5.1 Show that if $f(t) \rightleftharpoons g(\nu)$, then $g(t) \rightleftharpoons f(\nu)$ if f and g are even functions.

<u>Sol'n</u>

We have $f(t) = \int_{-\infty}^{\infty} g(\nu)\, e^{2\pi i \nu t} d\nu = \mathcal{F}^{-1}(g)$

= inverse F.T.

Let $\nu \rightarrow -t'$ and $t \rightarrow \nu'$ (change of notation).

Then $f(\nu') = -\int_{\infty}^{-\infty} g(-t')\, e^{-2\pi i \nu' t'} dt'$

$= \int_{-\infty}^{\infty} g(-t')\, e^{-2\pi \nu' t'} dt'$

$= \int_{-\infty}^{\infty} g(t')\, e^{-2\pi \nu' t'} dt'$

$= \mathcal{F}(g)$ = F.T. of g.

ie $g \rightleftharpoons f$ QED

5.2 Show ~~that~~ $e^{-\pi t^2} \rightleftharpoons e^{-\pi \nu^2}$.

Sol'n

$$\int_{-\infty}^{\infty} e^{-\pi t^2} e^{-2\pi i \nu t} \, dt = \text{F.T. of } e^{-\pi t^2} = F(\nu)$$

Now $t^2 + 2i\nu t = (t+i\nu)^2 - (i\nu)^2$

$$= (t+i\nu)^2 + \nu^2$$

$$\therefore F(\nu) = \int_{-\infty}^{\infty} e^{-\pi(t+i\nu)^2} e^{-\pi\nu^2} \, dt$$

Let $z = \sqrt{\pi}(t+i\nu) \Rightarrow dz = \sqrt{\pi}\, dt$

$$\therefore F(\nu) = \int_{-\infty}^{\infty} e^{-\pi z^2} e^{-\pi\nu^2} \frac{dz}{\sqrt{\pi}}$$

$$= \frac{e^{-\pi\nu^2}}{\sqrt{\pi}} \underbrace{\int_{-\infty}^{\infty} e^{-z^2} \, dz}_{= \sqrt{\pi}}$$

$$= e^{-\pi\nu^2}$$

QED

5.3 Using 5.1 and the transforms developed in the course notes thus far, complete the following table of Fourier Transforms: