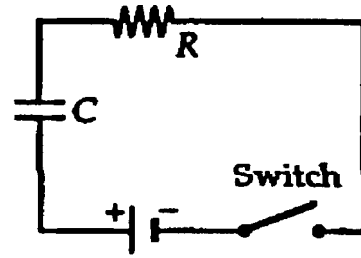




2. [30 marks] Consider a series RC circuit, as shown in the figure, for which  $R = 1.00 \text{ M}\Omega$ ,  $C = 5.00 \text{ }\mu\text{F}$ , and  $\varepsilon = 30.0 \text{ V}$ .
- Find the time constant of the circuit.
  - Find the maximum charge on the capacitor after the switch is closed.
  - If the switch is closed at  $t = 0$ , find the current in the resistor  $10.0 \text{ s}$  later.



Sol'n:

We have  $\varepsilon - q/c - IR = 0$ ,  $I = \frac{dq}{dt}$

$$\Rightarrow \frac{dq}{dt} = \varepsilon/R - q/Rc \Rightarrow q = \varepsilon C (1 - e^{-t/\tau})$$

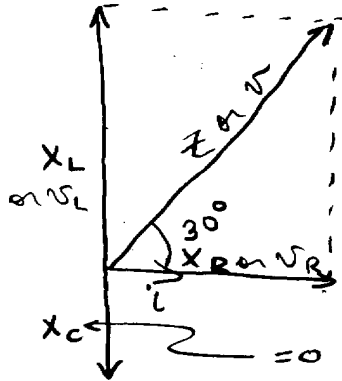
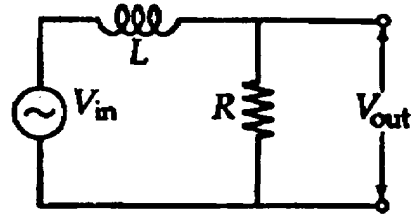
$$\text{and } I = \varepsilon/R e^{-t/Rc} \quad ; \quad \tau = RC$$

$$\therefore \text{a) } \tau = RC = 1 \times 10^6 \times 5 \times 10^{-6} = \underline{\underline{5.00 \text{ s}}}$$

$$\text{b) at } t = \infty, \quad q = \varepsilon C = 5 \times 10^{-6} \times 30 \\ = \underline{\underline{150 \text{ }\mu\text{C}}}$$

$$\text{c) } I = \frac{30}{1 \times 10^6} e^{-10/5} = 4.06 \times 10^{-6} \text{ A} \\ = \underline{\underline{4.06 \text{ }\mu\text{A}}}$$

3. [40 marks] Consider the phase-shifter circuit shown in the figure. The input voltage is described by the expression  $v = (10 \text{ V}) \sin 200t$  (in SI units). Assuming that  $L = 500 \text{ mH}$ .
- Find the value of  $R$  such that the output voltage  $v_{\text{out}}$  lags behind the input voltage by  $30^\circ$ .
  - Find the amplitude of the output voltage.



$$a) \tan 30^\circ = \frac{X_L - X_C}{X_R} = \frac{X_L}{X_R}$$

$$\therefore R = \frac{X_L}{\tan 30^\circ} = \frac{\omega L}{\tan 30^\circ} = \frac{200 \text{ rad/s} \times 0.5 \text{ H}}{\tan 30^\circ}$$

$$= 173 \Omega$$

$$b) \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{iR}{iZ} = \frac{R}{\sqrt{R^2 + X_L^2}}$$

$$\therefore V_{\text{out}} = \frac{10 \times 173}{\sqrt{173^2 + 100^2}} = 8.66 \text{ V}$$