**ENGINEERING PHYSICS 3W4**

DAY CLASS Dr. Wm. Garland

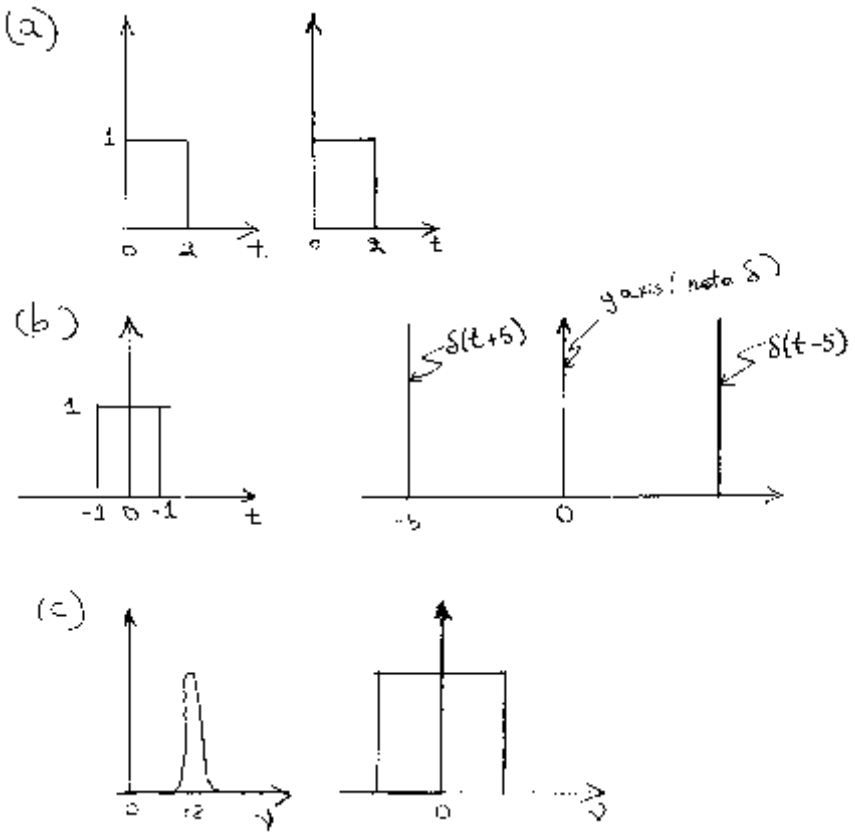
DURATION: 90 minutes

McMASTER UNIVERSITY MIDTERM March 2, 2000

**Special Instructions**: Closed Book. All calculators and up to 3 double sided 8 ½" by 11" crib sheets are permitted.

**THIS EXAMINATION PAPER INCLUDES 2 PAGES AND 5 QUESTIONS WORTH A TOTAL OF 70 MARKS.**

1. [15 marks] Graphically compute the convolution of the following function pairs:




2. [15 marks]

   a. What properties must f(t) have for the Fourier Transform (FT) to exist?

   b. If f(t) = E(t) + O(t), ie is the sum of its even and odd parts, show that

$$E(t) = \frac{f(t)+f(-t)}{2} \quad \text{and} \quad O(t) = \frac{f(t)-f(-t)}{2}$$

   c. Show that the FT of O(t) is imaginary.

3. [10 marks]

   a. If f(t) $\leftrightarrow$ F(<), what is the FT of f(t+a)? A proof is not required.
   b. What is the FT of $A_a(t)$? A proof is not required.
   c. What is the FT of exp(-Bt$^2$)? A proof is not required.
   d. What is the FT of *(t)? A proof is not required.
   e. What is the FT of *(t-a)? A proof is not required.

4. [15 marks]

   a. We have shown that *(t-$t_0$)+ *(t+$t_0$) $\leftrightarrow$ 2 cos(2 B< $t_0$). The converse is also true, ie, cos(2 B<$_0$ t) $\leftrightarrow$ ½ [*(<-<$_0$)+ *(<+<$_0$)]. Why? A detailed proof is not necessary. Just defend your statements by noting what you can infer from the properties of the above functions.
   b. Sketch what x(t) cos(2 B<$_0$ t) would look like in the frequency domain if x(t) were some function, say a Gaussian?
   c. We can think of x(t) as an amplitude modulation function and cos(2 B<$_0$ t) as a carrier signal. How could this be used to transmit a voice signal (kHz frequency range) over several radio stations in the MHz range?

5. [15 marks]

   a. Compute the FT of $\frac{d}{dt}\left[e^{-Bt^2}(A_a(t)\right]$. Hint: What is $\frac{d}{dt}\left[A_a(t)\right]$?
   b. Sketch the FT for 'a' large compared to B and for 'a' small compared to B.

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