ENGINEERING 2C03

DAY CLASS — Dr. Wm. Garland

DURATION: 3 hours

McMASTER UNIVERSITY FINAL EXAMINATION — April 20, 1999

Special Instructions:

1. Closed Book. All calculators and up to 6 single sided 8 ½" by 11" crib sheets are permitted.
2. Do all questions.
3. The value of each part is as indicated. TOTAL Value: 100 marks

THIS EXAMINATION PAPER INCLUDES 2 PAGES AND 10 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

1. [10 marks] A certain toaster has a heating element made of Nichrome resistance wire. When first connected to a 120V voltage source (and the wire is at a temperature of 20.0 °C), the initial current is 1.80 A but begins to decrease as the resistive element heats up. When the toaster has reached its final operating temperature, the current has dropped to 1.53 A. The temperature coefficient of resistivity, α, for Nichrome is 0.4×10^{-3} / °C.
   a. (3 marks) Find the power the toaster consumes when it is at its operating temperature.
   b. (7 marks) What is the final temperature of the heating element?

Solution

(a) $P = VI = (120\ \text{V})(1.53\ \text{A}) = 184\ \text{W}$ ◊

(b) The resistance at 20°C is $R_0 = \dfrac{V}{I} = \dfrac{120\ \text{V}}{1.8\ \text{A}} = 66.7\ \Omega$

At operating temperature, $R = \dfrac{120\ \text{V}}{1.53\ \text{A}} = 78.4\ \Omega$

Neglecting thermal expansion, we have

$$R = \frac{\rho L}{A} = \frac{\rho_0(1+\alpha(T-T_0))L}{A} = R_0(1+\alpha(T-T_0))$$

$$T = T_0 + \frac{R/R_0 - 1}{\alpha} = 20°\text{C} + \frac{78.4\ \Omega/66.7\ \Omega - 1}{0.4\times10^{-3}/\text{C}°} = 461°\text{C} \quad ◊$$

2. [10 marks] In figure 1, find
   a. the current in the 20 Ω resistor and
   b. the potential difference between points *a* and *b*.

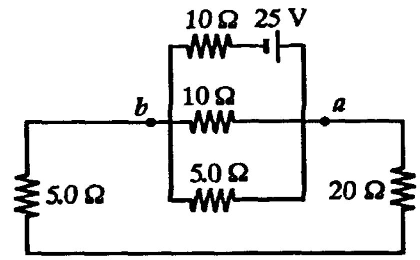


Figure 1

Solution If we turn the given diagram on its side, we find that it is the same as figure (a). The 20-Ω and 5.0-Ω resistors are in series, so the first reduction is as shown in (b). In addition, since the 10-Ω, 5.0-Ω, and 25-Ω resistors are then in parallel, we can solve for their equivalent resistance as:

$$R_{eq} = \frac{1}{\left(\frac{1}{10\ \Omega} + \frac{1}{5.0\ \Omega} + \frac{1}{25\ \Omega}\right)} = 2.94\ \Omega.$$

This is shown in figure (c), which in turn reduces to the circuit shown in (d).

Next, we work backwards through the pictures, applying $I = V/R$ and $V = IR$. The 12.94-Ω resistor is connected across 25-V, so the current through the voltage source in every diagram is

$$I = \frac{V}{R} = \frac{25\ \text{V}}{12.94\ \Omega} = 1.93\ \text{A}$$

In figure (c), this 1.93 A goes through the 2.94-Ω equivalent resistor to give a voltage drop of:

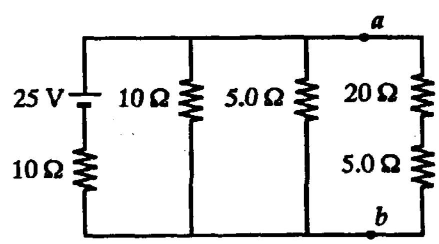
$$V = IR = (1.93\ \text{A})(2.94\ \Omega) = 5.68\ \text{V}$$

From figure (b), we see that this voltage drop is the same across V_{ab}, the 10-Ω resistor, and the 5.0-Ω resistor.

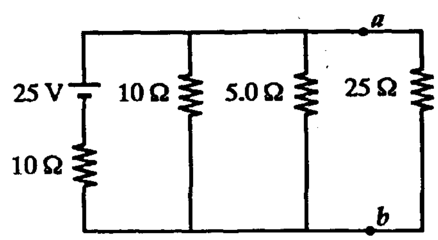
(b) Therefore, $V_{ab} = 5.7\ \text{V}$ ◊

Since the current through the 20-Ω resistor is also the current through the 25-Ω line *ab*,

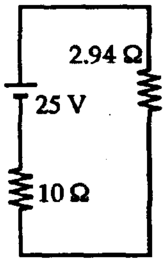
(a) $I = \frac{V_{ab}}{R_{ab}} = \frac{5.68\ \text{V}}{25\ \Omega} = 0.227\ \text{A}$ ◊



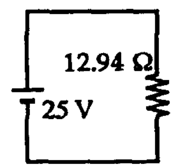
(a)



(b)



(c)



(d)

3. [10 marks] An inductor (L =400 mH), a capacitor (C =4.43 µF), and a resistor (R = 500 Ω) are connected in series. A 50.0 Hz ac generator produces a peak current of 250 mA in the circuit.

a. Calculate the required peak voltage *Vmax* .

b. Determine the angle by which the current leads or lags the applied voltage.

Solution

(a) **We first find the impedence of the capacitor and the inductor:**

$$X_L = \omega L = 2\pi(50.0\text{ Hz})(400\times10^{-3}\text{ H}) = 126\ \Omega$$

and $$X_C = \frac{1}{\omega C} = \frac{1}{(2\pi)(50.0\text{ Hz})(4.43\times10^{-6}\text{ F})} = 719\ \Omega$$

Then, we substitute these values into the equation for a series *LRC* circuit:

$$V_{\text{max}} = I_{\text{max}}Z = I_{\text{max}}\sqrt{R^2 + (X_L - X_C)^2}$$

Thus, $$Z = \sqrt{(500\ \Omega)^2 + (126\ \Omega - 719\ \Omega)^2} = 776\ \Omega$$

and $$V_{\text{max}} = I_{\text{max}}Z = (0.25\text{ A})(776\ \Omega) = 194\text{ V} \quad \Diamond$$

(b) $$\tan\phi = \frac{X_L - X_C}{R}$$

$$\phi = \tan^{-1}\left(\frac{126 - 719}{500}\right) = -49.9° \quad \Diamond$$

The current *leads* the voltage by 49.9°.

4. [10 marks] Determine the current (magnitude and direction) in each branch of figure 2.

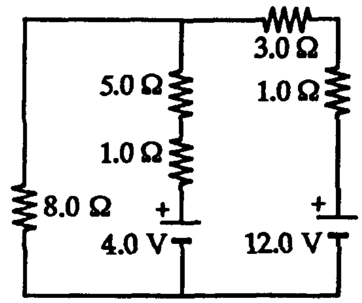


Figure 2

Solution First, we arbitrarily define the initial current directions and names, as shown in the figure below.

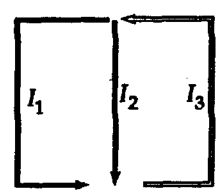
The current rule then says that $I_3 = I_1 + I_2$. (1)

The voltage rule says that in a clockwise trip around the left-hand loop,

$$+I_1(8.0\ \Omega) - I_2(5.0\ \Omega) - I_2(1.0\ \Omega) - 4.0\ \text{V} = 0 \qquad (2)$$

and in a clockwise trip around the right-hand mesh,

$$4.0\ \text{V} + I_2(1.0\ \Omega + 5.0\ \Omega) + I_3(3.0\ \Omega + 1.0\ \Omega) - 12.0\ \text{V} = 0 \qquad (3)$$



Solving by substitution rather than by determinants has the advantage that (just as when a cat has kittens) the answers, after the first, come out much more easily. Thus we substitute $(I_1 + I_2)$ for I_3, and reduce our three-equations to:

$$\left\{\begin{array}{l}(8.0\ \Omega)I_1 - (6.0\ \Omega)I_2 - 4.0\ \text{V} = 0 \\ \\ 4.0\ \text{V} + (6.0\ \Omega)I_2 + (4.0\ \Omega)(I_1 + I_2) - 12.0\ \text{V} = 0\end{array}\right\} \quad \text{or} \quad \left\{\begin{array}{l}I_2 = \dfrac{(8.0\ \Omega)I_1 - 4.0\ \text{V}}{6.0\ \Omega} \\ \\ -8.0\ \text{V} + (4.0\ \Omega)I_1 + (10.0\ \Omega)I_2 = 0\end{array}\right\}$$

Again substituting for I_2, we end up with: $-8.0\ \text{V} + (4.0\ \Omega)I_1 + \frac{10.0}{6.0}((8.0\ \Omega)I_1 - 4.0\ \text{V}) = 0$

$(17.3\ \Omega)I_1 - (14.7\ \text{V}) = 0,$

and I_1 0.85 A down in the 8 Ω resistor ◊

Thus, $I_2 = \dfrac{(8.0\ \Omega)(0.846\ \text{A}) - 4.0\ \text{V}}{6.0\ \Omega} = 0.46\ \text{A}$ down in middle branch ◊

$I_3 = 0.85\ \text{A} + 0.46\ \text{A} = 1.3\ \text{A}$ up in the right-hand branch ◊

It is a good idea to check the math by substituting back into the voltage-rule equations. If you are new at this, it is worth your time to solve the whole problem again, by taking counterclockwise trips around the loops.

5. [5 marks] The concrete sections of a certain superhighway are designed to have a length of 25.0 m. The sections are poured and cured at 10 °C. What minimum spacing should the engineer leave between the sections to eliminate buckling stress if the concrete is to reach a temperature of 50 °C? The linear expansion coefficient, α, for concrete is 12 x 10^{-6} / °C.

Solution:

The concrete will expand $\Delta L = \alpha \Delta T\, L$ = 12 x 10-6 / °C x 40° C x 25.0 m = 0.012 m = <u>1.2 cm.</u>

6. [10 marks] Around a crater formed by an iron meteorite, 75.0 kg of rock has melted under the impact of the meteorite. The rock has a specific heat of 0.800 kcal/kg °C, a melting point of 500°C, and a latent heat of fusion of 48.0 kcal/kg. The original temperature of the ground was 0.0°C. If the meteorite hit the ground while moving at 600 m/s, what is the minimum mass of the meteorite? Assume no heat loss to the surrounding unmelted rock or the atmosphere during the impact. Disregard the heat capacity of the meteorite. Recall that 1 calorie = 4.186 J.

Solution Assume that the total kinetic energy of the meteorite is converted into thermal energy of the rock.

$$K = Q_1 + Q_2$$

where $Q_1 = m_{\text{rock}} c_{\text{rock}} \Delta T_{\text{rock}}$ (to increase temperature to melting point)

and $Q_2 = m_{\text{rock}} L_f$ (to melt the rock)

$$\tfrac{1}{2} m v^2_{\text{meteorite}} = m_{\text{rock}} \left(c_{\text{rock}} \Delta T_{\text{rock}} + L_f \right)$$

$$m_{\text{meteorite}} = \frac{2 m_{\text{rock}}}{v^2} \left(c_{\text{rock}} \Delta T_{\text{rock}} + L_f \right)$$

Use $c_{\text{rock}} = 0.800 \ \dfrac{\text{kcal}}{\text{kg}\cdot{}^\circ\text{C}} = 3.349 \ \dfrac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{C}}$ and $L_f = 48.0 \ \dfrac{\text{kcal}}{\text{kg}} = 201 \ \dfrac{\text{kJ}}{\text{kg}}$

$$m_{\text{meteorite}} = \frac{(2)(75.0\text{ kg})}{(600\text{ m/s})^2} \left[\left(3.349 \times 10^3 \ \frac{\text{J}}{\text{kg}\cdot{}^\circ\text{C}} \right) (500^\circ\text{C}) + 2.01 \times 10^5 \ \frac{\text{J}}{\text{kg}} \right]$$

$$m_{\text{meteorite}} = 781\text{ kg} \quad \Diamond$$

7. [10 marks] A Thermopane window of area 6.0 m^2 is constructed of two layers of glass, each 4.0 mm thick separated by an air space of 5.0 mm. If the inside is at 20°C and the outside is at -30 °C, what is the heat loss through the window? The thermal conductivity of air is 0.0234 W/m°C and that for glass is 0.80 W/m°C. Ignore the insulation value of the surrounding air.

Solution:

We have:

$$\text{Heat tranfer rate} = \text{H Watts} = \frac{\text{Area}\ \Delta T}{\sum_i \Delta x/k_i}$$

$$= \frac{6\text{m}^2 \text{ x } 50°\text{C}}{0.005/0.0234 + 2 \text{ x } 0.004/.8}$$

$$= 1.34 \text{ kW}$$

8. [10 marks] A house loses thermal energy through the exterior walls and roof at a rate of 5000 J/s = 5.00 kW when the interior temperature is 22°C and the outside temperature is -5°C. Calculate the electric power required to maintain the interior temperature at 22°C for the following two cases:
   a. (3 marks) The electric power is used in electric resistance heaters (which convert all of the electricity supplied to thermal energy).
   b. (7 marks) The electric power is used to operate the compressor of a heat pump (which has a coefficient of performance equal to 60% of the Carnot cycle value).

Solution

(a) $P_{\text{electric}} = \frac{\Delta E}{\Delta t}$ **and if all of the electricity is converted into thermal energy,** $\Delta E = \Delta Q$.

Therefore, $P_{\text{electric}} = \frac{\Delta Q}{\Delta t} = 5000 \text{ W}$ ◊

(b) **For a heat pump,** $(\text{COP})_{\text{Carnot}} = \frac{T_h}{\Delta T} = \frac{295}{27} = 10.92$

Actual COP $= (0.6)(10.92) = 6.55 = \frac{Q_h}{W} = \frac{Q_h/t}{W/t}$

Therefore, to bring 5000 W of heat into the house only requires input power

$$\frac{W}{t} = \frac{Q_h/t}{\text{COP}} = \frac{5000 \text{ W}}{6.56} = 763 \text{ W} \quad ◊$$

9. [15 marks] The circuit in figure 3 has been connected for a long time.
   a. (8 marks) What is the voltage across the capacitor?
   b. (7 marks) If the battery is disconnected, how long does it take the capacitor to discharge to 1/10 of its initial voltage?

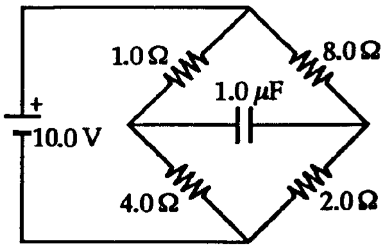


Figure 3

Solution

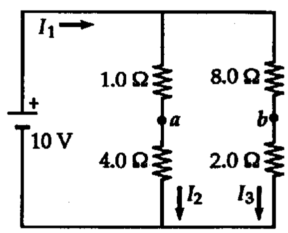
(a) After a long time the capacitor branch will carry negligible current. The current flow is as shown in Figure (a).

To find the voltage at point *a*, we first find the current, using the voltage rule:

$$10\text{ V} - (1.0\ \Omega)I_2 - (4.0\ \Omega)I_2 = 0$$

$$I_2 = 2.0\text{ A}$$

$$V_a = (4.0\ \Omega)I_2 = 8.0\text{ V}$$



(a)

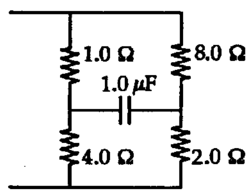
Similarly, $10\text{ V} - (8.0\ \Omega)I_3 - (2.0\ \Omega)I_3 = 0$

$$I_3 = 1.0\text{ A}$$

At point *b*, $V_b = (2.0\ \Omega)I_3 = 2.0\text{ V}$

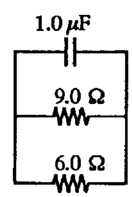
Thus, the voltage across the capacitor is

$$V_a - V_b = 8.0\text{ V} - 2.0\text{ V} = 6.0\text{ V} \quad \Diamond$$



(b)

(b) We suppose the battery is pulled out leaving an open circuit. We are left with Figure (b), which can be reduced to equivalent circuits (c) and (d).



(c)

From (d), we see that the capacitor sees 3.60 Ω in its discharge. According to $q = Qe^{-t/RC}$, we calculate that

$$qC = QCe^{-t/RC} \quad \text{and} \quad V = V_0e^{-t/RC}$$

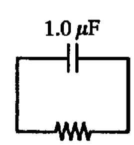
Solving, $\frac{1}{10}V_0 = V_0e^{-t/(3.60\ \Omega)(1.00\ \mu\text{F})}$

$$e^{-t/3.60\ \mu s} = 0.100$$

$$(-t/3.60\ \mu s) = \ln 0.100 = -2.30$$

$$\frac{t}{3.60\ \mu s} = 2.30$$

$$t = (2.30)(3.60\ \mu s) = 8.3\ \mu s \quad \Diamond$$



$$\frac{1}{\frac{1}{9}+\frac{1}{6}} = 3.60\ \Omega$$

(d)

10. A particular engine has a power output of 5.0 kW and an efficiency of 25%. If the engine expels 8000 J of thermal energy in each cycle, find
   a. the heat absorbed in each cycle and
   b. the time for each cycle.

Solution

(a) We have $e = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h} = 0.25$ and $Q_h = \frac{Q_c}{1-e}$, with $Q_c = 8000\ \text{J}$

We have $$Q_h = \frac{8000\ \text{J}}{1 - 0.25} = 10.7\ \text{kJ} \quad \lozenge$$

(b) $W = Q_h - Q_c = 2667\ \text{J}$ and from $P = \frac{W}{t}$, we have

$$t = \frac{W}{P} = \frac{2667\ \text{J}}{5000\ \text{J/s}} = 0.533\ \text{s} \quad \lozenge$$

The End