

5.1 Show that if  $f(t) \hat{=} g(v)$ , then  $g(t) \hat{=} f(v)$  if  $f$  and  $g$  are even functions.

Sol'n

$$\text{We have } f(t) = \int_{-\infty}^{\infty} g(v) e^{2\pi i v t} dv = \mathcal{F}^{-1}(g)$$

= inverse F.T.

Let  $v \rightarrow -t'$  and  $t \rightarrow v'$  (change of notation).

$$\begin{aligned} \text{Then } f(v') &= \int_{-\infty}^{\infty} g(-t') e^{-2\pi i v' t'} dt' \\ &= \int_{-\infty}^{\infty} g(t') e^{-2\pi i v' t'} dt' \\ &= \int_{-\infty}^{\infty} g(t') e^{-2\pi i v' t'} dt' \\ &= \mathcal{F}(g) = \text{F.T. of } g. \end{aligned}$$

ie  $g \hat{=} f$  QED

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5.2 Show that  $e^{-\pi t^2} \iff e^{-\pi v^2}$ .

Sol'n

$$\int_{-\infty}^{\infty} e^{-\pi t^2} e^{-2\pi i v t} dt = \text{F.T. of } e^{-\pi t^2} = F(v)$$

$$\begin{aligned} \text{Now } t^2 + 2i v t &= (t + i v)^2 - (i v)^2 \\ &= (t + i v)^2 + v^2 \end{aligned}$$

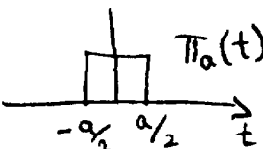
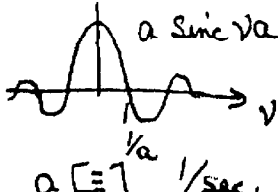
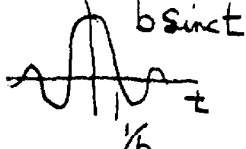
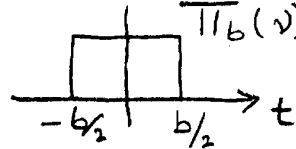
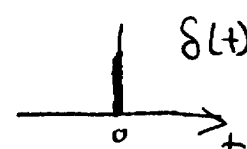
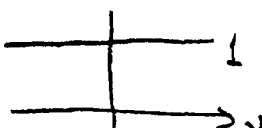

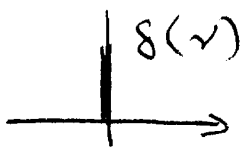
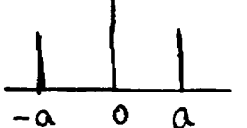
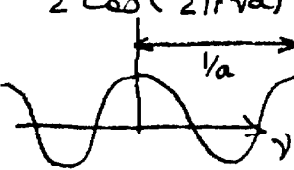
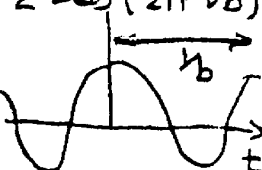
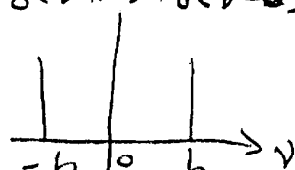
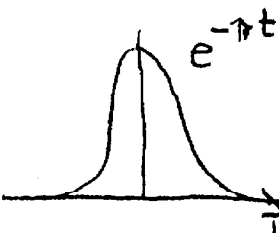
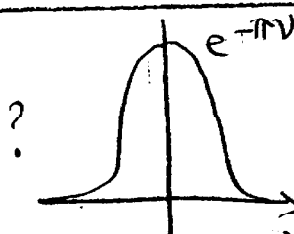
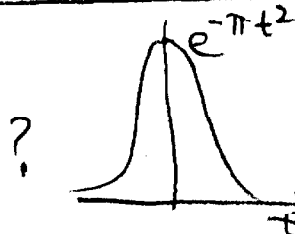
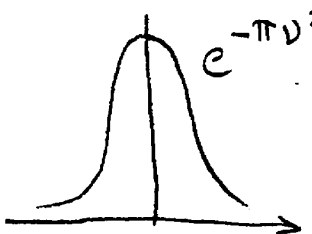

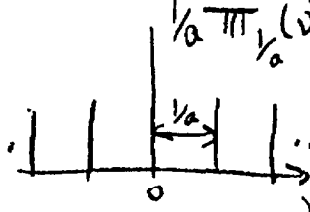
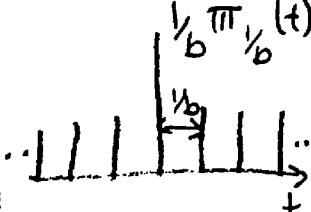
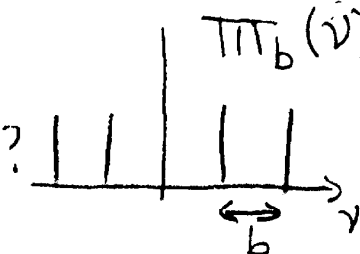
$$\therefore F(v) = \int_{-\infty}^{\infty} e^{-\pi(t+iv)^2} e^{-\pi v^2} dt$$

$$\text{Let } z = \sqrt{\pi}(t+iv) \Rightarrow dz = \sqrt{\pi} dt$$

$$\begin{aligned} \therefore F(v) &= \int_{-\infty}^{\infty} e^{-\pi z^2} e^{-\pi v^2} \frac{dz}{\sqrt{\pi}} \\ &= \frac{e^{-\pi v^2}}{\sqrt{\pi}} \underbrace{\int_{-\infty}^{\infty} e^{-z^2} dz}_{\sqrt{\pi}} \\ &= e^{-\pi v^2} \end{aligned}$$

QED

5.3 Using 5.1 and the transforms developed in the course notes thus far, complete the following table of Fourier Transforms:

$f(t) \rightleftharpoons F(\nu)$	$f(t) \rightleftharpoons F(\nu)$	$f(t) \rightleftharpoons F(\nu)$	$f(t) \rightleftharpoons F(\nu)$
 $\Pi_a(t)$	 $a \text{ sinc } \nu a$ $a [ ] \frac{1}{\nu} \text{ sec.}$	 $b \text{ sinc } \nu b$ $\frac{1}{b}$	 $\Pi_b(\nu)$ $b [ ] ? \frac{1}{\text{sec}}$
 $\delta(t)$	 $1$	 $1$	 $\delta(\nu)$
 $\delta(t+a) + \delta(t-a)$	 $2 \cos(2\pi \nu a)$ $\frac{1}{a}$	 $2 \cos(2\pi \nu b)$ $\frac{1}{b}$	 $\delta(\nu+b) + \delta(\nu-b)$
 $e^{-\pi t^2}$	 $e^{-\pi \nu^2}$	 $e^{-\pi t^2}$	 $e^{-\pi \nu^2}$
 $\Pi_a(t)$	 $\frac{1}{a} \Pi_{\frac{1}{a}}(\nu)$ $\frac{1}{a}$	 $\frac{1}{b} \Pi_{\frac{1}{b}}(t)$ $\frac{1}{b}$	 $\Pi_b(\nu)$ $b$