Transient Diffusion in a solid

$$\rho c \frac{\partial T}{\partial t} = g''' + \nabla \cdot k \nabla T$$

ρ: density; c: heat capacity; g''': heat source per unit volume

$$\frac{\partial T}{\partial t} \rightarrow \frac{T_i^{t+\Delta t} - T_i^{t}}{\Delta t}$$

$$\nabla \cdot k \nabla T = k \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) \text{ in 1-D with constant } k$$

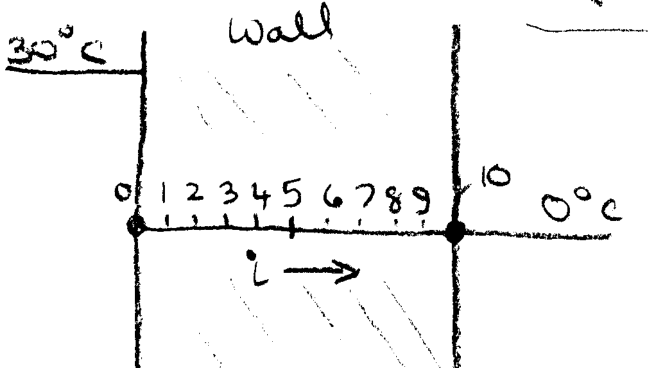
$$k \left[\frac{\left.\frac{\partial T}{\partial x}\right|_{i+\frac{1}{2}} - \left.\frac{\partial T}{\partial x}\right|_{i-\frac{1}{2}}}{\Delta x} \right]$$

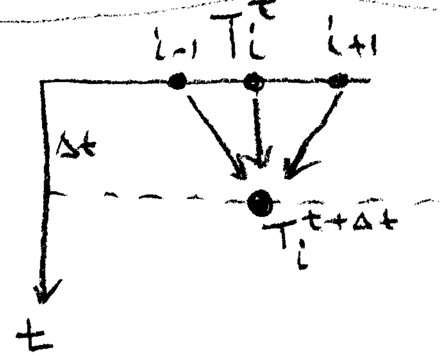
$$= k \left[\frac{\left(\frac{T_{i+1} - T_i}{\Delta x}\right) - \left(\frac{T_i - T_{i-1}}{\Delta x}\right)}{\Delta x} \right]$$

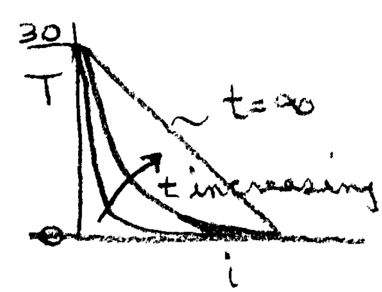
$$= \frac{k\left(T_{i+1} - 2T_i - T_{i-1}\right)}{\Delta x^2}$$

$$\therefore \rho c \frac{\left(T_i^{t+\Delta t} - T_i^{t}\right)}{\Delta t} = g''' + \frac{k}{\Delta x^2}\left(T_{i+1}^{t} - 2T_i^{t} + T_{i-1}^{t}\right)$$

$$\therefore T_i^{t+\Delta t} = T_i^{t} + \frac{\Delta t}{\rho c}\left[g''' + \frac{k}{\Delta x^2}\left(T_{i+1}^{t} - 2T_i^{t} + T_{i-1}^{t}\right)\right]$$







```c
#include <stdio.h>
#include <math.h>

#define PI 3.14159

//------------------------------------------------------------------------------
int main()
{
    float T[101], dx, L, t=0.0, dt, tfin, rho, c, k, Q=0.0, Tleft, Tright;
    int i, N;
    FILE *fp_in, *fp_out;

    if (( fp_in = fopen("input.txt","r")) == NULL)
    {
        /* File did not exist. Print message */
        printf("\nCould not open the input file\n");
        return 1;
    }

    if (( fp_out = fopen("results.txt","w")) == NULL)
    {
        /* File did not exist. Print message */
        printf("\nCould not open the output file\n");
        return 1;
    }
    fscanf(fp_in, "%d %f %f %f %f %f %f %f %f %f", &N, &L, &c, &rho, &k, &Q,
      &tfin, &dt, &Tleft, &Tright);


    //Set grid
    dx=L/(N-1);
    //Set initial temperatures
    for (i=0;i<N;i++)T[i]=Tright;
    T[0]=Tleft;

    //print initial output
    fprintf(fp_out, "#Temperature at grid points over time");
    //fprintf(fp_out, "\n# %f",t);
    //for (i=0;i<N;i++)fprintf(fp_out," %f",i*dx);
    //fprintf(fp_out, "\n %f",t);
    for (i=0;i<N;i++) fprintf(fp_out, "\n %f %f %f",t, i*dx, T[i]);


    //loop in time
    while (t<tfin)
    {
        //loop in space
        for (i=1;i<N-1;i++)
        {
            T[i] = T[i] + dt/rho/c*(Q + k*(T[i+1]-2.0*T[i]+T[i-1])/dx/dx);
        }
        t=t+dt;
        for (i=0;i<N;i++)fprintf(fp_out, "\n %f %f %f",t, i*dx, T[i]);
    }
    fclose(fp_in);
    fclose(fp_out);
    printf("All done");
    return 0;
}
```

Temperature Distribution in a Wall

$tfin := 15.0$

$dt := 0.1$

$L := 10$

$N := 21$

$dx := \frac{L}{N-1}$ $\qquad dx = 0.5$

$rho := 1.0$

$c := 1.0$

$k := 1.0$

$Q := 0.0$

$Tleft := 30.0 \qquad Tright := 0.0$

$numiter := \frac{tfin}{dt} \qquad numiter = 150 \qquad i := 0..N-1$

$input := (N\ \ L\ \ c\ \ rho\ \ k\ \ Q\ \ tfin\ \ dt\ \ Tleft\ \ Tright)$

input =

	0	1	2	3	4	5	6	7	8	9
0	21	10	1	1	1	0	15	0.1	30	0

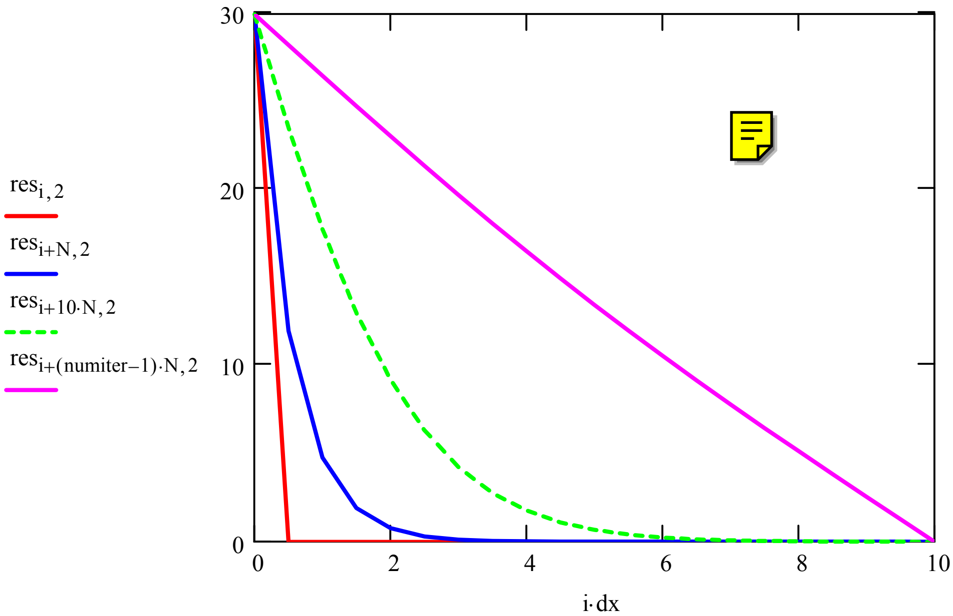
D:\..\input.txt

input

results :=

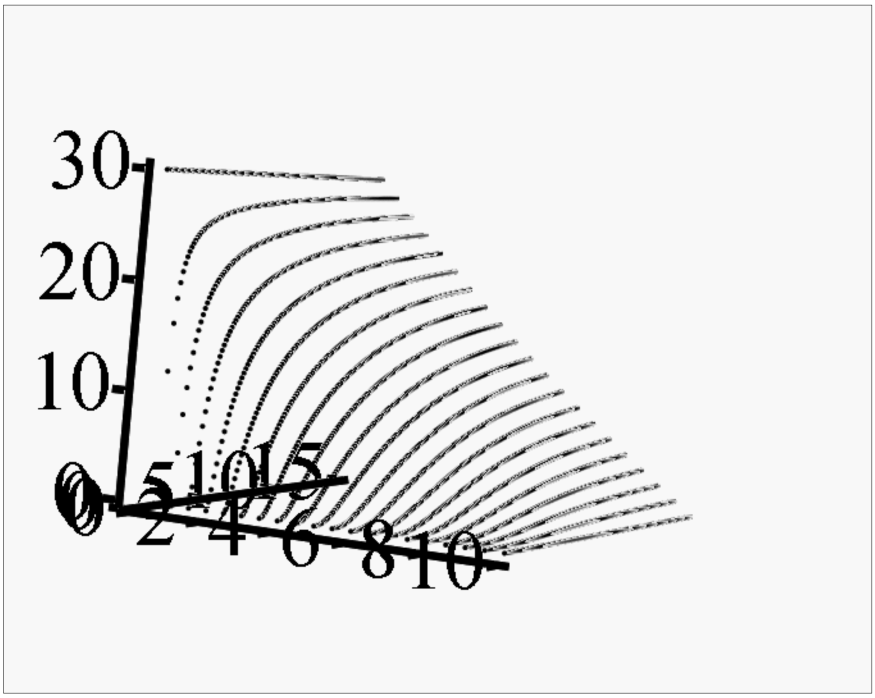
D:\..\results.txt

$res := submatrix(results, 1, rows(results) - 1, 0, cols(results) - 1)$



$N = 21 \qquad dt = 0.1$

$t := res^{\langle 0 \rangle} \qquad x := res^{\langle 1 \rangle} \qquad T := res^{\langle 2 \rangle}$



(x, t, T)

results =

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0		
1	0	0	30	0	0	0		
2	0	0.5	0	0	0	0		
3	0	1	0	0	0	0		
4	0	1.5	0	0	0	0		
5	0	2	0	0	0	0		
6	0	2.5	0	0	0	0		
7	0	3	0	0	0	0		
8	0	3.5	0	0	0	0		
9	0	4	0	0	0	0		
10	0	4.5	0	0	0	0		
11	0	5	0	0	0	0		
12	0	5.5	0	0	0	0		
13	0	6	0	0	0	0		
14	0	6.5	0	0	0	0		
15	0	7	0	0	0	0		

	0	1	2	3	4	5
0	0	0	30	0	0	0