ENGINEERING PHYSICS 4D3/6D3

DAY CLASS — Dr. Wm. Garland
DURATION: 3 hours —
McMASTER UNIVERSITY FINAL EXAMINATION — December 6, 2000

Special Instructions:

1. Closed Book. All calculators and up to 8 single sided 8 ½" by 11" crib sheets are permitted.
2. Do all questions.
3. The value of each question is as indicated.
4. Point form is sufficient for discussion type questions.

TOTAL Value: 100 marks

THIS EXAMINATION PAPER INCLUDES 4 PAGES AND 8 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

1. [10 marks]
   a. Define the following:
      i. Neutron Current
      ii. Neutron Flux
      iii. Neutron Density
      iv. Neutron Fluence
   b. Relate B_g^2 to B_m^2 for the 3 distinct criticality classifications (critical, sub-critical and super-critical). Explain the physical meaning of each case.
   c. Construct a general and comprehensive depletion / buildup rate equation for a nuclide, accounting for self decay, parental decay, neutron capture and transmutation. Explain each effect briefly.

2. [10 marks] The general multigroup neutron diffusion equations with delayed precursors and associated equations are given at the end of this exam paper.
   a. Define each variable. Explain the significance of each term. Be brief; a few words per variable or term is sufficient.
   b. Briefly describe how poisons and fuel depletion affect the flux equations. How is this compensated for in an operating reactor?

3. [10 marks] Starting with the general multigroup neutron diffusion equations with delayed precursors and associated equations are given at the end of this exam paper, derive the following simplifications:
   a. Two group (fast and thermal) neutron diffusion, two dimensional, transient equations for a heterogeneous reactor in Cartesian coordinates. Make reasonable assumptions and defend them. Ignore the poison and fuel equations.
   b. As in (a) but steady state. Show that the precursors drop out of the flux equations; what does this mean physically?

continued on page 2...

4. [10 marks] Consider the overly simple case of an infinite, homogeneous reactor modelled using one group theory with no delayed precursors. There is a constant source term, S, providing a small number of neutrons per second. The reactor has fuel and absorbing material. We are interested in the startup behaviour of the reactor. The governing flux equation is:

$$\frac{1}{v}\frac{\partial \phi}{\partial t} = (\nu \Sigma_f - \Sigma_a)\phi + S$$

a. Starting from an initial condition of zero flux and a super-critical reactor, what is the initial time behaviour of the flux?

b. As time goes on, the flux rises significantly so that the source, S, is now negligible. What is the time pehaviour of the flux now?

c. If the source, S, existed only briefly at t=0, thus creating an initial small flux at t=0, what would the initial time behaviour of the flux be?

d. Given that in real reactors there is a finite detection limit for neutron flux, what are the implications for startup of having a fixed source of neutrons available?

5. [10 marks] From the information given below, estimate the diffusion length, L, for H_2O and D_2O based on one group diffusion theory. Discuss the differences between two reactors of the same size, both critical, but one contains H_2O moderator while the other contains D_2O moderator. What does this imply for reactor design?

Parameter	H_2O	D_2O
Σ_a	0.022 cm^{-1}	3.3 x 10^{-5} cm^{-1}
Σ_s	3.45 cm^{-1}	0.449 cm^{-1}
μ_0	0.676	0.884

[Hint: Compare $B^2 = \frac{(\nu\Sigma_f/\Sigma_a - 1)}{L^2}$]

[Recall: $D = \frac{1}{3(\Sigma_t - \mu_0\Sigma_s)}$]

continued on page 3...

6. [20 marks] A reactor is undergoing a power excursion with a measured period, T, seconds ($\omega = 1/T$). Using the point kinetics model and the inhour equation:
   a. Show that for small reactivity insertions ($\rho << \beta$), the period is dominated by the delayed precursor time constant, not the neutron lifetime, ℓ.
   b. Show that for large reactivity insertions ($\rho >> \beta$), the period is dominated by the neutron lifetime, ℓ, not the delayed precursors.
   c. In light of the above, compare the transient response of the CANDU reactor to the PWR reactor under normal operational transients and under accident conditions. What are the safety implications? Some relevant characteristics of these reactor types are given in the following table.

Reactor Type	Neutron Lifetime	Control Rods	Void Feedback
PWR, pressure vessel, batch refuelling	1×10^{-4} seconds	large worth, penetrates pressure boundary	negative
CANDU, pressure tube, on-line refuelling	1×10^{-3} seconds	low worth, does not penetrate pressure boundary	positive

7. [20 marks] As a young and upcoming nuclear engineer, you have been assigned the task of building an "All Singing, All Dancing" reactor physics computer program, named ASAD, to solve the general multigroup neutron diffusion equations with delayed precursors and associated equations as listed at the end of this exam paper. The code is to be three-dimensional, Cartesian geometry, transient reactor core solver for MNR. Assume the cell properties are known. In the x and y direction, each cell is a fuel assembly, control assembly, or reflector assembly, etc. roughly 7 cm by 7 cm. The vertical dimension, z, is to be divided up into 10 grid points or so. Model the pool by a few exterior water cells. The core is a 6 by 9 grid of assemblies. Don't get hung up on the details. I am looking for the overall approach.
   a. Derive the difference equations for the numerical solver for the time and space dependent equations. Don't forget the precursors, poisons and fuel depletion equations. And don't forget to include a reactivity control scheme.
   b. Show how you can efficiently and effectively use the same code to solve a variety of problems ranging from short term flux response to reactivity perturbations like prompt jumps and reactor trips, to flux control simulations where the delayed precursors need to be tracked accurately, to poison calculations, to long term fuel management and associated reactivity control. Focus on adjusting the time constants of phenomena that does not need to be tracked accurately for the problem at hand. It's trivial really.

8. [10 marks] You have been asked to augment ASAD by adding a thermal hydraulics model. Coolant enters the top of the core and exits at the bottom. The flow, W kg/s per fuel assembly, is known. Assume uniform properties within each 'cell' of the x-y plane. Also assume that the fuel plates are thin, highly conductive and with negligible heat capacity so that you need not be concerned with the temperature distribution within the fuel plates. Derive the difference equations for the axial temperature distribution of the coolant as a function of time.

continued on page 4...

Additional Information:

The general multigroup neutron diffusion equations with delayed precursors are given by:

$$\frac{1}{v_g}\frac{\partial \phi_g}{\partial t} = \nabla\cdot D_g \nabla \phi_g - \Sigma_{a_g}\phi_g - \Sigma_{s_g}\phi_g + \sum_{g'=1}^{G} \Sigma_{s_{g'g}}\phi_{g'}$$

$$+ \chi_g(1-\beta)\sum_{g'=1}^{G} \nu_{g'}\Sigma_{f_{g'}}\phi_{g'} + \chi_g^C \sum_{i=1}^{N}\lambda_i C_i + S_g^{ext}$$

$$\frac{\partial C_i}{\partial t} = -\lambda_i C_i + \beta_i \sum_{g'=1}^{G} \nu_{g'}\Sigma_{f_{g'}}\phi_{g'}$$

Note that ϕ_g and C_i are functions of $\underline{r}$ and t but the notation has been dropped for clarity. The poison equations are:

$$\frac{\partial I(\underline{r},t)}{\partial t} = \gamma_I \sum_{g'=1}^{G} \Sigma_{f_{g'}}\phi_{g'}(\underline{r},t) - \lambda_I\, I(\underline{r},t)$$

$$\frac{\partial X(\underline{r},t)}{\partial t} = \gamma_X \sum_{g'=1}^{G} \Sigma_{f_{g'}}\phi_{g'}(\underline{r},t) + \lambda_I\, I(\underline{r},t) - \lambda_X\, X(\underline{r},t) - \sum_{g'=1}^{G} \sigma^X_{a_{g'}}\phi_{g'}(\underline{r},t)\; X(r,t)$$

and the fuel depletion equation is:

$$\frac{\partial N_f}{\partial t} = -\; N_f(\underline{r},t) \sum_{g'=1}^{G} \sigma^f_{a_{g'}}\phi_{g'}(\underline{r},t)$$

The point kinetic equations are:

$$\frac{dn}{dt} = \left(\frac{\rho-\beta}{\Lambda}\right)n(t) + \sum_{i=1}^{N}\lambda_i C_i$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{\Lambda}n(t) - \lambda_i C_i$$

The associated 'inhour equation" is:

$$\rho = \frac{\omega\ell}{(1+\omega\ell)} + \frac{1}{(1+\omega\ell)}\sum_{i=1}^{N}\frac{\omega\beta_i}{(\omega+\lambda_i)}$$

The "inverse method" equation is:

$$\rho(t) = \beta + \frac{\Lambda}{n(t)}\frac{dn}{dt} - \beta\int_0^{\infty}\frac{D(\tau)n(t-\tau)}{n(t)}d\tau \quad \text{where } D(\tau) = \sum_i \frac{\lambda_i\beta_i}{\beta}e^{-\lambda_i\tau}$$

Mathematical relationships:

$$\int_0^{\infty} te^{-\lambda t}dt = 1/\lambda^2$$

...The End...