{V_1}\right)^{k-1} = T_2\frac{1}{\left(\frac{V_1}{V_2}\right)^{k-1}} = T_2\cdot\frac{1}{r_k^{\;k-1}}$$

Since $\left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}} = \left(\frac{v_2}{v_1}\right)^{k-1}$ as also learned in the lesson on adiabatic expansion and compression, and $V_1/V_2 = r_k$.

The process 2-3 is a constant pressure process and therefore Charles Law applies. Hence:

$$\frac{T_3}{T_2} = \frac{V_3}{V_2} \quad \text{and} \quad T_3 = T_2\left(\frac{V_3}{V_2}\right)$$

From 3-4, we again have an adiabatic process and we get:

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}}$$

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = T_3\left(\frac{P_1}{P_2}\right)^{\frac{k-1}{k}} = T_3\left(\frac{V_2}{V_1}\right)^{k-1}$$

Substituting for T_3 and using r_k for V_1/V_2 we get:

$$T_4 = T_2\left(\frac{V_3}{V_2}\right)\frac{1}{r_k^{\;k-1}}$$

Substituting these various values for T_1, T_3 and T_4 in equation (15), we get:

$$e = 1 - \frac{\left[T_2\left(\frac{V_3}{V_2}\right)\frac{1}{r_k^{\;k-1}}\right] - \left[T_2\cdot\frac{1}{r_k^{\;k-1}}\right]}{\left[T_2\left(\frac{V_3}{V_2}\right)\right] - T_2}$$

T_2 cancels out, leaving:

$$e = 1 - \frac{\left(\frac{V_3}{V_2}\right)\frac{1}{(r_k)^{k-1}} - \frac{1}{r_k^{\;k-1}}}{\frac{V_3}{V_2} - 1} = 1 - \frac{\frac{1}{(r_k)^{k-1}}\left(\frac{V_3}{V_2} - 1\right)}{\left(\frac{V_3}{V_2} - 1\right)}$$

The last term cancels out, leaving:

$$e = 1 - \frac{1}{r_k^{\,k-1}} \quad \text{..........(16)}$$

This equation is the same as for the efficiency of an Otto cycle. After finding this equation for an Otto cycle, we drew the conclusion that the higher the compression ratio r_k, the greater the thermal efficiency of the ideal cycle and in general of the actual engine. While this conclusion is valid for the ideal Brayton cycle, it does not hold for the actual internal combustion turbine. In the actual Otto engine, the highest temperature is intermittent and exists only for a short time during each cycle.

Therefore, even if this highest temperature is carried to limits which are now thermodynamically possible, the metal parts can be maintained well below 700°F. On the other hand, certain metal parts of the gas turbine must be in continuous contact with the hottest gases. Consequently, there is a limiting temperature, set by the properties of metals and the desired life of the machine above which we cannot go.

The effect of this ceiling or limiting temperature on the work of the ideal cycle can be seen in Figure 6. Let the lowest and highest temperatures, T_1 and T_3 be fixed. Now, if there is a very small compression 1-a, the work done 1-a-b-c is also small. Also, if there is a large compression 1-d, carrying the temperature almost to T_3, the work again is small, represented by the area 1-d-e-f. Somewhere between these extremes, the ideal work is a maximum. Therefore, the rule that the higher the compression ratio, the greater the thermal efficiency does not apply to the Brayton cycle.

The next lesson will deal solely with the solution of problems on the three cycles covered in this lesson.

One last point regarding this lesson is that the student should be familiar with the equations given and how to apply them to a particular problem but they need not be memorized. On an examination, a list of equations to be used on questions regarding this lesson will be given.

D. Dueck.

NUCLEAR ELECTRIC G.S. TECHNICAL TRAINING COURSE

2 - Science Fundamentals - T.T.2

5 - Heat and Thermodynamics

-7 - Heat Engine Cycles

A - Assignment

1. a) Define heat engine.

   b) List the essential elements of any thermodynamic cycle.

2. Give the equation for:

   a) net work for any heat engine cycle.

   b) thermal efficiency for any heat engine cycle.

3. List the detailed processes involved in the Otto cycle. Draw the PV and TS diagrams for it.

4. List the processes involved in the Diesel cycle. Draw the PV and TS diagram for it.

5. List the processes involved in the Brayton cycle. Draw the PV and TS diagrams for it.

6. State in what type of engines each one of the cycles mentioned in questions 3, 4 and 5, is used.

NUCLEAR ELECTRIC G.S. TECHNICAL TRAINING COURSE

2 - Science Fundamentals - T.T. 2

5 - Heat & Thermodynamics

8 - Problems on Heat Engine Cycles

0.0 INTRODUCTION

This lesson will deal solely with the solution of problems on the Otto, Diesel and Brayton cycles which have been described in the previous lesson. It is assumed that the student is familiar with the use of common logarithms.

1.0 INFORMATION

Sample Problem #1.

Find the ideal thermal efficiency of an Otto cycle with a compression ratio of 5 and a specific heat ratio of 1.4.

Solution: For the Otto cycle the ideal thermal efficiency is

$$e = 1 - \frac{1}{r_k^{k-1}}$$

We are given that the compression ratio $r_k = V_1/V_2 = 5$ and the specific heat ratio: $k = c_p/c_v = 1.4$

Thus:

$$e = 1 - \frac{1}{(5)^{1.4-1}} = 1 - \frac{1}{(5)^{0.4}}$$

Solve the fraction $\frac{1}{(5)^{0.4}}$ by logarithms thus:

$$\log\left(\frac{1}{5^{0.4}}\right) = \log 1 - \log (5)^{0.4} = \log 1 - 0.4 \log 5.$$

Sample Problem # 1 (cont'd)

$$\begin{array}{rll} \log 1 = 0.000 = & & 10.000-10 \\ 0.4 \log 5 = 0.699 \times 0.4 = & & \underline{0.280} \\ \text{subtracting} & & 9.720-10 \end{array}$$

Antilog 9.720-10 = 0.525

$$\therefore \frac{1}{5^{0.4}} = 0.525$$

and $e = 1 - 0.525 = 0.475$

To express the answer in % we multiply by 100 to get:

$$e = 0.475 \times 100 = 47.5\%$$

Sample Problem # 2

Find the ideal thermal efficiency of an Otto cycle with a compression ratio of 10 and a specific heat ratio of 1.4. (Notice this compression ratio is double the compression ratio for problem # 1.)

Solution:

$$e = 1 - \frac{1}{r_k^{\,r-1}} = 1 - \frac{1}{(10)^{1.4-1}} = 1 - \frac{1}{(10)^{.4}}$$

$$\begin{array}{rll} \text{Log } 1 = & 0.000 = & 10.000-10 \\ .4 \text{ Log } 10 = & 1.000 \times .4 = & \underline{0.400} \\ & & 9.600-10 \end{array}$$

Antilog 9.600-10 = 0.398

$$\therefore e = 1 - 0.398 = 0.602$$

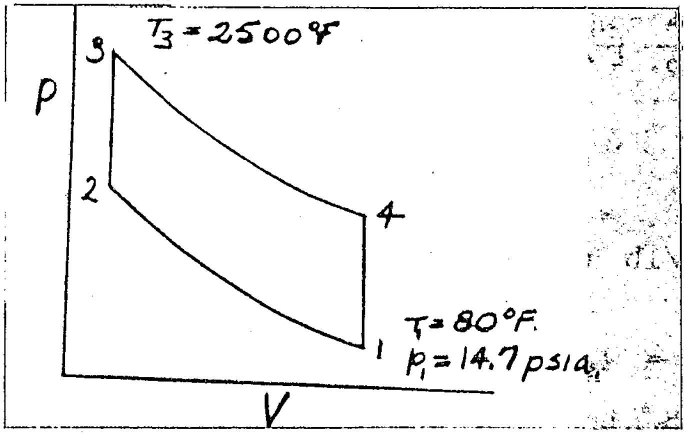
$$e = 0.602 \times 100 = 60.2\%$$

We have said in the lesson on Heat Engine Cycles that the efficiency of the Otto cycle depends solely on the compression ratio, providing that the specific heat ratio remains constant. Problems No. 1 & No. 2 illustrate this point. The specific heat ratio in both cases is 1.4 while the compression ratio is 5 for one and 10 for the other. Doubling the compression ratio increases the ideal efficiency from 47.5% to 60.2%.

Sample Problem # 3

Compute the a) heat added b) heat rejected and c) mechanical work produced in an ideal Otto cycle using air initially at 14.7 psi and 80°F. The maximum combustion temperature is 2500°F and the compression ratio is 12. Assume $k = 1.3$ for all constant entropy processes; $c_v = 0.225$

Solution



a) Heat added = $Q_A = c_v(T_3 - T_2)$

We know $c_v = 0.225$ and $T_3 = (2500 + 460) = 2960°R$, but do not know T_2 and must solve for it.

From our discussion on the the Otto cycle in the previous lesson, we know that 1-2 is an isentropic process and that:

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{k-1} = \frac{1}{\left(\frac{V_1}{V_2}\right)^{k-1}} = \frac{1}{(r_k)^{k-1}}$$

We've been given that $r_k = 12$ and $k = 1.3$

$$\therefore \quad T_2 = T_1(r_k)^{k-1} = (80 + 460)(12)^{1.3-1} = 540 \times 12^{0.3}$$

$\text{Log } 540 \times 12^{0.3} = \text{Log } 540 + 0.3 \log 12$

Log 540 =	2.732
0.3 Log 12 = 0.3 x 1.079 =	0.324
Adding=	3.056

Antilog 3.056 = 1138 $\quad \therefore \quad T_2 = 1138$ °R

$$\therefore \quad Q_A = 0.225(2960-1138) = 410 \text{ Btu/lb. air.}$$

b) Heat rejected $Q_R = c_v(T_4 - T_1)$

We know that $c_v = 0.225$ and $T_1 = (80 + 460) = 540$ but we must find T_4. Again from the previous lesson we have learned that:

$$T_4 = T_3\left(\frac{V_2}{V_1}\right)^{k-1} = T_3 \quad \frac{1}{\left(\frac{V_1}{V_2}\right)^{k-1}} = \frac{T_3}{(r_k)^{k-1}} = \frac{2960}{12^{0.3}}$$

$\text{Log}\left(\frac{2960}{12^{0.3}}\right) = \text{Log } 2960 - 0.3 \log 12$

$\text{Log } 2960 = 3.471$

$0.3 \log 12 = 0.324$

subtracting 3.147

Antilog $3.147 = 1403$

$\therefore \quad T_4 = 1403$ °R

$Q_R = 0.225(1403-540) = 194$ Btu/lb air.

c) Mechanical Work produced $W = Q_A - Q_R$

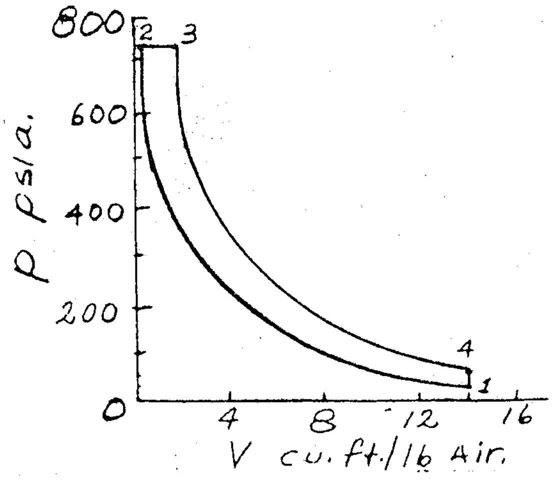
$W = 410 - 194 = 216$ Btu/lb air.

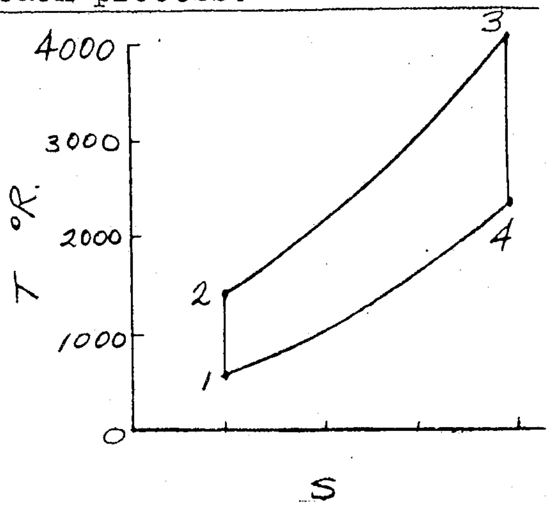
Sample Problem # 4

A diesel cycle takes in air at 100°F and 14.7 psia and works with a compression ratio of 20 and cutoff ratio $r_c = V_3/V_2 = 3$. Assuming $k = 1.3$ and $c_v = 0.225$, calculate the state of the air at the end of each process and plot the cycle on a p-v graph. Calculate a) the heat rejected per lb. b) work output per lb. c) overall thermal efficiency of the cycle.

Solution:

Calculating the state of air at end of each process.





Values which are given:

$r_k = \frac{V_1}{V_2} = 20 \quad r_c = \frac{V_3}{V_2} = 3 \quad k = 1.3 \quad c_v = 0.225$

$T_1 = (100 + 460) = 560°R \quad P_1 = 14.7$ psia

Calculate V_1 : from the gas laws we know that $P_1V_1 = RT_1$

The constant R varies slightly fro different conditons however, for this particular problem we will consider R = 53.3

$$\therefore \quad V_1 = V_4 = \frac{RT_1}{P_1} = \frac{53.3 \times (560)}{} = 14.1 \text{ cu. ft/lb. air}$$

Note that P_1 has to be multiplied by 144 to change it to lb/sq ft. air so the units will be consistent in the equation.

Calculate V_2 : $V_2 = \frac{V_1}{r_k} = \frac{14.1}{20} = 0.7$ cu.ft./lb.

Calculate T_2 : equation (8) in the previous lesson tells us that

$$T_2 = T_1 \, r_k^{k-1}$$

$$T_2 = 560(20)^{1.3-1} = 560(20)^{0.3}$$

$\text{Log } 560(20)^{0.3} = \log 560 + 0.3 \log 20.$

Log 560	=		2.748
0.3 Log 20	=	0.3 x 1.301	0.390
		adding	3.138

Antilog 3.138 = 1374 $\therefore T_2 = 1374°R$

Calculate P_2 : $P_2 \, V_2 = RT_2$

$$\therefore \quad P_2 = \frac{RT_2}{V_2} = \frac{53.3(1374)}{0.7} = 104{,}800 \text{ lb/sq.ft.abs.}$$

$$= \frac{104{,}800}{144} = 727 \text{ psia}$$

Calculate V_3

$$V_3 = V_2 \times 3 = 0.7 \times 3 = 2.1 \text{ cu. ft./lb.}$$

Calculate P_3

From the definition of a diesel cycle it follows that:

$P_3 = P_2 = 104{,}800$ lbs/sq. ft. abs.

Calculate T_3

The process from 2-3 is a constant pressure process and there Charle's Law applies from which we get:

$$\frac{T_3}{T_2} = \frac{V_3}{V_2} = r_c = 3$$

$$\therefore \quad T_3 = T_2 \times 3 = 1374 \times 3 = 4{,}122°R$$

Find V_4

From the P_v diagram it is apparent that $V_4 = V_1 = 14.1$ cu.ft./lb air.

Calculate P_4

The process from 3-4 is an adiabatic process and, therefore, as we learned in the lesson on adiabatic compression and expansion:

$$\frac{P_4}{P_3} = \left(\frac{V_3}{V_4}\right)^k$$

$$P_4 = P_3 \left(\frac{V_3}{V_4}\right)^k = 104{,}800 \quad \left(\frac{2.1}{14.1}\right)^{1.3}$$

$$\text{Log } 104{,}800\left(\frac{2.1}{14.1}\right)^{1.3} = \text{Log } 104{,}800 + 1.3 \log 2.1 - 1.3 \log 14.1$$

$$\begin{array}{llrl}
\text{Log } 104{,}800 & = & & 5.020 \\
1.3 \log 2.1 & = 1.3 \times 0.322 & = & \underline{0.419} \\
 & \text{adding} & = & 5.439 \\
1.3 \log 14.1 & = 1.3 \times 1.149 & = & \underline{1.494} \\
 & \text{subtracting} & & 3.945
\end{array}$$

Antilog 3.945 = 8811

$\therefore$ P_4 = 8,811 lb/sq.ft.abs.

= 61.2 psia.

Calculate T_4

Since we already know P_4 and V_4 we can use the equation $P_4V_4 = RT_4$;

$$T_4 = \frac{P_4\ V_4}{R} = 8{,}811 \times \frac{14.1}{53.3} = 2335°R$$

Summarizing we have:

P_1 = 14.7 psia V_1 = 14.1 cu.ft./lb. T_1 = 560°R	P_2= 727 psia V_2= 0.7 cu.ft./lb. T_2= 1374°R
P_3 = 727 psia V_3 = 2.1 cu. ft./lb. T_3 = 4,122°R	P_4 = 61.2 psia V_4 = 14.1 cu.ft./lb. T_4 = 2,335°R

a) Heat Rejected per lb.

$Q_R = c_v(T_4 - T_1) = 0.225(2{,}335 - 560) = 399$ Btu/lb.

b) Work output per lb.

$$W = Q_A - Q_R = c_P(T_3 - T_2) - 399$$

$$k = \frac{c_P}{c_v} \qquad c_P = k\ c_v = 1.3 \times .225$$

$\therefore \quad W = 1.3 \times .225(4{,}122 - 1374) - 399 = 805 - 399$

$W = 406$ Btu/lb air.

c) Overall Thermall Efficiency

From equation (13) of the previous lesson we have:

$$e = 1 - \frac{1}{r_k^{\,k-1}} \left[\frac{r_c^{\,k} - 1}{k(r_c - 1)} \right] = 1 - \frac{1}{(20)^{0.3}} \left[\frac{3^{1.3} - 1}{1.3(3-1)} \right]$$

$\text{Log } 3^{1.3} = 1.3 \log 3 = 1.3 \times 0.477 = 0.620$

Antilog $0.620 = 4.16$

$$\therefore \quad e = 1 - \frac{1}{20^{0.3}} \left[\frac{4.16 - 1}{1.3 \times 2} \right] = 1 - \frac{1}{20^{0.3}} \left[\frac{3.16}{2.6} \right] = 1 - \frac{3.16}{(20)^{0.3} \times 2.6}$$

$$\text{Log } \frac{3.16}{20^{0.3} \times 2.6} = \log 3.16 - 0.3 \log 20 - \log 2.6$$

$$\begin{array}{llll}
\text{Log } 3.16 & = & 0.500 & = 10.500-10 \\
0.3 \log 20 = 0.3 \times 1.301 & = & 0.390 & \\
\text{Log } 2.6 = & & \underline{0.415} & \\
 & \text{Adding} & & \underline{0.805} \\
 & \text{Subtracting} & & 9.695-10
\end{array}$$

Antilog $9.695 - 10 = 0.495$

$\therefore \quad e = 1 - 0.495 = .505$

$e = 0.505 \times 100 = 50.5\%$

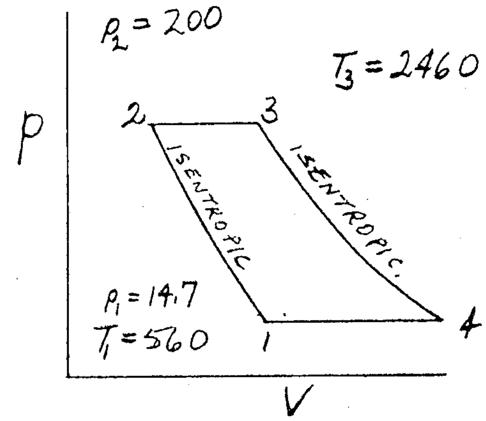
Sample Problem # 5

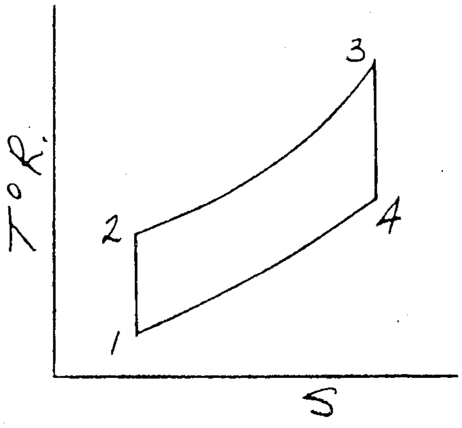
At the beginning of compression in an air-standard Brayton cycle the pressure is 14.2 psia and the temperature 100°F. At the end of compression the pressure is 200 psia and the temperature at the end of compression, the pressure is 200 psia, and the temperature at the end of combustion is 2000°F. a) How much heat is added? b) How much heat is rejected? c) What is net work per lb. of air passing through the cycle? d) What is the thermal efficiency? e) What is the compression ratio?

Items Given: P_1 = 14.2 psia. P_2 = 200 psia

T_1 = (100+460)=560°R T_3 = (2000+460)=2460°R

Assume: $k = 1.4$; $c_p = 0.24$





a) <u>How much heat is added?</u>

$$Q_A = c_p(T_3 - T_2)$$

We need to calculate T_2. From the discussion on the Brayton cycle in the previous lesson we know that:

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{k-1}{k}} \qquad \therefore \quad T_2 = \frac{T_1}{\left(\frac{P_1}{P_2}\right)^{\frac{k-1}{k}}}$$

$$T_2 = \frac{560}{\left(\frac{14.7}{200}\right)^{\frac{1.4-1}{1.4}}} = \frac{560}{\frac{(14.7)^{.286}}{(200\ )^{.286}}} = 560 \text{ x } \frac{(200)^{.286}}{(14.7)^{.286}}$$

Log 560 = 2.748

.286 Log 200= .286 x 2.301 = 0.658

Adding 3.406

.286 Log 14.7 = .286 x 1.167 = 0.334

Subtracting 3.072

Antilog 3.072 = 1,181 $\therefore$ $T_2 = 1{,}181°R$

$Q_A = 0.24\ (2460 - 1181) = 307$ Btu/lb air.

b) How much heat is rejected

$$Q_R = c_P(T_4 - T_1)$$

The only unknown here is T_4, which we can get from the equation given in the previous lesson:

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}}$$

$P_4 = P_1 = 14.7$ psia.

$P_3 = P_2 = 200$ psia.

$$\therefore\quad T_4 = T_3\left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = 2460\left(\frac{14.7}{200}\right)^{\frac{1.4-1}{1.4}} = 2460\left(\frac{14.7}{200}\right)^{.286}$$

$$\begin{array}{lllll}
\text{Log } 2460 & = & & & 3.391 \\
.286 \text{ Log } 14.7 & = & .286 \times 1.167 & = & \underline{0.333} \\
 & & \text{Adding} & = & \underline{3.724} \\
.286 \text{ log } 200 & = & .286 \times 2.301 & = & \underline{0.658} \\
 & & \text{Subtracting} & = & 3.066
\end{array}$$

Antilog 3.066 = 1164

$$Q_R = 0.24(1164 - 560) = 145 \text{ Btu/lb. air}$$

c) Net Work Done

$$W = Q_A - Q_R = 307-145 = 162 \text{ Btu/lb.}$$

d) Thermal Efficiency

$$e = 1 - \frac{1}{r_k^{\,k-1}} =$$

The unknown is $r_k = \frac{V_1}{V_2}$. From the previous lesson on the Brayton cycle we know that $\frac{T_1}{T_2} = \frac{1}{r_k}{}^{k-1}$

$$\therefore \quad e = 1 - \frac{T_1}{T_2} = 1 - \frac{560}{1181} = 1 - .474 = .526$$

$$e = 0.526 \times 100 = 52.6\ \%$$

e) Compression Ratio r_k

Since: $$\frac{T_1}{T_2} = \frac{1}{r_k^{\,k-1}} = \left(\frac{1}{r_k}\right)^{k-1}$$

$$\left(\frac{T_1}{T_2}\right)^{\frac{1}{k-1}} = \frac{1}{r_k} \text{ and } r_k = \frac{1}{\left(\frac{T_1}{T_2}\right)^{\frac{1}{k-1}}} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{k-1}}$$

$$\therefore \quad r_k = \left(\frac{1181}{560}\right)^{\frac{1}{1.4-1}} = (2.11)^{2.5}$$

$\text{Log } (2.11)^{2.5} = 2.5 \log 2.11 = 2.5 \times 0.324 = 0.810$

Antilog 0.810 = 6.45

$$\therefore \quad \text{Compression Ratio } r_k = \frac{(V_1)}{(V_2)} = 6.45$$

Conclusion

The student should be familiar with the equations used in the lesson and be able to transpose them to suit a particular problem but they need not be memorized. On a check-out a list of equations covered in this course will be given.

D. Dueck

NUCLEAR ELECTRIC G.S. TECHNICAL TRAINING COURSE

2 - Science Fundamentals - T.T.2
5 - Heat & Thermodynamics
8 - Problems on Heat Engine Cycles
A - Assignment

1. Find the thermal efficiency of an Otto cycle with a compression ratio of 5 and a specific heat ratio of 1.3.

2. If maximum combustion temperature in an Otto cycle is 2500°F and the engine has a compression ratio of 12, what will be the maximum cylinder pressure? Assume an air temperature of 80°F, and 14.7 psia.

3. For an air-standard Diesel cycle which uses 1 lb. air, $T_1 = 110°F$ $T_2 = 1300°F$, and 250 Btu of heat is added during combustion.
   a) the heat rejected; b) compression ratio.
   b) thermal efficiency of this cycle.

4. In an air-standard Brayton cycle, the absolute pressure at discharge from the compressor is eight times that at the suction end of this unit. What is its thermal efficiency?

NUCLEAR ELECTRIC G.S. TECHNICAL TRAINING COURSE

2 - Science Fundamentals - T.T.2

5 - Heat and Thermodynamics

-9 - Definition of Symbols used in T.T.2 Level

A = area, sq. in. or sq. ft.
B.H.P. = brake horsepower
c = specific heat, Btu/lb/°F.
c_p = specific heat at constant pressure, Btu/lb/°F.
c_v = specific heat at constant volume, Btu/lb/°F.
C = a constant as in $PV^k = C$
e = efficiency
E_M = engine mechanical efficiency
E_{th} = engine thermal efficiency
F = force, lbs.
F.W. = flow work, ft. lbs.
g = acceleration of gravity 32.2 ft/sec/sec.
h = enthalpy, Btu/lb.
H = enthalpy for w lbs, Btu.
H.H.V. = Higher heating value of fuel, Btu/lb.
I.H.P. = engine indicated horsepower
J = Joule's constant; 778 ft.lbs. = 1 Btu
$k = c_p/c_v$ = specific heat ratio
°K = Kelvin absolute temperature scale (273 + °Centigrade)
K.E. = kinetic energy, ft. lbs.
ℓ = distance, ft.
p = pressure, psi or lb/ft^2 as applicable
P = pressure, psi or lb/ft^2 as applicable
Q = amount of heat transferred, Btu.
Q_A = heat added, Btu.
Q_R = heat rejected, Btu.
r_c = fuel cut-off ratio = $^{v}3/v_2$

r_k = adiabatic compression ratio = V_1/V_2
°R = Rankine absolute temperature scale (460 + °Fahrenheit)
s = entropy for 1 lb. of a substance.
s_f = entropy of saturated liquid
s_{f_g} = entropy of water and steam mixture
s_g = entropy of saturated steam
S = entropy of w lb. of a substance
t = temperature °F
T = absolute temperature
u = internal energy, Btu/lb.
U = internal energy for w lbs., Btu.
v_s = velocity ft/sec.
v = volume, cu. ft/lb.
V = volume, for w lbs., cu. ft.
w = weight of substance lbs.
W = work done, ft. lbs., or Btu.
W_f = engine fuel consumption, lbs/hr.
x = % steam quality
y = % moisture in steam
z = elevation above a datum line, ft.

Constants and Conversion Factors

1 horsepower = 33,000 ft. lbs/min.
= 550 ft. lbs/sec.
= 746 watts
= 2545 Btu/hr. (1 Btu = 778 ft.lbs)
1 kilowatt = 1000 watts = 1.34 H.P.
= 3412 Btu/hr

For air at 80°F $c_p = 0.24$
$c_v = 0.17$
$k = 1.4$

D. Dueck.

NUCLEAR ELECTRIC G.S. TECHNICAL TRAINING COURSE

2 - Science Fundamentals - T.T.2

5 - Heat and Thermodynamics

-10 - Summary of Equations Applicable to T.T.2 Level

Work: $W = F \times \ell$ ft-lbs.

Engine Mechanical Efficiency: $E_M = \frac{B.H.P.}{I.H.P.} \times 100\%$

Engine Thermal Efficiency: $E_{th} = \frac{B.H.P.}{W_f \times H.H.V.} \times 100\%$

General Energy Equation:

Energy in = Energy out

$$P.E._1 + K.E._1 + U_1 + F.W._1 + Q = P.E._2 + K.E._2 + U_2 + F.W._2 + W$$

or

$$\frac{wz_1}{J} + \frac{wv_{s1}^2}{2gJ} + U_1 + \frac{P_1V_1}{J} + Q = \frac{wz_2}{J} + \frac{wv_{s2}^2}{2gJ} + U_2 + \frac{P_2V_2}{J} + W$$

$$P.E. = \frac{w.z}{J}; \quad K.E. = \frac{wv_s^2}{2gJ}; \quad F.W. = \frac{pV}{J}$$

$h = u + \frac{pv}{J}$ for one lb.

$H = U + \frac{pV}{J}$ for w lbs.

$$\frac{wv_{s1}^2}{2gJ} + H_1 + Q = \frac{wv_{s2}^2}{2gJ} + H_2 + W$$

Internal Energy: $u = cT$ Btu/lb.

$W = c(T_B - T_A)\ J$ ft-lbs/lb.

$k = \frac{c_p}{c_v}$

Reversible Adiabatic Process: $PV^k = C$

$$\frac{T_A}{T_B} = \left(\frac{V_B}{V_A}\right)^{k-1} = \left(\frac{P_A}{P_B}\right)^{\frac{k-1}{k}} \quad \text{or} \quad \frac{P_A}{P_B} = \left(\frac{V_B}{V_A}\right)^{k}$$

Isothermal Expansion or Compression Process:

$$W = (P_B{''}V_B{''})\ 2.3 \log_{10}\left(\frac{V_A}{V_B}\right) \text{ ft-lbs.}$$

Efficiency of a Heat Engine:

$$e = \frac{Q_{in} - Q_R}{Q_{in}} \times 100\%$$

Entropy:

$$\Delta s = s_2 - s_1$$

$$\Delta s = \frac{Q}{T}$$

% Quality of Steam: $s = s_f + \frac{x}{100} \cdot s_{fg}$

% Moisture in Steam: $s = s_g - \frac{y}{100} \cdot s_{fg}$

Otto Cycle:

$$Q_A = wc_v(T_3 - T_2) \text{ Btu.}$$

$$Q_R = wc_v(T_4 - T_1) \text{ Btu.}$$

$$W = Q_A - Q_R = wc_v(T_3 - T_2) - wc_v(T_4 - T_1) \text{ Btu.}$$

$$r_k = V_1/V_2 = (T_2/T_1)^{1/k-1} = \left(\frac{p_2}{p_1}\right)^{1/k}$$

$$e = 1 - \frac{1}{(r_k)^{k-1}}$$

Diesel Cycle:

$$Q_A = wc_p(T_3 - T_2) \text{ Btu.}$$

$$Q_R = wc_v(T_4 - T_1) \text{ Btu.}$$

$$W = wc_p(T_3 - T_2) - wc_v(T_4 - T_1) \text{ Btu.}$$

$r_c = V_3/V_2 = \frac{T_3}{T_2}$ = fuel cut-off ratio.

$$e = 1 - \frac{1}{r_k^{k-1}}\left[\frac{r_c^{k} - 1}{k(r_c - 1)}\right]$$

Brayton Cycle:

$Q_A = wc_p\ (T_3 - T_2).$

$Q_R = wc_p\ (T_4 - T_1).$

$W = wc_p\ (T_3 - T_4 - T_2 + T_1)$

$$e = 1 - \frac{1}{r_k^{k-1}}$$