

Name: A. Einstein

Student ID: $E = mc^2$

EP 2H04 / Phy 2H04

DAY CLASS

Dr. Wm. Garland

DURATION: 50 minutes

McMASTER UNIVERSITY

TEST # 1

February 15, 2001

Special Instructions:

1. Closed Book. All calculators and up to 4 single sided 8 1/2" by 11" crib sheets are permitted.
2. Do all questions. Place your answers on the exam sheets; use additional pages if necessary.
3. The value of each part is as indicated. TOTAL Value: 100 marks

THIS EXAMINATION PAPER INCLUDES 4 PAGES AND 7 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

1. [20 marks total] Express mathematically the zeroth, first, second and third laws of thermodynamics and state their physical meanings in one sentence each.

(5) 0th Law: Temperatures of bodies in equilibrium are equal, ie $T_A = T_B$ if A+B are in equilibrium.

(5) 1st Law: $dE = dQ - dW + \mu dN$

This is conservation of energy, change in energy is that due to heat change, work done + diffusive changes

(5) 2nd Law: $\Delta S \geq 0$, Entropy ($\equiv k \ln \Omega$) of a system increases as it approaches equilibrium. $\Delta S = 0$ at equilibrium.

(5) 3rd Law: $S \rightarrow 0$ as $T \rightarrow 0$. Only 1 state at $T = 0$.

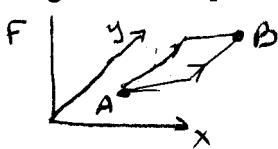
2. [12 marks total]

a. How would you determine if a certain differential, $g(x,y) dx + h(x,y) dy$, is exact?

(3) $dF = g dx + h dy$ is exact if $\frac{\partial g}{\partial y} = \frac{\partial h}{\partial x}$

- b. What is the physical meaning of the concept of exact and inexact differentials and give an example of each?

(3)



As you move from A to B, the value of F will change. The change is independent of the path taken for an exact differential. It is path dependent for an inexact diff.

Example of exact: $F = x^2 y \Rightarrow dF = 2xy dx + x^2 dy$

Example of inexact: $dF = 3xy dx + x^2 dy$

- c. Convert e^{25} to base 10.

(3)

$$e^{25} = 10^y \quad ; \quad y = 25 \ln(e) / \ln(10) = 25 / 2.3026 = 10.85$$

$$\therefore e^{25} = 10^{10.85}$$

- d. Convert $2^{10^{25}}$ to base 10

(3)

$$2^{(10^{25})} = 10^{10^{25} \times \ln(2) / \ln(10)} = 10^{0.301 \times 10^{25}}$$

3. [9 marks total]

- a. For a small system of 100 experimental trials, 10 successes were found. What is the standard deviation, σ ?

(3)

$$p = \bar{n} / N = 10 / 100 = 0.1 \Rightarrow q = 0.9$$

$$\sigma = \sqrt{Npq} = \sqrt{100 \times 0.1 \times 0.9} = 3 \quad \underline{\underline{\sigma = 3}}$$

- b. What is the relative standard deviation, σ / \bar{n} ?

(3)

$$\frac{\sigma}{\bar{n}} = \frac{3}{10} = \underline{\underline{0.3}}$$

- c. What would you expect to happen to σ and σ / \bar{n} as more experiments are performed?

(3)

σ would increase $\propto \sqrt{N}$

$\frac{\sigma}{\bar{n}}$ would decrease $\propto \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}}$

But it also would be refined as \bar{n} estimate improves.

4. [12 marks total]

a. Starting with the standard definition of work, show that $P dV$ is a work term.

$$(3) \quad dW = F \cdot ds = P \cdot \underbrace{A \cdot ds}_{dV} = P \cdot dV$$

b. How is the quantity "entropy" defined? What is its physical significance?

$$(3) \quad S \equiv k \ln \Omega. \text{ It is a measure of the number of states of a system.}$$

c. State the Fundamental Postulate.

$$(3) \quad \text{An isolated system in equilibrium is equally likely to be in any of its accessible states. } \left(P_i = \frac{\Omega_i}{\Omega_0} \right)$$

d. State the Equipartition Theorem.

$$(3) \quad \text{On average, the internal energy is distributed equally among the degrees of freedom whose energies are expressible in the form } b \nu^2$$

5. [20 marks total]

a. What is the probability, P_i , of being in any particular state, given Ω_0 , the number of states in a system?

$$(5) \quad P_i = \frac{1}{\Omega_0}$$

b. What is the probability, P_i , of being in any particular sub-set of states, given Ω_0 , the number of states in a system and Ω_i , the number of states in a sub-system?

$$(5) \quad P_i = \frac{\Omega_i}{\Omega_0}$$

c. How is Ω_0 , the number of states in a system, related to energy, E , typically?

$$(5) \quad \Omega_0 = \text{const } E^{\mathcal{R}/2}, \quad \mathcal{R} = \text{total \# of d.o.f.}$$

where $\nu = \# \text{ of dof per particle}$, $N = \# \text{ of particles}$

d. Illustrate that the number of available states increases dramatically for even a small increase in energy in a system.

$$(5) \quad \text{Let } E_2 = 1.001 E_1$$

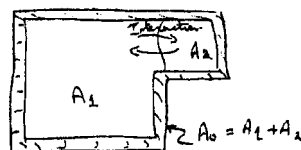
$$\therefore \frac{\Omega_2}{\Omega_1} = \left(\frac{1.001 E_1}{E_1} \right)^{\mathcal{R}/2} = (1.001)^{\mathcal{R}/2}$$

D:\TEACH\EP2H4\test1-2002.doc February 14, 2002

$$\mathcal{R} \text{ is typically } \sim 10^{28} \therefore \frac{\Omega_2}{\Omega_1} = (1.001)^{10^{28}}$$

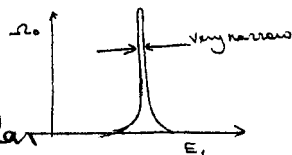
which is a huge increase.

6. [15 marks total] Recalling the system defined by the figure, as discussed in class, we found that the total number of available states as a function of the energy partition, E_1 , peaked at a particular energy. Yet we also know that the number of available states of a system, as a function of energy, increases as E increases, with no peak. How can both these facts be true?



$$E_0 = E_1 + E_2 = \text{constant}$$

$$\Omega_0 = \Omega_1 \Omega_2$$



(15)

The peak refers to the fact that at a particular partition of energy (ie in this case, a particular value of $E_1 + E_2$), the number of possible states is maximized. The total energy, E_0 , is held constant in that exercise.

If E_0 were to increase, many more states become available. For this new E_0 , there is still a partition of energy that maximizes the number of states available.

7. [12 marks total]

a. Show that $dS = dQ/T$ when V and N are held constant.

$$S(E+dQ) = S(E) + \frac{\partial S}{\partial E} dQ + \frac{1}{2} \frac{\partial^2 S}{\partial E^2} (dQ)^2 + \dots$$

$\uparrow \equiv \frac{1}{T}$

\nearrow drop higher order terms

(6)

$$\therefore S(E+dQ) - S(E) = dS = \frac{dQ}{T}$$

Note: Because we hold $V+N$ const, $dE = dQ - dW + \mu dN$

b. If a system with entropy, S_1 , and number of states, Ω_1 , is somehow changed to S_2

and Ω_2 , what is $\frac{\Omega_2}{\Omega_1}$ in terms of $\Delta S = S_2 - S_1$?

(6)

$$S = k \ln \Omega \therefore S_2 - S_1 = k \ln \Omega_2 - k \ln \Omega_1 \equiv \Delta S$$

$$\therefore \Delta S = k (\ln \Omega_2 - \ln \Omega_1) = k \ln (\Omega_2 / \Omega_1)$$

$$\therefore \frac{\Omega_2}{\Omega_1} = e^{\Delta S/k}$$

---THE END---