

Name:

Solution

Student ID: _____

EP 2H04 / Phy 2H04

DAY CLASS

Dr. Wm. Garland

DURATION: 50 minutes

McMASTER UNIVERSITY

TEST # 2

March 19, 2001

Special Instructions:

1. Closed Book. All calculators and up to 4 single sided 8 1/2" by 11" crib sheets are permitted.
2. Do all questions. Place your answers on the exam sheets; use additional pages if necessary.
3. The value of each part is as indicated. TOTAL Value: 100 marks

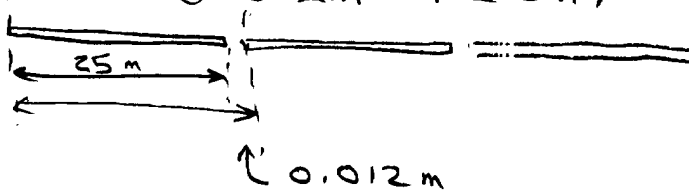
THIS EXAMINATION PAPER INCLUDES 6 PAGES AND 8 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

1. [10 marks] The concrete sections of a certain superhighway are designed to have a length of 25.0 m. The sections are poured and cured at 10 °C. What minimum spacing should the engineer leave between the sections to eliminate buckling stress if the concrete is to reach a temperature of 50 °C? The linear expansion coefficient, α , for concrete is $12 \times 10^{-6} / ^\circ\text{C}$.

The concrete will expand $\Delta L = \alpha \Delta T L$

$$= 12 \times 10^{-6} / ^\circ\text{C} \times 40^\circ\text{C} \times 25.0 \text{ m}$$

$$= 0.012 \text{ m} = 1.2 \text{ cm},$$



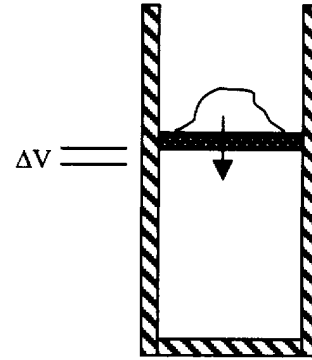
2. [10 marks] A Thermopane window of area 6.0 m^2 is constructed of two layers of glass, each 4.0 mm thick separated by an air space of 5.0 mm. If the inside is at 20°C and the outside is at -30°C , what is the heat loss through the window? The thermal conductivity of air is $0.0234 \text{ W/m}^\circ\text{C}$ and that for glass is $0.80 \text{ W/m}^\circ\text{C}$. Ignore the insulation value of the surrounding air.

Heat Transfer rate = H watts = $\frac{\text{Area } \Delta T}{\sum_i \Delta x_i / k_i}$

$$= \frac{6 \text{ m}^2 \times 50^\circ\text{C}}{\frac{0.005}{0.0234} + 2 \times \frac{0.004}{0.8}} = 1.34 \text{ kW}$$

3. [15 marks total]

- a. Sand is added slowly to the top of the piston in the figure so that the volume is reduced by 0.001 m^3 as shown. The insulated container volume is big so that the pressure does not change during the process. Pressure is 1 atm. There are 10^{+26} ideal gas molecules in the container. No particles are added or removed. What is ΔE , ΔS , ΔT ? [$k = 1.381 \times 10^{-23} \text{ J/K}$]



$$\Delta E = \Delta Q - P\Delta V + \mu\Delta N$$

$$= -1 \text{ atm} \times \frac{1.01 \times 10^5 \text{ Pa}}{\text{atm}} \times (-0.001 \text{ m}^3) = +101 \text{ J}$$

done on the system

$$\Delta S = dQ/T = 0$$

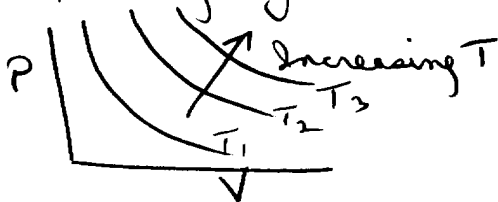
$$R = \gamma N = 3 \times 10^{+28}$$

$$\Delta T = \frac{\Delta E}{\frac{1}{2} Rk} = \frac{101 \text{ J}}{\frac{3}{2} \times 10^{28} \times 1.381 \times 10^{-23}} = +0.0486 \text{ }^\circ\text{C}$$

Note: although $P = \text{const}$ because V is large, we cannot say that $P\Delta V = 0$. Thus $PV = NkT \Rightarrow P\Delta V + V\Delta P = Nk\Delta T$, not $P\Delta V = Nk\Delta T$. You cannot use the gas law to help here.

- b. You should have found in part (a) that T went up when V went down. But for an ideal gas, we know that $PV = NkT$, implying that T should go down as V goes down for constant P and N . How come it didn't?

We have shown in the recuse that $PV = NkT$ is true at constant $E + N$, i.e., $PV = \text{const}$ at constant E or T , forming a family of lines. In part (a), T & E are changing,



so we are going from one isotherm to another.

4. [5 marks total] Typically, the volumetric coefficient of expansion is three times the linear coefficient of expansion, i.e., $\beta = 3\alpha$. If it is not for a particular material, what does this imply?

We derived $\beta = 3\alpha$ from $V' = V + \Delta V = V(1 + \alpha\Delta T)^3$.

Implicit in that derivation is that α is the same in all 3 directions (x, y, z), i.e., the material is amorphous. If $\beta \neq 3\alpha$, then the material is not amorphous. It is perhaps a crystal or a laminate.

5. [15 marks total] We have shown in class that as an isolated system, composed of subsystem 1 and subsystem 2, approaches equilibrium,

$$\Delta S = \left(\frac{1}{T_1} - \frac{1}{T_2} \right) \Delta Q_1 + \frac{1}{T_2} (P_1 - P_2) \Delta V_1 - \frac{1}{T_2} (\mu_1 - \mu_2) \Delta N_1 > 0.$$

(10)

- a. Show that this implies that heat flows from hot to cold, that higher pressure regions expand and that particles flow to lower potential areas.

Since we can vary ΔQ_1 , ΔV_1 , & ΔN_1 independently, it follows that when $\Delta V_1 = \Delta N_1 = 0$ for finite ΔQ_1 , then

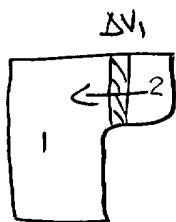
$\left(\frac{1}{T_1} - \frac{1}{T_2} \right) \Delta Q_1 > 0$, which gives $T_1 < T_2$ when $\Delta Q_1 > 0$ (ie 1 receives heat), ie heat flows from hot to cold.

Similarly, $(P_1 - P_2) \Delta V_1 > 0$ for $\Delta Q_1 = \Delta N_1 = 0$ implies $P_1 > P_2$ when $\Delta V_1 > 0$, ie higher pressure region expands.

$(\mu_1 - \mu_2) \Delta N_1 < 0 \Rightarrow \mu_1 < \mu_2$. When $\Delta N_1 > 0$, particles flow to the lower μ region.

(5)

- b. It is cold outside and you are in a stuffy, hot examination room. A window is opened to let in a fresh breeze of cold air. We designate the room as '1' and the outside as '2'. Does this event violate (a)? Show why or why not. [Hint: Assume $\mu_1 = \mu_2$. Consider this a 2-stage process. Stage 1: The cold incoming air acts like a piston, compressing the air in the room quickly. Stage 2: thermal relaxation follows more slowly.]



This does not violate (a) because in stage 1:

$$\Delta Q_1 = 0, \Delta N_1 = 0, \Delta V_1 < 0, P_2 > P_1$$

$\therefore (P_1 - P_2) \Delta V_1 > 0$. This agrees with (a).

Region 2 expands in region 1.

In stage 2: $\Delta V_1 = 0, \Delta N_1 = 0, \Delta Q_1 < 0, T_2 < T_1$

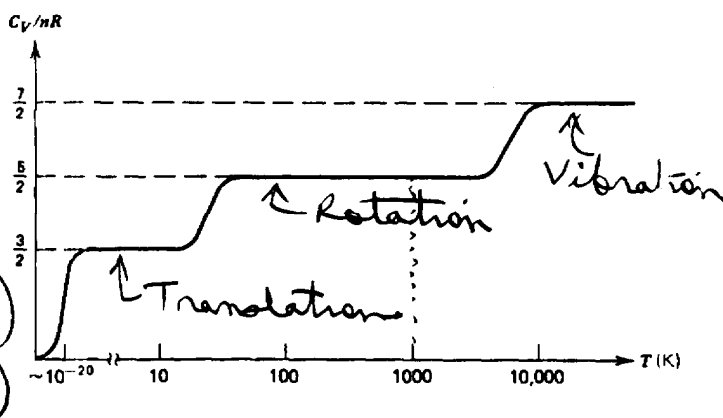
$\therefore \left(\frac{1}{T_1} - \frac{1}{T_2} \right) \Delta Q_1 > 0$. This agrees with (a).

Region 1 cools and region 1 warms.

Stage 2 is much slower than stage 1.

6. [10 marks total] We measure the C_v of a gas for various temperatures and get the results as shown in the figure.

- a. What must be happening to get such a result?



$$C_v = \frac{\nu}{2} n R \quad \left(= \frac{\nu}{2} N k \right)$$

$$\left(= \frac{\nu R k}{2} \right)$$

$$\therefore \frac{C_v}{nR} = \frac{\nu}{2}$$

\therefore # of d.o.f. must be increasing as T increases.
Also: close to $T=0$ K, $C_v \rightarrow 0$ as expected.

- b. What is the number of degrees of freedom at 1000 K?

(3)

$$\text{at } 1000 \text{ K} \quad \frac{C_v}{nR} = \frac{\nu}{2} = \frac{5}{2} \Rightarrow \nu = \underline{\underline{5}}$$

7. [20 marks total] Starting with $\Omega = \text{const } V^N E^{3N/2}$, show that for an ideal gas:

a. $E = \frac{3}{2} NkT$ $dE = dQ - PdV + \mu dN = Tds - PdV + \mu dN$

$$\therefore ds = \frac{1}{T} (dE + PdV - \mu dN) \quad \text{--- ①}$$

$$S = k \ln \Omega = k [\ln \text{const} + \ln V^N + \ln E^{3N/2}]$$

$$= k \ln \text{const} + N \ln V + \frac{3}{2} N \ln E$$

From ① $\left. \frac{\partial S}{\partial E} \right)_{V,N} = \frac{1}{T} = k \frac{3}{2} \frac{\partial \ln E}{\partial E} = \frac{k \cdot 3}{2E}$

$\therefore E = \frac{k \cdot 3}{2} N T = \frac{3}{2} N k T$ for ideal gas ($\nu = 3$)

b. $PV = NkT$

From ①

$$\left. \frac{\partial S}{\partial V} \right)_{E,N} = \frac{P}{T} = k N \frac{\partial \ln V}{\partial V} = \frac{k N}{V}$$

$$\therefore PV = kNT = nRT$$

c. $C_V = \frac{3}{2} Nk$

$$C_V \equiv \left. \frac{\partial Q}{\partial T} \right|_{V,N} = \left. \frac{\partial E}{\partial T} \right|_{V,N} = \frac{\partial \left(\frac{3}{2} NkT \right)}{\partial T} = \frac{3}{2} Nk$$

d. $C_p = \frac{5}{2} Nk$ $dQ = dE + PdV - \mu dN$

$$\begin{aligned} C_p &\equiv \left. \frac{\partial Q}{\partial T} \right|_{P,N} = \left. \frac{\partial E}{\partial T} \right|_{P,N} + P \left. \frac{\partial V}{\partial T} \right|_{P,N} \\ &= \frac{3}{2} Nk + P \left. \frac{\partial (NkT/P)}{\partial T} \right|_{P,N} = \frac{3}{2} Nk + Nk = \frac{5}{2} Nk \end{aligned}$$

8. [15 marks total] Show that $\left. \frac{\partial T}{\partial V} \right|_{S,N} = - \left. \frac{\partial P}{\partial S} \right|_{V,N}$. [Hint: start with the first law with $dQ = TdS$.]

$$dE = TdS - PdV + \mu dN$$

$$\left. \frac{\partial E}{\partial S} \right|_{V,N} = T \Rightarrow \frac{\partial}{\partial V} \left(\left. \frac{\partial E}{\partial S} \right|_{V,N} \right) = \left. \frac{\partial T}{\partial V} \right|_{S,N}$$

$$\left. \frac{\partial E}{\partial V} \right|_{S,N} = -P \Rightarrow \frac{\partial}{\partial S} \left(\left. \frac{\partial E}{\partial V} \right|_{S,N} \right) = - \left. \frac{\partial P}{\partial S} \right|_{V,N}$$

Since $\frac{\partial}{\partial V} \left(\left. \frac{\partial E}{\partial S} \right|_{V,N} \right) = \frac{\partial^2 E}{\partial V \partial S} = \frac{\partial}{\partial S} \left(\left. \frac{\partial E}{\partial V} \right|_{S,N} \right)$

we have $\left. \frac{\partial T}{\partial V} \right|_{S,N} = - \left. \frac{\partial P}{\partial S} \right|_{V,N}$

Table 13.1 Maxwell's Relations—
Those Most Commonly
Used

$$\left(\frac{\partial T}{\partial V}\right)_{S,N} = -\left(\frac{\partial p}{\partial S}\right)_{V,N} \quad (\text{M1})$$

$$-\left(\frac{\partial p}{\partial N}\right)_{S,V} = \left(\frac{\partial \mu}{\partial V}\right)_{S,N} \quad (\text{M2})$$

$$\left(\frac{\partial T}{\partial N}\right)_{S,V} = \left(\frac{\partial \mu}{\partial S}\right)_{V,N} \quad (\text{M3})$$

$$\left(\frac{\partial S}{\partial V}\right)_{T,N} = \left(\frac{\partial p}{\partial T}\right)_{V,N} \quad (\text{M4})$$

$$-\left(\frac{\partial p}{\partial N}\right)_{T,V} = \left(\frac{\partial \mu}{\partial V}\right)_{T,N} \quad (\text{M5})$$

$$-\left(\frac{\partial S}{\partial N}\right)_{T,V} = \left(\frac{\partial \mu}{\partial T}\right)_{V,N} \quad (\text{M6})$$

$$\left(\frac{\partial T}{\partial p}\right)_{S,N} = \left(\frac{\partial V}{\partial S}\right)_{p,N} \quad (\text{M7})$$

$$\left(\frac{\partial V}{\partial N}\right)_{S,p} = \left(\frac{\partial \mu}{\partial p}\right)_{S,N} \quad (\text{M8})$$

$$\left(\frac{\partial T}{\partial N}\right)_{S,p} = \left(\frac{\partial \mu}{\partial S}\right)_{p,N} \quad (\text{M9})$$

$$-\left(\frac{\partial S}{\partial p}\right)_{T,N} = \left(\frac{\partial V}{\partial T}\right)_{p,N} \quad (\text{M10})$$

$$\left(\frac{\partial V}{\partial N}\right)_{T,p} = \left(\frac{\partial \mu}{\partial p}\right)_{T,N} \quad (\text{M11})$$

$$-\left(\frac{\partial S}{\partial N}\right)_{T,p} = \left(\frac{\partial \mu}{\partial T}\right)_{p,N} \quad (\text{M12})$$

For nondiffusive interactions:

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial p}{\partial S}\right)_V \quad (\text{M1})$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V \quad (\text{M4})$$

$$\left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial V}{\partial S}\right)_p \quad (\text{M7})$$

$$-\left(\frac{\partial S}{\partial p}\right)_T = \left(\frac{\partial V}{\partial T}\right)_p \quad (\text{M10})$$

---THE END---