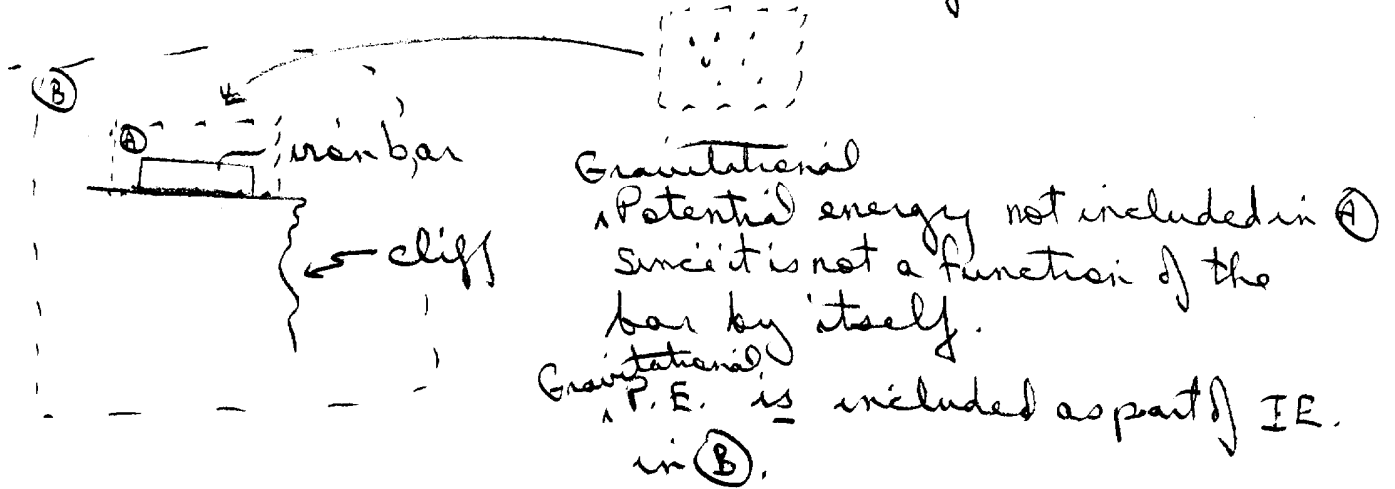


# Chapter 5 Internal Energy + Equipartition

Internal energy =  $\sum$  energies of individual elements of a system  
(I.E.)

But what is a system? Anything inside a "box" that you draw.

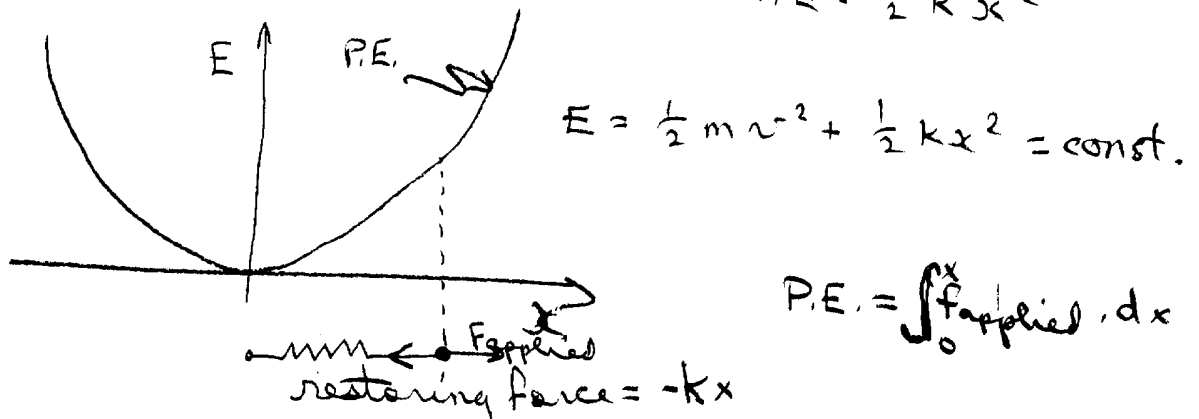


Typically,

$$E = b q^2$$

$\uparrow$  const.      $\nwarrow$  position or momentum coordinate.

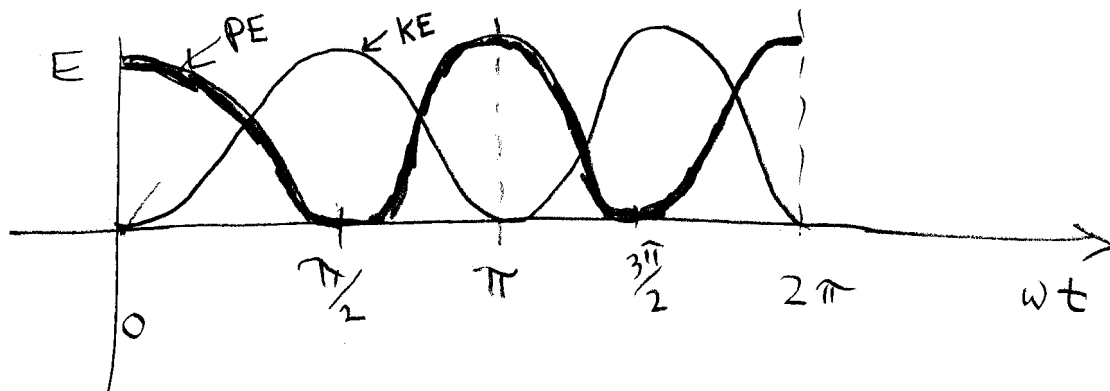
Examples: harmonic oscillator:  $KE = \frac{1}{2} m v^2 = \frac{p^2}{2m}$   
 $PE = \frac{1}{2} k x^2$



For harmonic oscillator:

$$x = A \cos \omega t, \quad \omega = \sqrt{k/m}$$

$$\begin{aligned} \therefore E &= \frac{1}{2} k A^2 \cos^2 \omega t + \frac{1}{2} m A^2 \omega^2 \sin^2 \omega t \\ &= \frac{1}{2} A^2 k (\cos^2 \omega t + \sin^2 \omega t) = \frac{1}{2} A^2 k = \text{constant.} \end{aligned}$$



Note: Average PE =  $\int_0^{2\pi} \frac{1}{2} k A^2 \cos^2 \omega t d(\omega t) / \text{period}$

$$= \frac{1}{2} \frac{k A^2}{2\pi} \left[ \frac{\omega t}{2} + \frac{1}{4} \sin 2\omega t \right]_0^{2\pi}$$

$$= \frac{1}{4} k A^2 \frac{2\pi}{2\pi} = \frac{k A^2}{4}$$

Average KE =  $\int_0^{2\pi} \frac{1}{2} k A^2 \sin^2 \omega t d(\omega t) / \text{period}$

$$= \frac{1}{2} \frac{k A^2}{2\pi} \left[ \frac{\omega t}{2} - \frac{1}{4} \sin 2\omega t \right]_0^{2\pi}$$

$$= \frac{k A^2}{4}$$

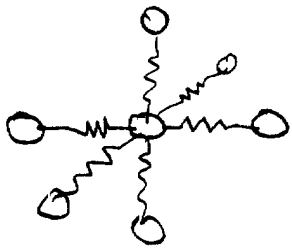
i.e. average P.E. = average K.E. =  $\frac{1}{4} k A^2 = \frac{1}{2} E_{\text{Total}}$

This occurs in all common systems:

Equipartition Theorem: On average, I.E. is distributed equally among the degrees of freedom whose energies are expressible in the form  $bq^2$ .

## Degrees of Freedom:

Examples: atoms in a solid



$$P.E. \equiv U = \frac{1}{2} k_x^2 + \frac{1}{2} k_y^2 + \frac{1}{2} k_z^2$$

$$K.E. = E_K = \frac{1}{2} m v_x^2 + \frac{1}{2} m v_y^2 + \frac{1}{2} m v_z^2$$

D.o.F. = number of degrees of freedom = 6

ie all the energy can be in  $\frac{1}{2} k_x^2$  or  $\frac{1}{2} k_y^2$  etc.

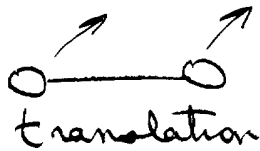
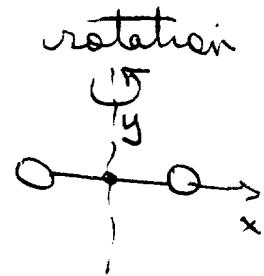
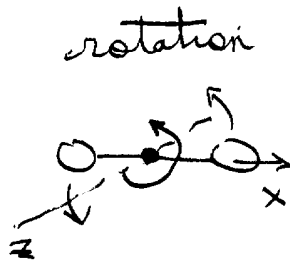
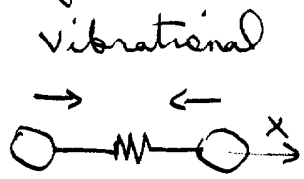
The energy is free to be anywhere in these 6 "bins" - or spread out in all 6.

On average - each degree of freedom contains  $\frac{1}{6}$  of the total energy.

For  $N$  atoms  $\rightarrow 6N$  degrees of freedom.

## Degrees of freedom (cont'd):

Gas of diatomic molecules.



$$E = E_{\text{translation}} + E_{\text{rotation}} + E_{\text{vibration}}$$

$$= \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} + \frac{L_y^2}{2I_y} + \frac{L_z^2}{2I_z} + \frac{1}{2} m v_{\text{rel}}^2 + \frac{1}{2} k x^2$$

$$= 7 \text{ degrees of freedom.}$$

moment of inertia  
angular momentum.

Each system has its own degrees of freedom.

It can be shown that the average energy per degree of freedom is:

$$\bar{\epsilon} = \frac{1}{2} kT$$

$$= 0.846 \times 10^{-4} \text{ eV/K}$$

$$k = 1.381 \times 10^{-23} \text{ J/K}$$

= Boltzmann's constant

T = temperature in Kelvin.

Caution:

Phase changes (melting, boiling ... ) involve a change in the number of DDF, ie atoms in a solid are no longer bound rigidly, etc.

Thus  $\bar{E} \uparrow$  while  $T = \text{const.}$

ie added energy goes into adding DDF or raising the reference point of the P.E.

More on this next!

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