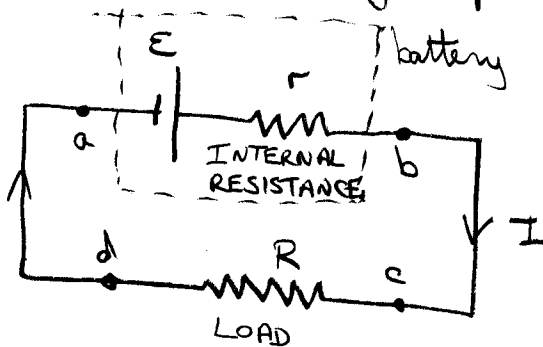


## Chapter 28    Direct Current Circuits

### 28.1 Electromotive Force (emf)

Source of energy: emf. Produces electric field  
 eg: battery  
 generator  
 ↓  
 change motion  
 is current.

Like a charge "pump".



$$V = V_b - V_a = \mathcal{E} - Ir$$

$$= IR$$

$$\therefore \mathcal{E} = IR + Ir = I(R+r)$$

$$\therefore I = \frac{\mathcal{E}}{R+r}$$

$$\text{Power delivered by battery} = I\mathcal{E} = I^2(R+r)$$

Example Terminal Voltage of a Battery

Battery:  $\mathcal{E} = 12.0 \text{ V}$ ,  $r = 0.05 \Omega$

Load:  $3.00 \Omega$ .

What is  $I$  and terminal voltage of the battery?  
 Power dissipated in load? In  $r$ ? Total?

$$I = \frac{\mathcal{E}}{R+r} = \frac{12.0 \text{ V}}{3.05 \Omega} = 3.93 \text{ A}$$

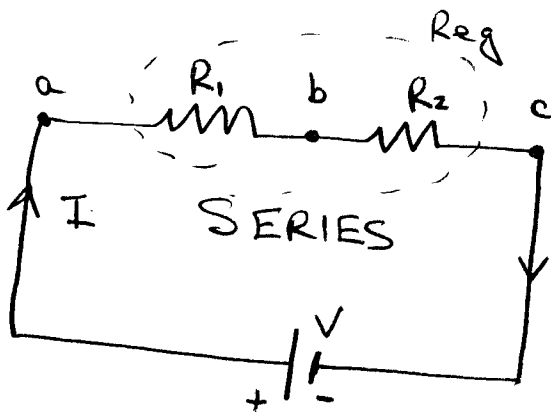
$$V = \mathcal{E} - Ir = 12.0 - 3.93 \times 0.05 = 11.8 \text{ V}$$

$$P_R = I^2 R = (3.93)^2 (3.00) = 46.3 \text{ W}$$

$$P_r = I^2 r = (3.93)^2 (0.05) = 0.772 \text{ W}$$

$$P_{\text{Total}} = P_r + P_R = 47.1 \text{ W} = I\mathcal{E}$$

## 28.2 Resistors in Series and in Parallel



$$\begin{aligned}
 V &= IR_1 + IR_2 \\
 &= I(R_1 + R_2) \\
 &= I R_{eq} \\
 &\quad \underbrace{\hspace{2cm}} \\
 &\quad \text{equivalent resistance}
 \end{aligned}$$

In general:

$$R_{eq} = R_1 + R_2 + R_3 + \dots$$

Power in each  $R$ :  $I^2 R_1$  vs  $I^2 R_2$

If  $R_1$  were a "60 W" bulb, ie it dissipates 60 W when  $V=120$ ,

$$P_{R_1} = IV = \frac{V^2}{R_1} = \frac{(120)^2}{R_1} = 60 \quad \text{if wired separately}$$

$$\therefore R_1 = \frac{(120)^2}{60} = 240 \Omega.$$

$$\text{For a "120 W" bulb, } R_2 = \frac{(120)^2}{120} = 120 \Omega.$$

$\therefore$  If a "60 W" bulb and a "120 W" bulb were wired in series to a 120 V source,

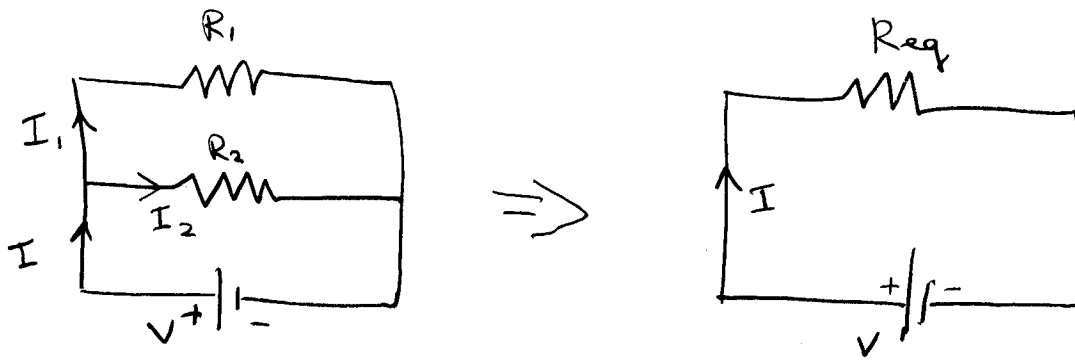
$$I = \frac{120}{(240 + 120)} = \frac{120}{360} = \frac{1}{3} \text{ A.}$$

$$\therefore P_{R_1} = I^2 R_1 = \left(\frac{1}{3}\right)^2 \times 240 = \frac{240}{9} = 26.6 \text{ W} \Leftrightarrow 60 \text{ W}$$

$$P_{R_2} = I^2 R_2 = \left(\frac{1}{3}\right)^2 \times 120 = \frac{120}{9} = 13.3 \text{ W} \Leftrightarrow 120 \text{ W}$$

$P_{\text{total}} = 40 \text{ W} \leftarrow$

$$R_{eq} = 240 + 120 = 360 \Omega, \Rightarrow P_{\text{total}} = I^2 R_{eq} = \left(\frac{1}{3}\right)^2 \times 360 = 40 \text{ W}$$



$$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} = V \left( \frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{V}{R_{eq}}$$

$$\therefore \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

or in general:  $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$

If  $R_1$  were a "60W" bulb, the power dissipated when  $V = 120V$  is  $\frac{V^2}{R_1} = \frac{(120)^2}{240} = 60W$  (duh!)

↑ from before

If  $R_2$  were a "120W" bulb, the power dissipated is 120W.

$$\Rightarrow P_{Total} = 60 + 120 = 180W$$

$$I_1 = \frac{V}{R_1} = \frac{120}{240} = 0.5A$$

$$I_2 = \frac{V}{R_2} = \frac{120}{120} = 1.0A$$

$$I = I_1 + I_2 = 1.5A$$

$$\frac{1}{R_{eq}} = \frac{1}{240} + \frac{1}{120} = \frac{3}{240} \Rightarrow R_{eq} = 80\Omega$$

$$\left. \begin{aligned} P_{Total} &= I V = 1.5 \times 120 = 180W \\ &= \frac{V^2}{R_{eq}} = \frac{(120)^2}{80} = 180W \end{aligned} \right\}$$

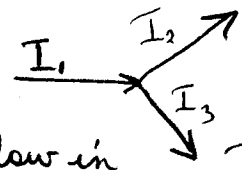
## 28.3 Kirchhoff's Rules

1.  $\sum_j I_j = 0$  at a junction

CONSERVATION  
OF  
CHARGE

ie  $I_1 - I_2 - I_3 = 0$  [note: +ve if flow in  
-ve if flow out.]

or  $I_1 = I_2 + I_3$  (flow in = flow out)



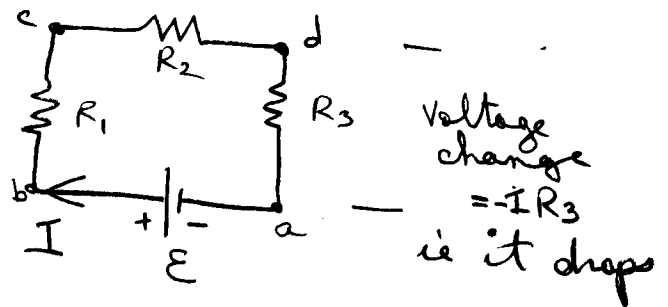
Pick a current direction and stick with it for the duration of the problem.

2.  $\sum_j \Delta V_j = 0$  around a loop

CONSERVATION  
OF  
ENERGY

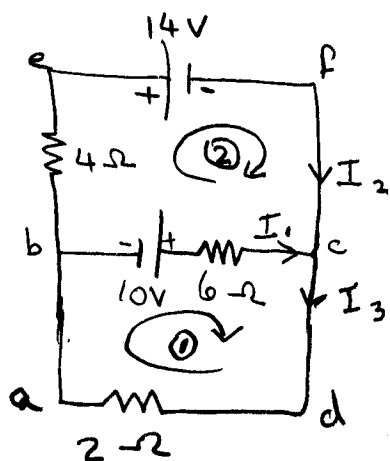
↑ changes in potential energy  
ie  $+E - IR_1 - IR_2 - IR_3 = 0$

or  $E = IR_1 + IR_2 + IR_3$



Note: in steady state, a capacitor is an open circuit since charge builds up to just balance the potential difference across it.

### Example: Applying Kirchhoff's Rules



Junction c  
 $I_1 + I_2 = I_3$

(junction b doesn't add anything new)

Loop ①  
 $10 - 6I_1 - 2I_3 = 0$

Loop ②  
 $-14 + 6I_1 - 10 - 4I_2 = 0$

Have 3 eqn's with 3 unknowns

Eqn 1  $I_1 + I_2 - I_3 = 0$

Eqn 2  $-6I_1 - 2I_3 = -10$

Eqn 3  $6I_1 - 4I_2 = 24$

$$\begin{bmatrix} 1 & 1 & -1 \\ 6 & 0 & 2 \\ 6 & -4 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 24 \end{bmatrix}$$

[Matlab gives  $I_1 = 2A$ ,  $I_2 = -3A$ ,  $I_3 = -1A$ .]

or manually:

Eqn 1  $\rightarrow$  Eqn 2:  $+6I_1 + 2(I_1 + I_2) = +10$   
 $\therefore 8I_1 + 2I_2 = 10$

(Eqn 3)/2 + Eqn 2:  
 $3I_1 - 2I_2 = 12$   
 $8I_1 + 2I_2 = 10$   


---

 $11I_1 = 21 \Rightarrow I_1 = \underline{\underline{2A}}$

$\therefore$  from Eqn 3:  $I_2 = \frac{12 - 24}{4} = \underline{\underline{-3A}}$

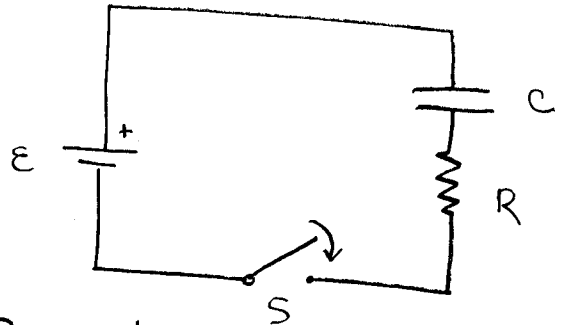
+ from Eqn 1:  $I_3 = 2 - 3 = \underline{\underline{-1A}}$

28.4 RC CircuitsCharging:

After the switch is closed:

$$\sum_j \Delta V_j = 0$$

$$\text{ie } \mathcal{E} - \frac{q}{C} - IR = 0 \quad [C = \text{Capacitance} - \text{see note below}]$$

note:  $q = q(t)$ ,  $I = I(t)$ .at  $t=0$ , no charge has accumulated on  $C$ ,

$$\therefore I(0) = \frac{\mathcal{E}}{R}$$

at  $t=\infty$ , the charge has built up on  $C$  causing  $I(\infty) = 0$ .This implies that  $\mathcal{E} = \frac{q(\infty)}{C}$  at  $t=\infty$ .

$$\text{Since } I \equiv \frac{dq}{dt} \Rightarrow \mathcal{E} - \frac{q}{C} - \frac{dq}{dt} R = 0$$

$$\Rightarrow \frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{q}{RC} \quad \Leftarrow \text{Need to solve.}$$

Note: Recall (Serway 26.2) for a parallel plate capacitor:

$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A} \quad (\text{from Gauss' Law } \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0})$$

$$+ V = Ed = \frac{qd}{\epsilon_0 A} \equiv \frac{q}{C}$$

To solve, Let  $q = y + \epsilon c$

$$\begin{aligned} \text{Thus: } \frac{dy}{dt} &= \frac{\epsilon}{R} - \frac{y}{RC} - \frac{\epsilon t}{RC} \\ &= -y/RC \end{aligned}$$

$$\therefore \frac{dy}{y} = - \frac{dt}{RC} \equiv - dt/\tau \quad (\tau \equiv RC)$$

$$\therefore \int_{y_0}^y \frac{dy}{y} = - \int_0^t dt/\tau$$

$$\begin{aligned} & \left[ \equiv \frac{\text{Volts} \times Q}{\text{current} \times V} \right] \\ & \left[ = \frac{Q}{C/\tau} = \text{time} \right] \end{aligned}$$

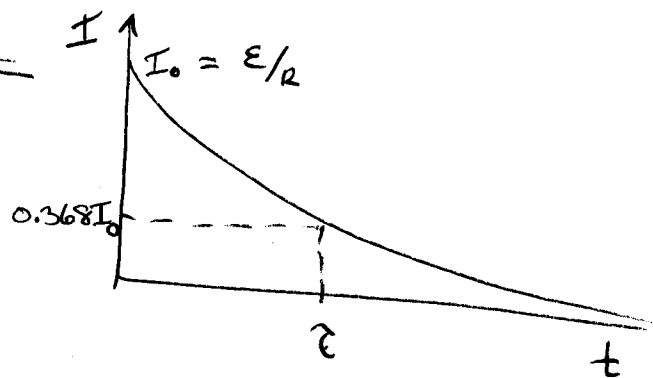
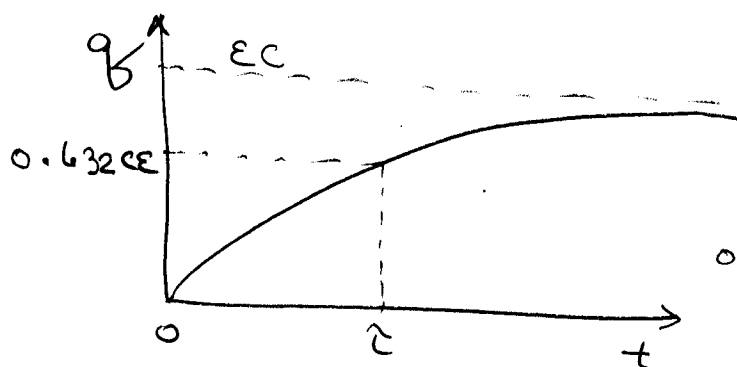
$$y_0 = q_0 - \epsilon c = -\epsilon c \quad (\text{no initial charge})$$

$$\therefore \ln(y/y_0) = -t/\tau \Rightarrow y = y_0 e^{-t/\tau} = -\epsilon c e^{-t/\tau}$$

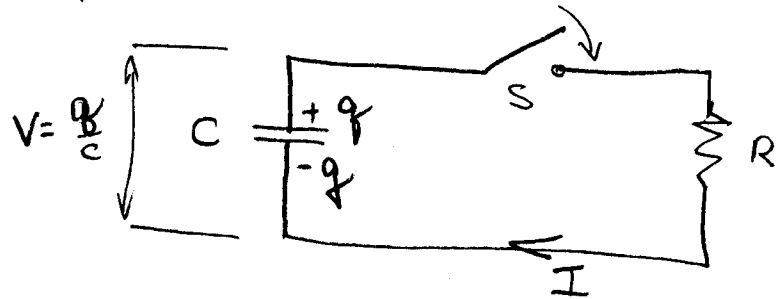
$$\begin{aligned} \therefore q &= \epsilon c (1 - e^{-t/\tau}) \\ &= q(\infty) (1 - e^{-t/\tau}) \end{aligned}$$

$$\text{Since } I = \frac{dq}{dt} \Rightarrow I = \epsilon c (-e^{-t/\tau}) (-1/\tau)$$

$$\begin{aligned} \therefore I &= \frac{\epsilon}{R} e^{-t/\tau} \\ &= \epsilon/R e^{-t/RC} \end{aligned}$$



## Discharging a charged Capacitor



Since  $\sum_j \Delta V_j = 0$ ,

$$IR = q/c, \quad I = I(t), \quad q = q(t)$$

now  $I = -\frac{dq}{dt}$  (since  $I = \text{rate of decrease of charge on capacitor}$ .  $I$  is still  $\frac{dq_{\text{wire}}}{dt}$  where  $q_{\text{wire}} = \text{charge flowing through the wire}$ .)

$$\therefore -R \frac{dq}{dt} = q/c \Rightarrow \frac{dq}{q} = -\frac{1}{Rc} dt$$

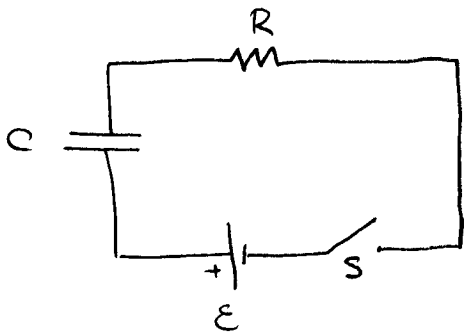
Thus:  $\int_{q(0)}^q \frac{dq}{q} = -\int_0^t \frac{dt}{Rc}$

$$\Rightarrow q = q(0) e^{-t/Rc} = q(0) e^{-t/\tau}$$

$$+ I = -\frac{dq}{dt} = \frac{q(0)}{Rc} e^{-t/\tau} = I(0) e^{-t/\tau}$$

↑  
decay just like the charging case.

Example: Charging a Capacitor in an RC circuit



$$\mathcal{E} = 12 \text{ V}$$

$$C = 5 \mu\text{F}$$

$$R = 8 \times 10^5 \Omega$$

Find  $\tau$ ,  $q_{\text{max}}$ ,  $I_{\text{max}}$ ,  $q(t)$ ,  $I(t)$

$$\tau = RC = 8 \times 10^5 \Omega \times 5 \times 10^{-6} \text{ F} = 4.0 \text{ s}$$

$$q_{\text{max}} = C\mathcal{E} = 5 \times 10^{-6} \text{ F} \times 12 \text{ V} = 60 \mu\text{C} \text{ (coulombs)}$$

$$I_{\text{max}} = \frac{\mathcal{E}}{R} = \frac{12 \text{ V}}{8 \times 10^5 \Omega} = 15 \mu\text{A}$$

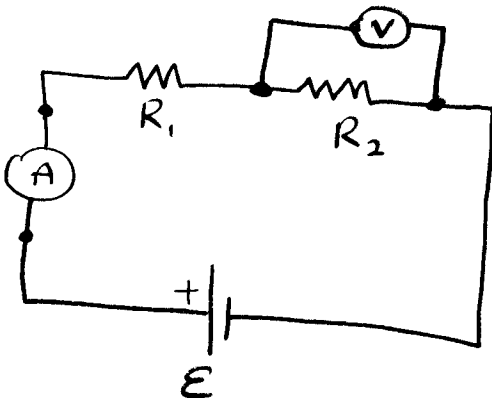
$$\therefore q(t) = 60 [1 - e^{-t/4}] \mu\text{C}$$

$$I(t) = 15 e^{-t/4} \mu\text{A}$$

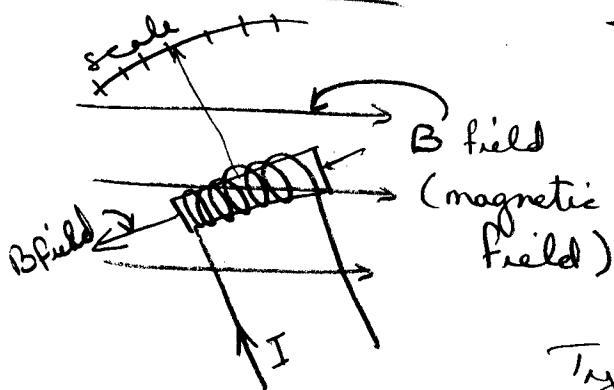
## 28.5 Electrical Instruments

- Ammeter
- measures current.
  - wire in series
  - ideally meter resistance = 0 so as to not disturb the current.
  - in reality,  $\Omega_{\text{meter}} \ll R_{\text{circuit}}$

- Voltmeter
- measures voltage
  - wired in parallel to component
  - ideally meter resistance =  $\infty$  so no current passes through it.
  - in reality  $\Omega_{\text{meter}} \gg R_{\text{component}}$



### Galvanometer

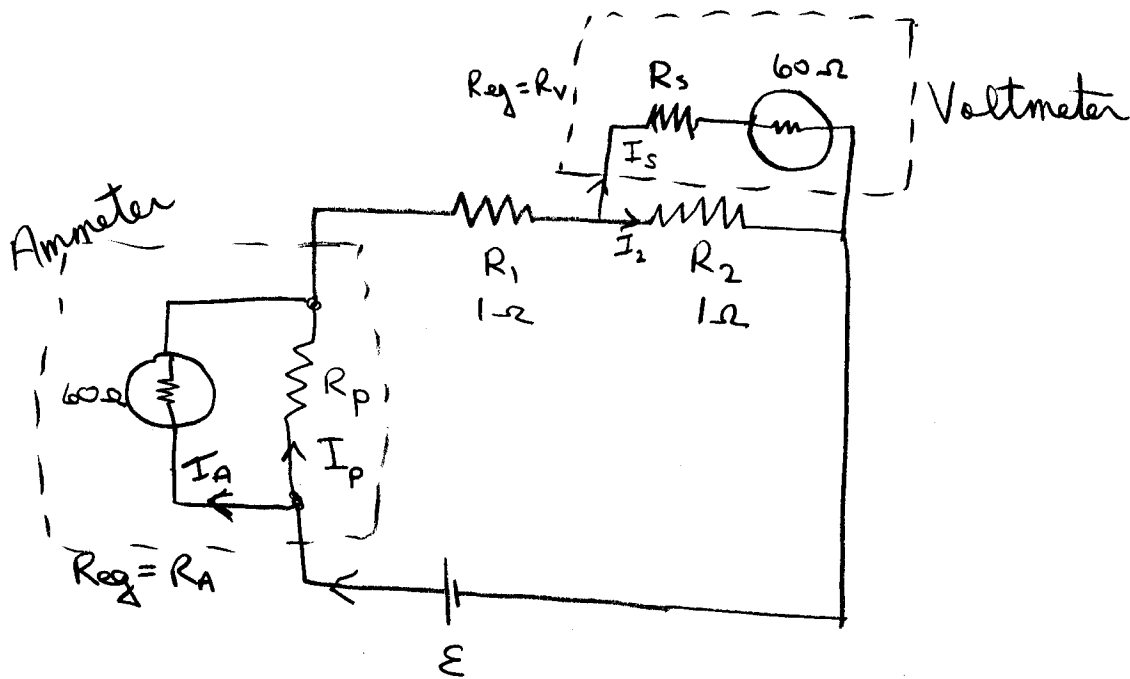


The coil is mounted on a torsional spring. Current loop makes a magnetic field  $\propto$  to current.

$$\text{Force} \propto B_{\text{field}} \propto I$$

$$\therefore \text{measurement} \propto I$$

Typical  $\Omega$  of coil  $\sim 60 \Omega$ .  
Get full scale movement for  $I \sim 1 \text{ mA}$ .



Let's say :  $R_1 = R_2 = 1 \Omega \therefore I = \frac{\mathcal{E}}{2} A$

Ammeter just have the ammeter hooked up?  
Want  $R_p$  small compared to  $2 \Omega$

Say Ammeter shows full scale at  $1 \text{ mA}$ . Pick  $R_p = 0.01 \Omega$ .

$$\text{Thus } R_{eq} = R_A = \frac{1}{\frac{1}{60} + \frac{1}{.01}} = 0.00999 \approx .01 \Omega$$

$$\therefore I_{\text{Actual}} = \frac{\mathcal{E}}{2.01} A = I_A + I_P$$

← 1 part in 200 error = 0.5%

$$+ \Delta V_{\text{ammeter}} = I_A R_A = I_P R_p$$

$$\therefore \frac{I_A}{I_P} = \frac{R_p}{R_A} = \frac{.01}{60} = .000167 \text{ i.e. } I_A \ll I_P$$

$$\therefore \frac{\mathcal{E}}{2.01} = I_A + \frac{60}{.01} I_A$$

$$= 6001 I_A \Rightarrow I_A = 8.29 \times 10^{-5} \mathcal{E} A$$

$$\therefore \text{scale} = I_{\text{measured}} \times \frac{I_{\text{Actual}}}{I_A} = I_{\text{meas}} \times \frac{\mathcal{E}/2.01}{8.29 \times 10^{-5} \mathcal{E}} = 6001 I_{\text{meas}}$$

Voltmeter:

If just have the voltmeter hooked up:

Want  $R_s$  large compared to  $R_2$ , say  $940 \Omega$ .

Then  $R_{eq} = R_v = R_s + 60 = 1000 \Omega$ .

Thus the total resistance of the circuit is:

$$R_{Total} = R_1 + \frac{1}{\frac{1}{1000} + \frac{1}{1}} = 1 + \frac{1}{1.001} = 1.999 \Omega$$

$$\therefore I_{Actual} = \frac{\mathcal{E}}{1.999 \Omega}$$

$$\text{Now } \Delta V_{\text{voltmeter}} = I_s (R_s + 60) = I_2 R_2$$

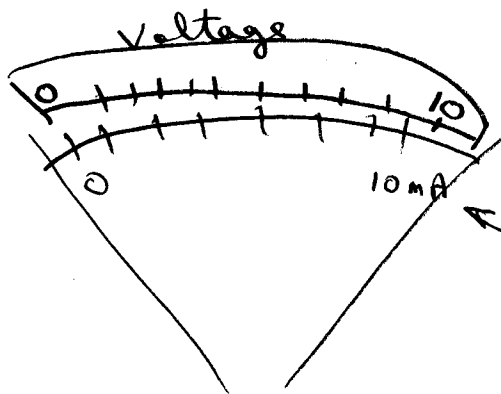
$$\therefore \frac{I_s}{I_2} = \frac{R_2}{R_s + 60} = \frac{1}{10.00}$$

$$\text{and } I_{Actual} = I_s + I_2 = I_s + 10.00 I_s = 10.01 I_s$$

$$\therefore I_s = \frac{\mathcal{E}}{1.999 \times 10.01} = 1.001 I_2$$

$$\text{Voltage across } R_2 = I_s (R_s + 60) = \frac{1000 \mathcal{E}}{1.999 \times 1001} = 0.49975 \mathcal{E}$$

$$\therefore \text{Scale} = I_{measured} \times \frac{V_{R_2}}{I_s} = \frac{I_s (R_s + 60)}{I_s} = I_{meas.} \times 1000$$



Say the galvanometer reads full scale when it passes a current of 10 mA

Error

$$\sim 0.1\%$$

(i.e.  $I_{meas.} \sim 1.001 I_2$ )

Of course, in general you won't know what  $R_1 + R_2$  are, so you can only guess at the real error.

But as the previous example shows,

for an ammeter, a shunt resistance of  $1/200$  of the circuit resistance gives an error of  $\sim 0.5\%$ . If the circuit resistance were higher, the error would drop. Conversely, a low resistance circuit would cause large errors with this  $60 \Omega$  galvanometer movement.

For the voltmeter, a shunt resistance of  $\sim 1000$  times the load resistance gives an error  $\sim 0.1\%$ .

For highly resistive loads, the errors would rise.

## Wheatstone Bridge

Used to measure an unknown resistor,  $R_x$ .

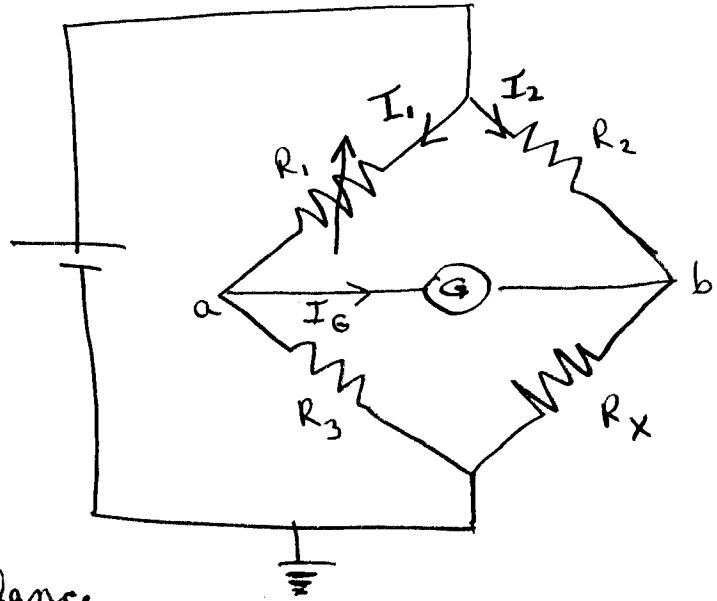
Vary  $R_1$  until  $I_G = 0$

Thus  $I_1 R_1 = I_2 R_2$

+  $I_1 R_3 = I_2 R_x$

Since  $V_a = V_b$  at the balance condition.

$$\therefore R_x = \frac{R_2 R_3}{R_1}$$



## Potentiometer

Used to measure an unknown  $\mathcal{E}_x$ .

Slide  $R_x$  until  $I_x = 0$ .

Thus  $\mathcal{E}_x = I R_x$ .

But you don't know  $I$  accurately  
since  $\mathcal{E}$  has some internal  $R$

So replace  $\mathcal{E}_x$  with a standard.

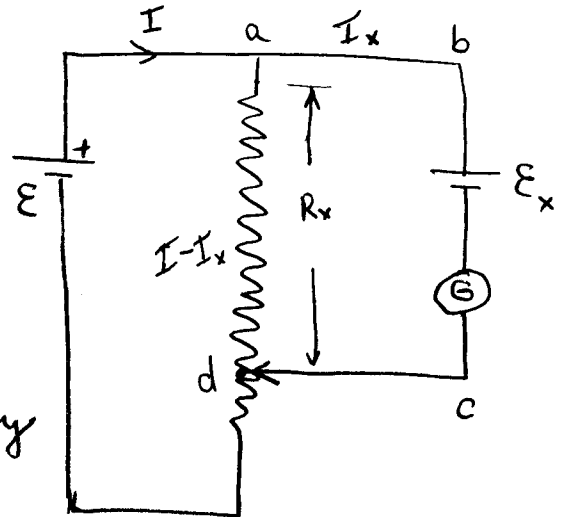
Repeat to give  $\mathcal{E}_s = I R_s$

$$\therefore \mathcal{E}_x = \frac{R_x}{R_s} \cdot \mathcal{E}_s$$

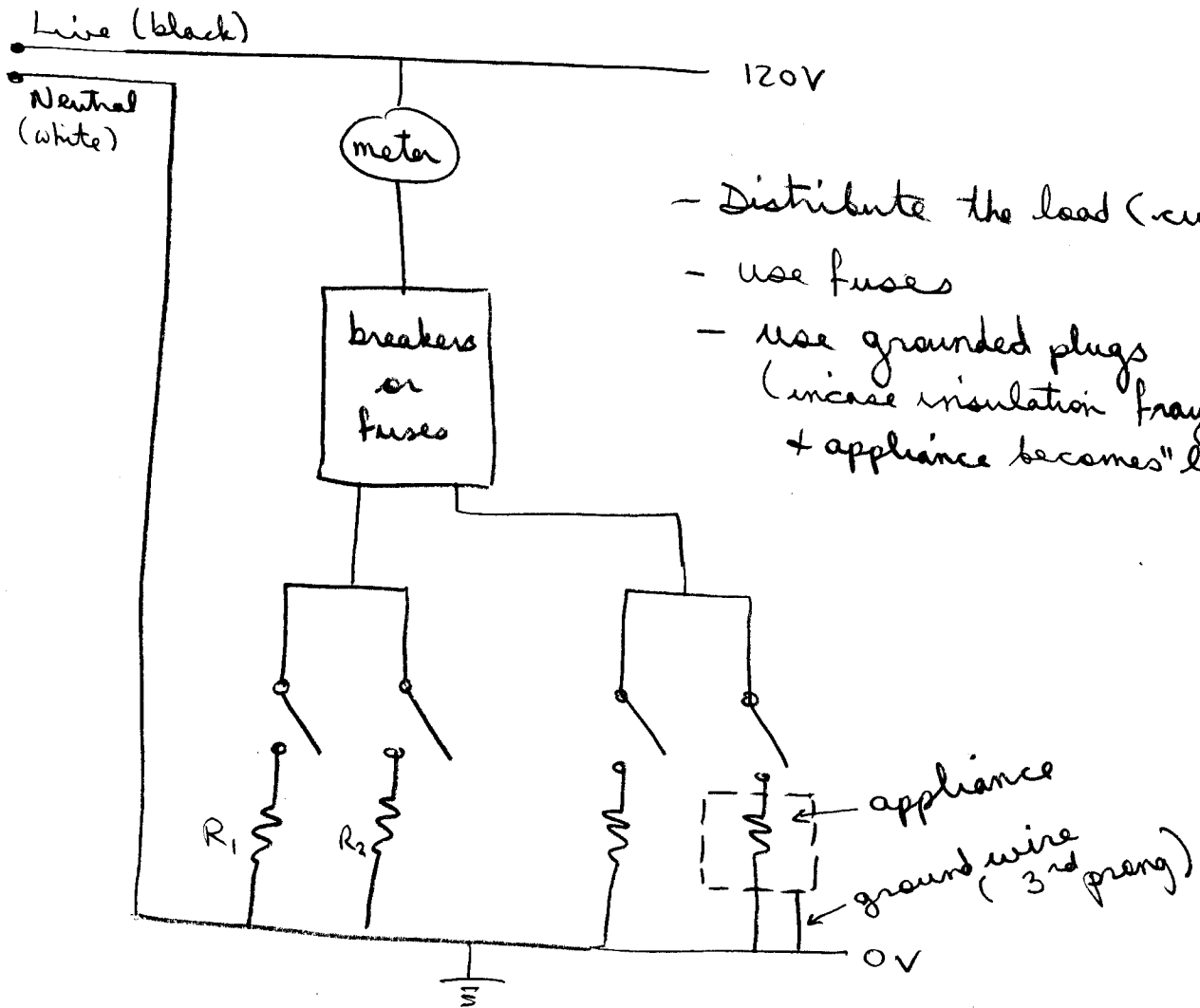
↑ known Emf at zero current.

Since  $R \propto \text{length, } L$ , on a sliding wire,

$$\text{we have } \mathcal{E}_x = \frac{L_x}{L_s} \cdot \mathcal{E}_s.$$



# 28.6 Household Wiring and Electrical Safety



- Distribute the load (current)
- use fuses
- use grounded plugs (in case insulation frays + appliance becomes "live")

