Summary:
The basic definitions and perspectives for the behaviour of free neutrons as they interact with their surrounding media are introduced. This forms the basis for the detailed study to follow.

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1 Introduction

1.1 Overview

Figure 1 Course Overview
1.2 Learning outcomes

1.2.1 To understand the following physical processes
- fission
- neutron life cycle
- the neutron environment
- neutron energy distribution

1.2.2 To understand the basics of neutron processes
- decay
- absorption and scattering
- kinematics

1.2.3 To understand the main issues for reactor modelling
2 The Life and Times of the Neutron

2.1 The Fission Event

The neutron, which is uncharged, can interact with a U\textsuperscript{235} nucleus leading to fission.

The result is the creation of fission products (which may be radioactive), radiation (usually $\gamma$’s and $\beta$’s) and 2 to 3 neutrons at high energy (1-2 MeV).

The probability of the event is a strong function of the neutron energy, as shown next.

![Figure 2: The fission event](image)

$$Figure\ 2\ The\ fission\ event$$

![Figure 3: Fission cross section of U-235](image)

$Figure\ 3\ Fission\ cross\ section\ of\ U-235\ [source:\ DUD1976,\ figure\ 2-17]$  

$\sigma \equiv \text{microscopic cross section [cm}^2\text{]} = \text{effective interaction area}$  

1 barn $\equiv 1 \times 10^{-24}$ cm$^2$  

$\sigma$ is usually quoted in units of barns since the effective area is so small.
2.2 Neutron Life Cycle in CANDU

Figure 4 Neutron life cycle [source: unknown]
Neutrons Slowing Down

- When the number of slow neutrons is constant, the system is critical.
- Delayed neutrons appear after ~10 seconds.
- Fast neutrons slow down in ~1 millisecond.

Leaked Neutrons

- Prompt neutrons from fission
- "ASHES" (Fission Products)
- Heat
- U235 Fission
- Slow neutrons
- Captured neutrons (some neutrons are captured in U238 and so produce useful fuel - Pu239)
- Neutrons diffusing
- Delayed neutrons (from fission)
- Neutrons slowing down
- Leaked neutrons

Figure 1 – The Neutron Cycle in a Thermal Reactor

Figure 5 – Another view of the neutron life cycle [source: EP712 course notes, chapter 2]
2.3 Density of neutrons required to produce 1 watt/cm$^3$

Consider a beam of neutrons moving at velocity, $v$ cm/s.

The average distance travelled before a fission interaction is $\bar{X}$ cm (in U$^{235}$)

\[ \therefore \text{Average time per interaction} = \frac{\bar{X}}{v} \text{ seconds and frequency of interaction} = \frac{v}{\bar{X}} \text{ s}^{-1} \text{ per neutron.} \]

If the density of neutrons is $n$ neutrons/cm$^3$, then interaction rate $= \frac{nv}{\bar{X}}$ interactions/s-cm$^3$.

For

\[ \frac{1 \text{ watt}}{\text{cm}^3} = \frac{1 \text{ Joule}}{s - \text{cm}^3}, \]

\[ 1 \frac{\text{J}}{s - \text{cm}^3} = \text{energy} \times \frac{\# \text{fissions}}{\text{fission}} \times \text{s}^{-1} \text{ cm} \]

\[ = 200 \times 10^6 \text{ eV} \times 1.602 \times 10^{-19} \frac{\text{Joules}}{\text{eV}} \times \frac{nv}{\bar{X}} \]

\[ \therefore \text{n} = \frac{\bar{X} \sim 1 \text{ cm}}{2 \times 10^8 \times 1.6 \times 10^{-19} \times v \sim 2 \times 10^5 \text{ cm/s}} \]

\[ = 1.5 \times 10^5 \text{ n/cm}^3 \]

Compare this to the typical nuclei density $\sim 10^{22}$/cm$^3$

Conclusion: Neutrons do not interact with each other.

This is an important conclusion.
2.4 Neutron Energy

Thermal distribution:

\[ n(v) = 4\pi \left( \frac{m}{2\pi kT} \right)^{3/2} n_0 v^2 e^{-mv^2/2kT} \]

\[ \downarrow \]

\[ n(E) = \frac{2\pi n_0}{(\pi kT)^{3/2}} E^{1/2} e^{-E/kT} \]

\[ \phi(E) \equiv n(E) v = v n_0 M(E) \]

\[ = \frac{2\pi n_0}{(\pi kT)^{3/2}} \left( \frac{2}{m} \right)^{1/2} E e^{-E/kT} \]

Maxwellian Distribution

Now, \( n_o = \int_0^\infty n(E) dE = \int_0^\infty n(v) dv \)

Note: \( n(E) = \# \) of neutrons in interval dE [\# / eV]

\( n(v) = \# \) of neutrons in interval dv [\# / (m/s)]

Thus \( n \left( \frac{1}{2} mv^2 \right) \neq n(v) \) since interval size is different

But \( n(E) \ d(E) = n(v) \ dv \) so that \( \int_0^\infty n(E) dE = \int_0^\infty n(v) dv \)

Most probable vel:

\[ \frac{dn(v)}{dv} = 0 \Rightarrow v_p = \sqrt{\frac{2kT}{m}} \]

\[ = 2200 \text{ m/s} \]

\[ \Rightarrow E(p) = kT = 0.025 \text{ eV at 20° C} \]

Most probable energy:

\[ \frac{dn(E)}{dE} = 0 \Rightarrow E_p = \frac{1}{2} kT \]

\[ \bar{E} = \frac{3}{2} kT \]

\[ \frac{1}{v} = \sqrt{\frac{8kT}{\pi m}} \]
Figure 6 Neutron Energy Distribution
2.5 Units

\[ V_p = \sqrt{\frac{2kT}{m}} = \sqrt{\frac{2 \times 1.3806 \times 10^{-23} \text{ Joules} / \text{K} \times 293.13 \text{K}}{1.67 \times 10^{-27} \text{kg}}} \]

= 2201 m/s

\[ E \equiv \text{gm} \frac{\text{cm}^2}{\text{sec}^2} = \text{erg} \quad \text{or} \quad \text{kg} \frac{\text{m}^2}{\text{sec}^2} = \text{Joules} \]

Recall:

\[ F = ma \Rightarrow \text{dyne} = \text{gm cm/sec}^2 \]

\[ \therefore E = F \cdot x = \text{dyne - cm = erg} = 10^{-7} \text{ J.} \]
3 1/E Spectrum

3.1 Derivation of 1/E spectrum (equation 8-14 of D & H)

Assume the neutron is slowing down in H in the absence of absorption. Further assume that there is no upscatter.

\[
\frac{[\Sigma_s(E) + \Sigma_a(E)]\phi(E)}{\text{# of neutrons leaving energy } E} = \int_0^\infty \Sigma_s(E') \phi(E') \, dE' + S(E)
\]

Since \(\Sigma_a(E) = 0\), we have

\[
\frac{\Sigma_s(E)\phi(E)}{\text{equal probability of scatter (isotropic)}} = F(E)
\]

\[
\therefore F(E) = \int_{E_0}^{E_a} \frac{F(E')}{E'} \, dE' + S_0 \delta(E - E_0)
\]

\[
\frac{d}{dE} F(E) = -\frac{1}{E} F(E)
\]

\[
F(E) = \frac{S_0}{E} + S_0 \delta(E - E_0)
\]

\[
\therefore \phi(E) = \frac{S_0}{\Sigma_s(E) E} + \frac{S_0}{\Sigma_s(E)} \delta(E - E_0)
\]

\[
\Sigma_s \sim \text{const} \Rightarrow \phi \sim \frac{1}{E}
\]

At this point you should be able to answer Questions 1, 2, 3 and 4 at the end of this chapter.
4 Decay

\[- \frac{dN(t)}{dt} = \lambda N(t)\]

\[
N(t) = N_0 e^{-\lambda t}
\]

\[
\therefore - \frac{dN(t)}{dt} = \text{RATE} = \lambda N_0 e^{-\lambda t}
\]

\# decaying in dt at \(t\) = \(-\frac{dN(t)}{dt}\) = \(\lambda N_0 e^{-\lambda t}\) dt

fraction of initial decaying in dt at \(t\) = \(\lambda e^{-\lambda t}\) dt

mean lifetime,

\(\bar{t} = \int_0^\infty p(t) dt = \lambda \int_0^\infty t e^{-\lambda t} dt = \frac{1}{\lambda}\)

\[
\therefore \bar{t} = \frac{1}{\lambda}
\]

Half Life, \(T_{1/2}\)

\(N(T_{1/2}) = \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}\)

\[
\Rightarrow T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}
\]
4.1 Math aside

If we have two functions $f(x) + g(x)$:

$$d(fg) = f'g + g'f \quad \Rightarrow \quad \int d(fg) = fg$$

$$= \int f'gdx + \int g'fdx$$

$$\therefore \int_0^\infty \frac{t}{f} e^{\lambda t} dt = -\frac{te^{-\lambda t}}{\lambda} \bigg|_0^\infty - \int_0^\infty \frac{e^{\lambda t}}{(-\lambda)} dt$$

$$= 0 + \frac{1}{\lambda^2}$$
4.2 Example (D&H #2.3)

Decay chain for an initially pure radioactive sample.

\[
\frac{dN_1}{dt} = -\lambda_1 N_1 \quad \Rightarrow \quad N_1(t) = N_1(0) e^{-\lambda_1 t}
\]

\[
\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \quad \Rightarrow \quad N_2(t) = \frac{\lambda_1 N_1(0)}{\lambda_2 - \lambda_1} [e^{-\lambda_1 t} - e^{-\lambda_2 t}]
\]

\[
\frac{dN_3}{dt} = \lambda_2 N_2 - \lambda_3 N_3 \\
\vdots \\
\frac{dN_N}{dt} = \lambda_{N-1} N_{N-1} - \lambda_N N_N
\]

\[\Rightarrow \text{messy solutions}\]

To solve for \(N_2\):

Rewrite to get:

\[
\frac{dN_2}{dt} + \lambda_2 N_2 = \lambda_1 N_1
\]

\[\Rightarrow \quad d N_2 + \lambda_2 N_2 dt = \lambda_1 N_1 dt\]

Multiply by \(e^{\lambda_2 t}\):

\[\Rightarrow \quad e^{\lambda_2 t} d N_2 + \lambda_2 e^{\lambda_2 t} N_2 dt = \lambda_1 N_1 e^{\lambda_2 t} dt\]

\[\Rightarrow \quad d(e^{\lambda_2 t} N_2) = \lambda_1 N_1(0)e^{(\lambda_2 - \lambda_1)t} dt\]

\[\therefore \quad e^{\lambda_2 t} N_2 = \frac{\lambda_1 N_1(0)}{\lambda_2 - \lambda_1} e^{(\lambda_2 - \lambda_1)t} + C\]
Now at $t = 0$, $N_2(0) = 0$ \[ \Rightarrow C = \frac{-\lambda_1 N_1(0)}{\lambda_2 - \lambda_1} \]

\[ \therefore N_2(t) = \frac{\lambda_1 N_1(0)}{\lambda_2 - \lambda_1} [e^{(\lambda_2 - \lambda_1)t} - 1] e^{-\lambda_2 t} = \frac{\lambda_1 N_1(0)}{\lambda_2 - \lambda_1} [e^{-\lambda_1 t} - e^{-\lambda_2 t}] \]

For fast decay of 1 and slow decay of 2 ($\lambda_1 >> \lambda_2$)

\[ N_2(t) \sim \frac{\lambda_1 N_1(0)}{\lambda_2 - \lambda_1} [e^{-\lambda_1 t} - e^{-\lambda_2 t}] \]

\[ \sim N_1(0) e^{-\lambda_2 t} \]

i.e. decay dominated by decay of 2.

For slow decay of 1 and fast decay of 2 ($\lambda_2 >> \lambda_1$)

\[ N_2(t) \sim \frac{\lambda_1 N_1(0)}{\lambda_2} e^{-\lambda_1 t} = N_1(t) \frac{\lambda_1}{\lambda_2} \]

i.e. $\lambda_2 N_2(t) = \lambda_1 N_1(t)$

This is called “secular equilibrium”.

(lasting a long time, indifferent, not religious)
5 Cross Section

5.1 Microscopic cross section, $\sigma$ [cm$^2$]

Consider a beam of neutrons incident on a target. The rate of interaction (neutron-nuclei) is

$$\text{Rate of interaction} = \sigma \frac{I}{n} \frac{N}{cm^2} [\equiv \frac{\#}{cm^3 \cdot s}]$$

Recall that 1 barn = $10^{-24}$ cm$^2$

The total cross section, $\sigma_{\text{total}} = \sigma_{\text{scatter}} + \sigma_{\text{absorption}}$

ie. $\sigma_T = \sigma_s + \sigma_a$

Diagram:

- Total
- Scatter
  - Inelastic
  - Elastic
- Absorption
  - Fission
  - Capture $(n, \gamma)$
  - $(n, 2n)$
  - $(n, 3n)$
  - $(n, p)$
  - $(n, \alpha)$
5.2 Example (D & H 2.7)

Question: How long, on average for a given nuclei to suffer a neutron interaction?

\[
\text{Rate} = \frac{\sigma I}{N} = 4 \times 10^{-24} \times 10^{12} \text{ interactions/sec} = 4 \times 10^{-12} \text{ interactions/sec for 1 nuclei}
\]

\[
\therefore \text{ seconds/interactions for 1 nuclei} = \frac{1}{4 \times 10^{-12}} \text{ s} = 2.5 \times 10^{11} \text{ seconds}
\]
5.3 Macroscopic Cross Section, $\Sigma$ [cm$^{-1}$]

Rate $= \sigma I N \equiv \Sigma I$

$$= -\frac{dI}{dx}$$

$$\Sigma \equiv \sigma N \left[ \frac{cm^2 \cdot \#}{cm^3} \right]$$

$$= cm^{-1}$$

$$\left( -\frac{dI}{I} \right) = \Sigma$$

= fractional change of $I$ in distance $dx$

= probability of reaction per unit length

$$\downarrow$$

$$I(x) = I_o e^{-\Sigma x}$$

$$\frac{I(x)}{I_o} = \text{probability of going } x \text{ with no interaction}$$

$$= e^{-\Sigma x}$$

Probability of interaction at $x$ in $dx$ is: $(p(x)dx)$

$$-\frac{dI}{I_o} = \frac{I(x)}{I_o} \cdot \Sigma dx = \int \frac{I(x)}{I_o} \cdot \Sigma e^{-\Sigma x} \, dx$$

At this point you should be able to answer Questions 5 at the end of this chapter.
5.4 Mean Free Path

\[ \bar{x} = \int_0^\infty p(x) \, dx = \int_0^\infty \Sigma e^{-\Sigma x} \, dx \]
\[ = \frac{1}{\Sigma} = \text{mean free path} \]

cf: \[ \bar{t} = \int_0^\lambda \lambda e^{-\lambda t} \, dt = \frac{1}{\lambda} \]

Mean time between collisions:

\[ \frac{\bar{x}}{\text{velocity}} = \frac{1}{v\Sigma} \]

Collision frequency = \( \frac{1}{\text{time}} = v\Sigma \)

5.5 Calculation of Nuclei Density

\[ \Sigma_t = \Sigma_a + \Sigma_s \]

\[ \Sigma_s = \sum_i \Sigma_{i_s} \]

\[ \Sigma_a = N_x \sigma_{a_x} + N_y \sigma_{a_y} + ... \]

\[ = \sum_i N_i \sigma_{a_i} \]

\[ N_i = A \left( \frac{\#}{\text{gm - mole}} \right) \cdot \rho \left( \frac{\text{gm}}{\text{cm}^3} \right) \]

where \( A = \text{Avogadro's number}, 6.0221367 \times 10^{23} \)
6 Nuclear Reactions

Reactions:

\[ a + b \rightarrow c + d \]
\[ a \ (b, c) \ d \]

Example:

\[ _{92}^{235}U + _{0}^{1}n \rightarrow _{92}^{236}U + \gamma \]

Radioactive Capture: \((n, \gamma)\)

Fission: \(n + X \rightarrow X_1 + X_2 + \gamma + n + \text{energy} \)

Scattering: \((n, n)\) elastic
\((n, n')\) inelastic

Source of neutrons:
1. Fission
   A. Initiated by cosmic radiation
   B. Spontaneous
   C. Neutron absorption
2. \((\alpha, n)\)

\[ _{4}^{9}Be + _{2}^{4}He \rightarrow _{6}^{12}C + _{0}^{1}n \]
from radium, Pu, polonium

3. \((\gamma, n)\) (photoneutrons)

\[ _{1}^{2}H + \gamma \rightarrow _{1}^{1}H + _{0}^{1}n \]
\[ _{9}^{4}Be + \gamma \rightarrow _{4}^{8}Be + _{0}^{1}n \]
Figure 7 Nuclear Transformations [Source: A. A. Harms, McMaster University]
Figure 8 Segment of the Chart of the Nuclides [Source: A. A. Harms, McMaster University]
Figure 9 Number of neutrons and protons in stable nuclei [Source: A. A. Harms, McMaster University]
7 Summary

7.1 Summary of key concepts

- neutrons do not interact with each other
- life cycle
- neutron energy spectrum
- decay
- chart of the nuclides
- $\sigma, \sum$
- mechanics of collision (addendum)
7.2 Summary of approximations

\textbf{Boltzmann Transport Eqn's}

\[ n (F, E, \Omega, t) \]

\[ c_0 (F, E, t) \]

\textit{became isotropic}

\[ n (F, E, t) \]

\[ c_i (F, E, t) \]

\textit{continuum of Energy \rightarrow neutron energy groups}

\[ n_g (F, t), g = 1, \ldots, G \]

\[ c_g (F, t), i, \ldots, N \]

\textit{number of energy groups}

\textit{number of delayed precursor groups}

\textit{Transport of neutrons approximated by Diffusion theory}

\textbf{Diffusion eqn's}

\[ (n_g (F, t), c_g (F, t)) \]

\textit{where} \[ n_0 = DU \nu \]

\textit{Choice of simplifications}

\textit{Multigroup neutron diffusion eqn's}

\textit{Fick's law}

\textit{Multi-group steady state diffusion one \textbf{delayed precursor} group (F, t)}

\[ n_g (F, t) \]

\[ c_g (F, t) \]

\textit{for short times (seconds)}

\textit{delayed precursor investigations (neutron poisoning) etc}

\[ t \sim \text{sec} \]

\textit{One group (F, t)}

\[ n_0 (F, t), c_0 (F, t) \]

\textit{space-time kinetics for longer times (minutes hours)}

\[ t \sim \text{min} \]
8 A Look Ahead

8.1 The neutron balance

Figure 10 Neutron processes

Neutron balance:

\[
\frac{\partial n(r, t)}{\partial t} \equiv \frac{1}{v} \frac{\partial \phi(r, t)}{\partial t} = S(r, t) - \sum_a \phi(r, t) - \nabla \cdot J(r, t) \tag{2}
\]

\[= + \nabla \cdot D \nabla \phi(r, t) \text{ from Fick's Law: } J \equiv -D \nabla \phi\]

Reactor physics is all about the calculation of the neutron density, n, or flux, \(\phi\).
8.2 The Central Role of Flux

- Fick’s Law
- Neutron balance eqn. (continuity eqn.)
- Sinks, $\Sigma_a, \Sigma_s$

# Dim.
SS or tran.
Delayed precursors
# of groups
variance in material properties.
-depletion
-inhomo
-control
-power dependence (Temperature effects)
Radiation damage on structure

Effect on samples
($\gamma$, heating, $\phi$, etc.)

t/h calc. $\Rightarrow$ T, P, density flows

Dimensions # of grid pts. Type of cell at each grid pt.

Radiation damage on structure

$\phi_g (r, t, E, \Omega)$

$N_i (r, t)$

$C_i (r, t)$

- Say 69 groups

Fuel Control

Moderator

Cell Materials & Composition

1

Cell Defn (group collapse)

Grid Prop Setup

Grid

Initial Flux

Flux calc’s
- Multigroup diffusion
- Transport
- Monte Carlo

Mat. Lib.

Burnup

Fuel Management

Fission Products
- Xenon
- delayed precursors

Detector Response

Control

Rod position

Effect on samples
($\gamma$, heating, $\phi$, etc.)

t/h calc. $\Rightarrow$ T, P, density flows

Dimensions # of grid pts. Type of cell at each grid pt.

Radiation damage on structure

$\phi_g (r, t, E, \Omega)$

$N_i (r, t)$

$C_i (r, t)$

- Say 69 groups

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Mat. Lib.

Burnup

Fuel Management

Fission Products
- Xenon
- delayed precursors

Detector Response

Control

Rod position
9 Some Questions

9.1 Question on characteristics

Given this brief look at neutrons and their life cycle, what are some of the issues/characteristics that you would expect to arise in the design of a nuclear power plant?

9.2 Reactor Modelling Issues

Imagine a reactor consisting of a central fuel region surrounded by a moderator. There is a variable absorber for control. What are some of the issues to consider in setting up a model of the reactor?

9.3 Question of n(E)

Illustrate on a graph of n(E) vs. ln(E) the life cycle of a neutron in a fission reactor.

9.4 Question of non-Maxwellian

Illustrate how the thermal neutron spectrum differs from a Maxwellian and explain why.

9.5 Question on Cross section

Consider:

\[ I(x) = I_0 e^{-\Sigma x} \]

What are some of the assumptions in or limitations of this equation?

What are some of the things implied by this equation?