ENGINEERING 2C3 Assignment #3 Solution

1. [based on Serway Chapter 33 Question 4, pg 984]
(a) Explain how the acronym "ELI the ICE man" can be used to recall whether current leads voltage or voltage leads current in RLC circuits.
b) Why does the lead / lag exist for capacitors and inductors? Explain using physical reasoning / analogies.
(c) What causes the lead / lag mathematically?

(a) In RLC circuits:
for inductors: voltage (emf) leads current (I)
for capacitors: current (I) leads voltage (emf).
Hence, emf (E) in an inductor (L) comes before I
and I in a capacitor (C) comes before E
ie "ELI the ICE man" is a convenient mnemonic.

(b) In physical terms, the inductor acts like an inertia in that the current responds to an applied voltage but with some lag, just like the velocity of an object when a force is applied.
For a capacitor, a current has to flow before the charges can build up on its plates, thus generating an electric field and a voltage. This takes some time, much like filling a tank with a flow of water.

(c) for inductors, $v = L\dfrac{di}{dt}$. If $v = V_{max} \sin \omega t$ we get:

$$\Rightarrow i = \frac{V_{max}}{\omega L} \sin(\omega t - \pi/2)$$

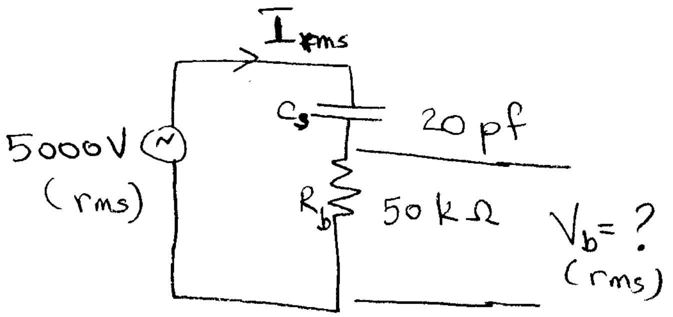
lag

for capacitors, $v = Q/C \Rightarrow \dfrac{dv}{dt} = i/C$

$$\Rightarrow i = \omega C \sin(\omega t + \pi/2)$$

lead.

2. [Serway Chapter 33 Problem 29, pg 987]
A person is working near the secondary of a transformer, as shown in the figure. The primary voltage is 120 V at 60.0 Hz. The capacitance C_s, which is the capacitance between hand and secondary winding, is 20.0 pF. Assuming the person has a body resistance to ground R_b = 50.0 kΩ, determine the rms voltage across the body. (Hint: Redraw the circuit with the secondary of the transformer as a simple ac source.)



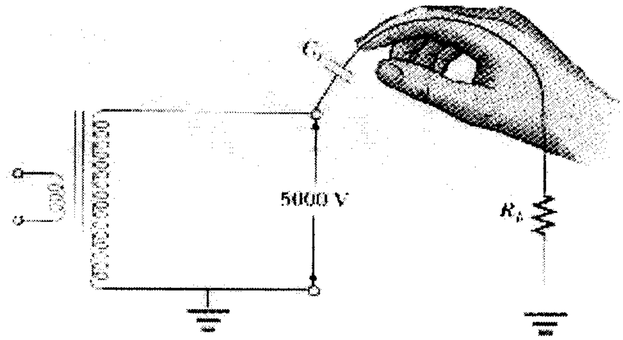


FIGURE P33.29

$$V_{b\text{-}(rms)} = I_{rms} R_b$$

$$I_{rms} = \frac{V_{rms} (= 5000\,\text{v})}{Z}$$

$$Z = \sqrt{R_b^2 + \left(\frac{1}{wC}\right)^2} = \sqrt{(50{,}000)^2 + \frac{1}{(377 \times 20 \times 10^{-12})^2}}$$

$$= \sqrt{(50{,}000)^2 + (132{,}626{,}000)^2}$$

$$\simeq 132{,}626{,}000\ \Omega.$$

$$\therefore I_{rms} = \frac{5000}{Z} = 3.77 \times 10^{-5}\ A$$

$$\therefore V_b = 50{,}000 \times 3.77 \times 10^{-5} = \underline{\underline{1.885\ V}}$$

3. [Serway Chapter 33 Problem 30, pg 988]
The voltage source in the figure has an output V_{rms} = (100V) cos(1000t). Determine (a) the current in the circuit and (b) the power supplied by the source. (c) Show that the power dissipated in the resistor is equal to the power supplied by the source.

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\phi = \tan^{-1} \frac{X_L - X_C}{X_R}$$

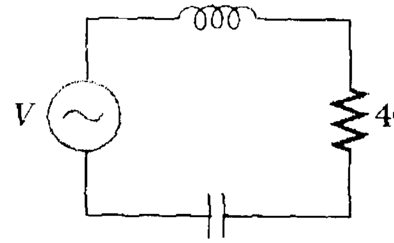


FIGURE P33.30

(a)

$$Z = \sqrt{(40)^2 + \left(1000 \times 0.05 - \frac{1}{1000 \times 50 \times 10^{-6}}\right)^2}$$

$$= \sqrt{40^2 + (50 - 20)^2}$$

$$= 50\,\Omega.$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{30}{40}\right) = 36.87°$$

$$\therefore I_{rms} = \frac{V_{rms}}{Z} = \frac{100}{50} = 2\,A.$$

(b) Power supplied = $V_{rms} I_{rms} = 100 \times 2 \cos\phi$
$= 100 \times 2 \times 0.8 = 16\,W$

(c) Power dissipated in resistor = $I_{rms}^2 R$

$= (2)^2 \times 40 = 16\,W.$

which equals the supplied power.

4. [Serway Chapter 33 Problem 48, pg 989]
A series RLC circuit has the following values: L = 20.0 mH, C = 100 μF, R = 20.0 Ω and V_{max} = 100 V, with v = V_{max} sin ωt. Find (a) the resonance frequency, (b) the amplitude of the current at the resonance frequency, (c) the Q of the circuit, and (d) the amplitude of the voltage across the inductor at resonance.

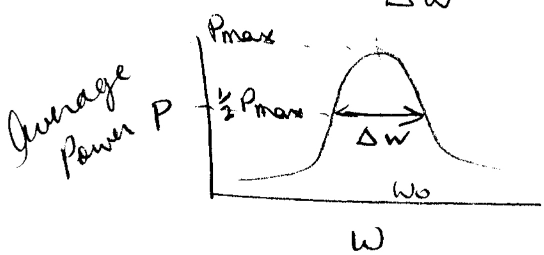
(a) $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20\times10^{-3} \times 100\times10^{-6}}} = 707.1 \ \frac{rad}{s}$$

$$\therefore f_0 = \frac{\omega}{2\pi} = \underline{\underline{112.5 \ Hz}}$$

(b) At resonance, $Z = R$

$$\therefore I_{MAX} = \frac{V_{max}}{R} = \frac{100}{20} = \underline{\underline{5 \ A.}}$$

(c) $Q = \frac{\omega_0}{\Delta\omega}$ where $\Delta\omega$ = width at ½ max, ie



$$P_{average} = \frac{V_{rms}^2 R}{R^2 + (\omega L - 1/\omega C)^2}$$

This is a maximum when

$\omega L = 1/\omega C \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$

The resonance condition.

$P \to \frac{1}{2} P_{max}$ when $R^2 = (\omega L - 1/\omega C)^2$

This occurs when $+R = \omega L - 1/\omega C$ — ①

__and__ when $-R = \omega L - 1/\omega C$ — ②

From ① : $\omega^2 - R/L \ \omega - 1/LC = 0 \Rightarrow \omega = \frac{1000 \pm 1732}{2}$

$\Rightarrow 1366$ rads/s
(only 1 real root)

From ② ; $\omega^2 + R/L \ \omega - 1/LC = 0 \Rightarrow \omega = \frac{-1000 \pm 1732}{2}$

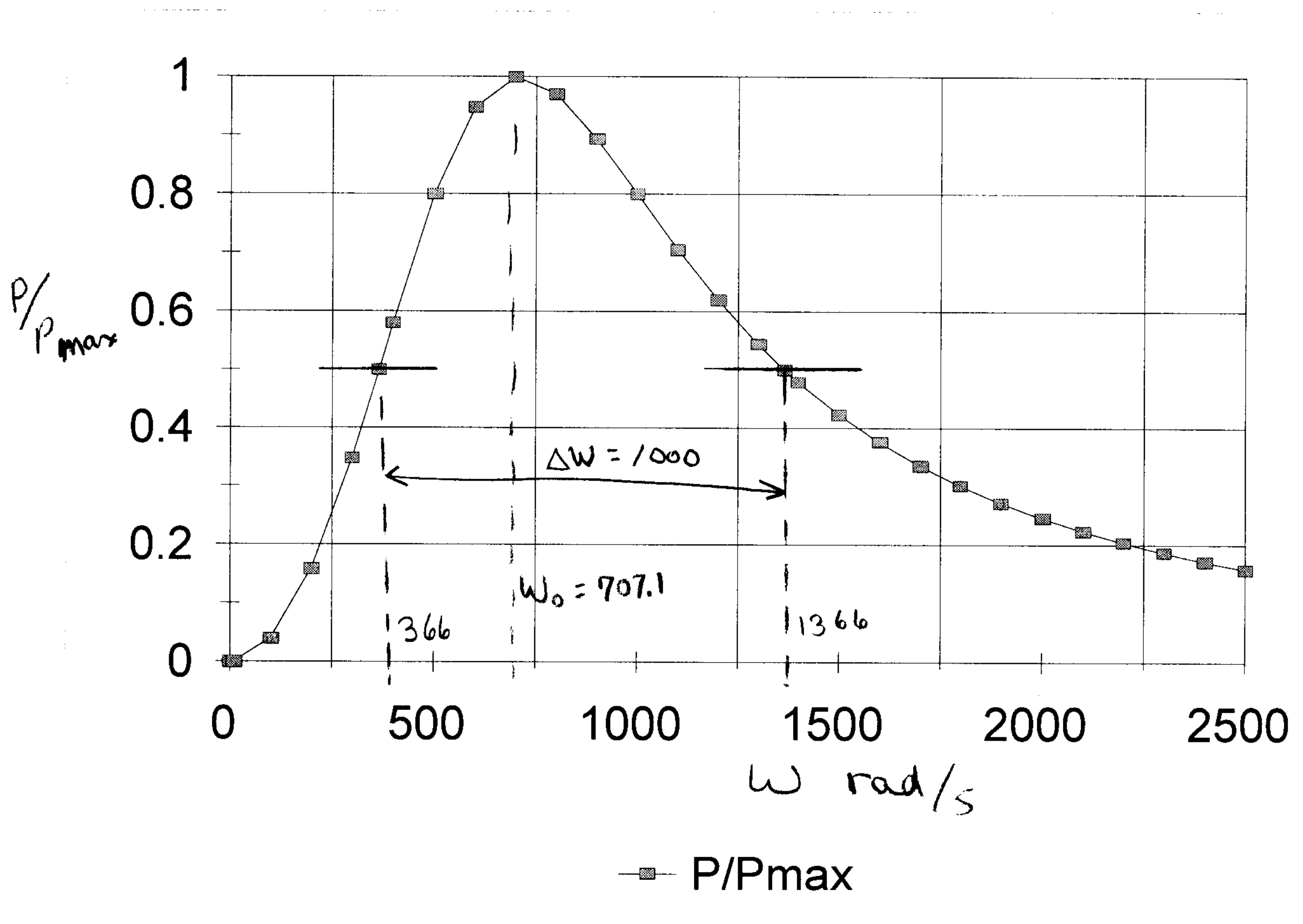
$= 366$ rads/s.

(c) continued

This is verified on the numerical plot below.

Hence $\Delta W = 1366 - 366 = 1000$ rads/s

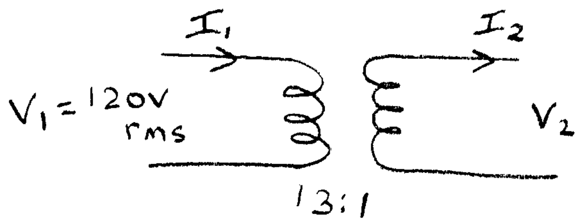
Hence $Q = \frac{707.1}{1000} = \underline{\underline{.707}}$



(d) $V_{L\,max} = I_{max} X_L = \frac{V_{max}}{R}(707.1 \times 20 \times 10^{-3})$
(at resonance)

$= \frac{100}{\cancel{20}} \times .7071 \times \cancel{20} = \underline{\underline{70.71 \text{ V}}}$

5. [based on Serway Chapter 33 Problem 61, pg 990]
A step-down transformer is used for running / recharging the batteries of portable devices such as tape players. The turns ratio inside the transformer is 13:1 and is used with 120-V (rms) household service. If a particular tape player draws 0.35 A from the household outlet, what are (a) the voltage and (b) the current supplied from the transformer? (c) How much power is delivered?



(a) $V_2 = \frac{N_2}{N_1} V_1 = \frac{1}{13} \times 120 = \underline{9.23\ V}$

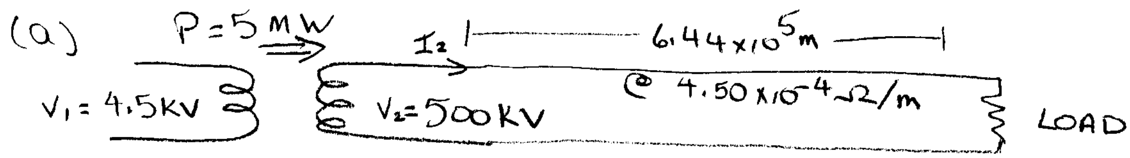
(b) $I_2 V_2 = I_1 V_1$

$\therefore I_2 = \frac{0.35 \times 120}{(1/13 \times 120)} = 13 \times 0.35 = \underline{4.55\ A.}$

(c) $P_1 = P_2 = 0.35 \times 120 = \underline{42\ W}$

6. [Serway Chapter 33 Problem 71, pg 990]
A transmission line that has a resistance per unit length of $4.50 \times 10^{-4}\ \Omega / \text{m}$ is to be used to transmit 5.00 MW over 400 miles (6.44×10^{5} m). The output voltage of the generator is 4.5 kV. (a) What is the line loss if a transformer is used to step up the voltage to 500 kV? (b) What fraction of the input power is lost to the line under these circumstances? (c) What difficulties would be encountered on attempting to transmit 5.00 MW at the generator voltage of 4.5 kV?

(a)



I_2 = current in transmission line (rms)

$$= \frac{P}{V_2} = \frac{5 \times 10^6\ \text{W}}{500 \times 10^3\ \text{V}} = 10\ \text{A}.$$

$\therefore$ line loss $= I_2^2 R = (10)^2 \times 6.44 \times 10^5\ \text{m} \times 4.5 \times 10^{-4}\ \Omega/\text{m}$

$$= \underline{\underline{28.98\ \text{kW}}}$$

(b) Fractional loss $= \dfrac{28.98 \times 10^3\ \text{W}}{5 \times 10^6\ \text{W}} = \underline{\underline{0.0058}}$

(c) If the voltage wasn't stepped up,

$$I_2 = \frac{5 \times 10^6\ \text{W}}{4.5 \times 10^3\ \text{V}} = 1{,}111\ \text{A}.$$

This is a lot of current.

The line loss would be $I^2 R$

$$= \underline{\underline{360\ \text{MW}}} \quad !!$$

This is impossible of course. The true is, the voltage drop over the line is >> 4.5 kV, hence the current could never get this high.