ENGINEERING PHYSICS 3W4

DAY CLASS | Dr. Wm. Garland
DURATION: 2 hours |
McMASTER UNIVERSITY FINAL EXAMINATION | April 2000

Special Instructions:

1. Closed Book. All calculators and up to 6 single sided (or 3 double sided) 8 ½" by 11" crib sheets are permitted.
2. Do all questions.
3. The value of each question is as indicated.
4. Point form is sufficient for discussion type questions.

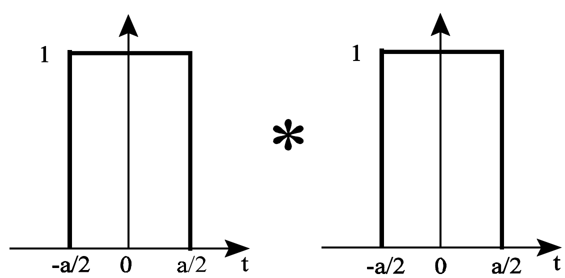
   TOTAL Value: 100 marks

THIS EXAMINATION PAPER INCLUDES 3 PAGES AND 7 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

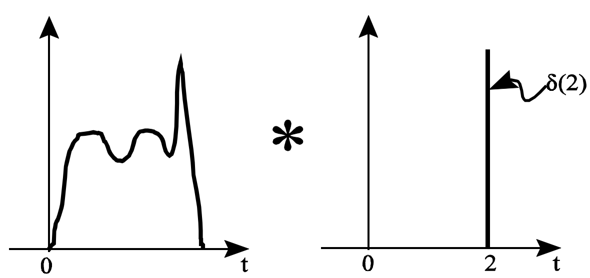
1. [15 marks] Define:
   a. Linear system
   b. Causal system
   c. Time invariant system
   d. Convolution integral
   e. Autocorrelation

2. [20 marks] Graphically compute the convolution of the following function pairs:

   a.



   b.



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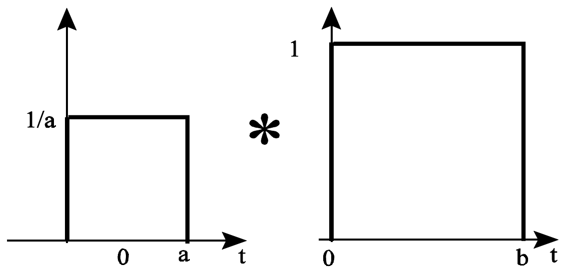
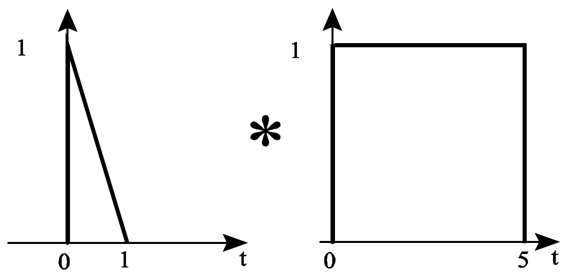
c.



d.



3. [15 marks] Evaluate the following expressions:

a. Evaluate $\int_{-\infty}^{+\infty} \delta(-2t-1)\,\mathrm{sinc}(\pi\nu t)\,dt$

b. Evaluate $\frac{d}{dt}\Pi_a(t)$

c. Show that $t\,f(t) \rightleftharpoons (\frac{i}{2\pi})\frac{dF(\nu)}{d\nu}$. [Don't panic; it's easy!]

4. [10 marks] Give brief answers to the following:

a. In signal analysis, what is the advantage of reflecting a time signal about the origin?

b. Show how the convolution integral results from making a measurement. Assume a linear, time invariant, causal system.

c. Why are Fourier Transforms so useful when dealing with convolutions?

d. Laplace and Fourier Transforms are similar. Why are Laplace transforms used more for solving differential equations and Fourier Transforms used more for signal analysis?

5. [10 marks] In the following, a proof is not required.

a. What is the FT of $[a(t)*b(t) + c(t).d(t)]^2$

b. What is the FT of $\Pi_2(t)$. Graph the result.

c. What is the FT of $\exp(-\pi t^2)$?

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6. [20 marks] An input step voltage, V_0 u(t), is passed through an ideal low pass filter. V_0 is the magnitude of the voltage and u(t) denotes the unit step.
   a. Sketch the input signal in time space.
   b. What does the ideal low-pass filter look like in frequency space?
   c. What does the ideal low-pass filter look like in time space?
   d. By working only in time space, determine the output voltage.
   e. What is the Fourier Transform of the input V_0 u(t)? We denote this as $V_0\, \varphi(\nu)$
   f. What is the output voltage in frequency space, $Y(\nu)$?
   g. By performing the Inverse Fourier Transform of $Y(\nu)$, determine the output voltage in time space, y(t).
   h. Referring to both the time space and the frequency space, describe briefly what happens to the signal as it passes through the filter.

7. [10 marks] We have shown that a sampled signal in frequency space is

$$\hat{S}(\nu) = \sum_{n=-\infty}^{+\infty} s(nT)e^{-2\pi i\nu nT}.$$

   a. Show that is a periodic function.
   b. Discuss the ramifications for setting the sampling rate.
   c. Find the critical sampling rate.

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