CHAPTER 3: ELEMENTARY PHYSICS OF REACTOR CONTROL

MODULE B: REACTOR KINETICS

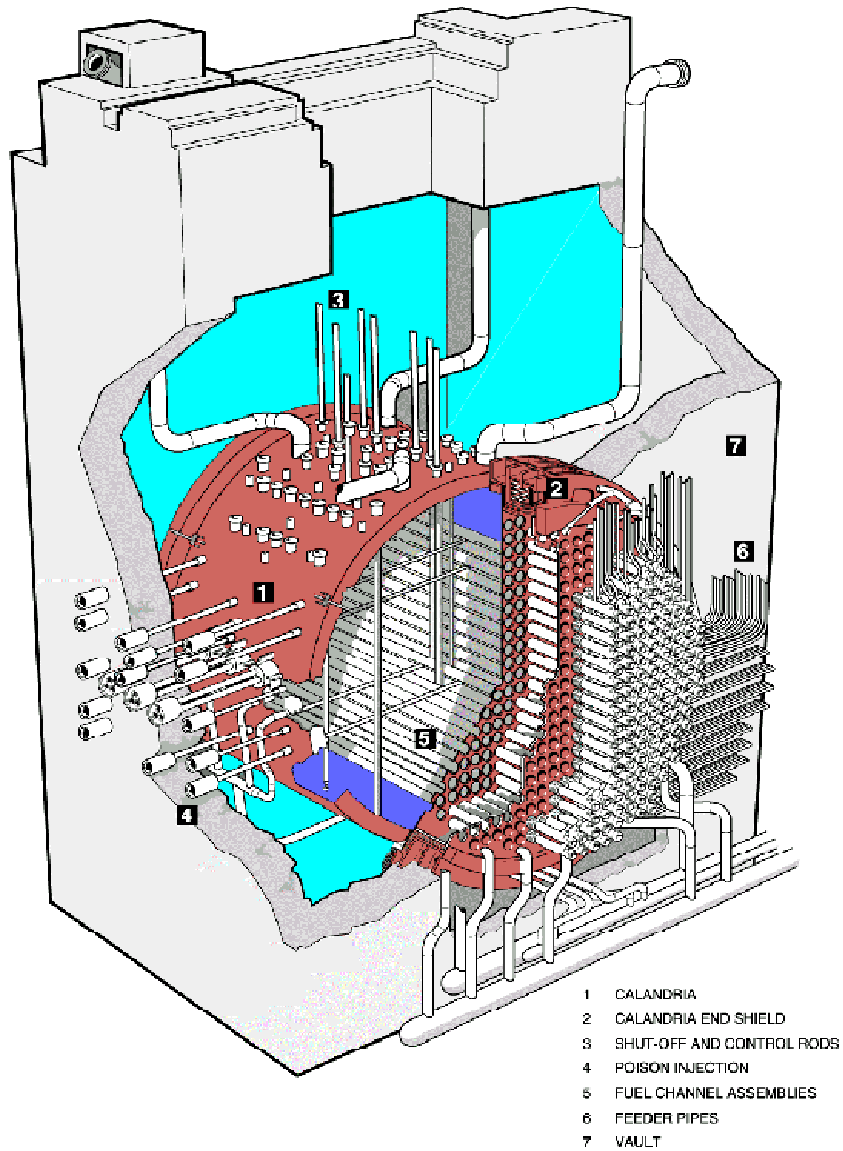
MODULE OBJECTIVES:

At the end of this module, you will be able to describe:

1. **the response of a nuclear reactor to a positive step change in reactivity**
2. **what is meant by 'reactor period'**
3. **the importance of delayed neutrons to reactor control**
4. **the prompt jump approximation**
5. **what is meant by 'prompt critical'**
6. **the response of a nuclear reactor to large negative reactivity insertions, such as reactor trips**
7. **the significance of the various terms in the point kinetics equations**

1. INTRODUCTION

- **in the previous module we looked at the factors that affect neutron multiplication in a reactor, and defined the term 'reactivity' as the measure of the deviation of the reactor from criticality**
- **in this module we develop simple physical and mathematical models for the rate of change of the neutron population as a function of reactivity**
- **we will find that if all the neutrons released in the fission process appeared at the instant fission takes place, then controlling the nuclear chain reaction would not be practical**
- **a small percentage of the neutrons appear some time after fission has taken place when certain fission products decay by neutron emission: the resultant neutrons are called 'delayed neutrons'**
- **a simple mathematical representation of the time dependent neutron flux response to various reactivity changes will be developed**
- **we will study the response of the reactor to large positive and negative step changes in reactivity, and understand why reactivity should only be changed at a slow rate during normal reactor operations**



2. REACTOR RESPONSE TO A POSITIVE STEP CHANGE IN REACTIVITY

We had seen in the previous module that the change in the neutron density over one generation can be written as:

$$\Delta N = kN - N$$

where N is the neutron density in one generation, and

kN is the neutron density in the next generation

By definition, this change takes place in one neutron lifetime 'ℓ' giving the rate of change in neutron population as:

$$\frac{\Delta N}{\Delta t} = \frac{kN - N}{\ell} \quad \text{or} \quad \frac{dN}{dt} = \frac{kN - N}{\ell} = \frac{N}{\ell}(k-1) = \frac{N}{\ell}\delta k$$

The solution to this equation is

$$N(t) = N_0 e^{\frac{\delta k}{\ell} t} \qquad \textit{(3B – 1)}$$

where N_0 is the neutron density at t = 0 and N(t) is the neutron density at time t; i.e. the neutron flux and therefore the power produced by the reactor will increase exponentially with time at a rate determined by the ratio of reactivity (δk) to the neutron lifetime (ℓ).

3. REACTOR PERIOD

In working with control systems one usually defines parameters called 'time constants' that indicate how long it takes for a given parameter to change by a specified factor. The parameter commonly used for nuclear reactors is the 'reactor period', which is the time taken for the neutron flux to increase by a factor of 'e' (base for natural logarithm, e = 2.7183). The symbol τ will be used for the reactor period.

The reactor period may be determined by setting $t = \tau$, which corresponds to a neutron flux increase from N_0 to eN_0. Substituting into equation (3B – 1), we get

$$e\,N_0 = N_0\, e^{\frac{\delta k}{\ell}\tau} \quad \text{or} \quad e = e^{\frac{\delta k}{\ell}\tau} \quad \text{and therefore} \quad \frac{\delta k}{\ell}\tau = 1$$

The expression for the reactor period in a supercritical reactor is therefore

$$\tau = \frac{\ell}{\delta k} \qquad \text{and equation } \textit{(3B – 1)} \text{ can now be written as} \qquad N = N_0\, e^{\frac{t}{\tau}}$$

Consider a step increase in reactivity of δk = 4 mk.
For heavy water moderated reactors ℓ is of the order of 0.001 sec

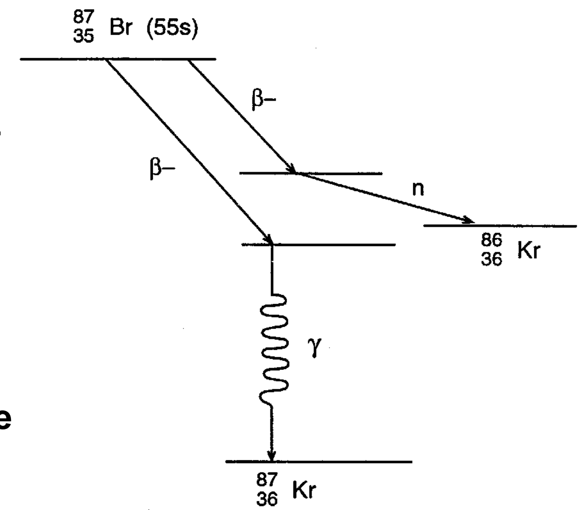
Under these conditions $$\tau = \frac{\ell}{\delta k} = \frac{0.001}{0.004} = 0.25\,\text{sec} \qquad \text{and} \qquad N = N_0 e^{\frac{1}{0.25}} = N_0 e^4 = 55\,N_0 \ \text{ in 1 sec}$$

It would be extremely difficult to construct a control system for a reactor that responded in this manner.

However, not all fission neutrons are 'prompt' i.e. released directly at the time the fissile nuclei fissioned. A small fraction of the neutrons are released during the neutron decay of fission products, and these have decay times that range from 0.3 to 80 seconds. Although only a small fraction (less than 1%) of fission neutrons are 'delayed' in this manner, since their half-lives are much longer than the neutron lifetime, the average lifetime of prompt plus delayed neutrons is much longer than 0.001 second.

4. DELAYED NEUTRONS

In most case the nuclei of fission products do not have sufficient excitation energy to emit a neutron. However, some nuclei that are formed following β decay may have an excitation energy that results in neutron decay.



One example is Br-87, which decays with a half-life of 55 sec to form Kr-87, and some of these Kr-87 nuclei may have excitation energies that are high enough for subsequent neutron decay. Although the neutron decay of Kr-87 is virtually instantaneous, the appearance of the neutron will have been delayed by the half-life of Br-87 relative to the time the fission that resulted in the formation of Br-87.

Nuclides such as Br-87 are known as 'delayed neutron precursors', and the neutrons emitted by these nuclides are called 'delayed neutrons'.

Although there is a wide variety of nuclei that produce delayed neutrons, the half-lives of all these decaying nuclei form six distinct groups. A weighted average of these six groups and of the prompt neutrons can be used to calculate an average lifetime for all neutrons.

Let β_i be the fraction of the fission neutrons which appear in the ith group (i = 1, 6)

The total fraction of neutrons which are delayed in then given by

$$\beta = \sum_{i=1}^{6} \beta_i$$

and therefore the fraction (1 - β) of the fission neutrons is emitted as prompt neutrons.

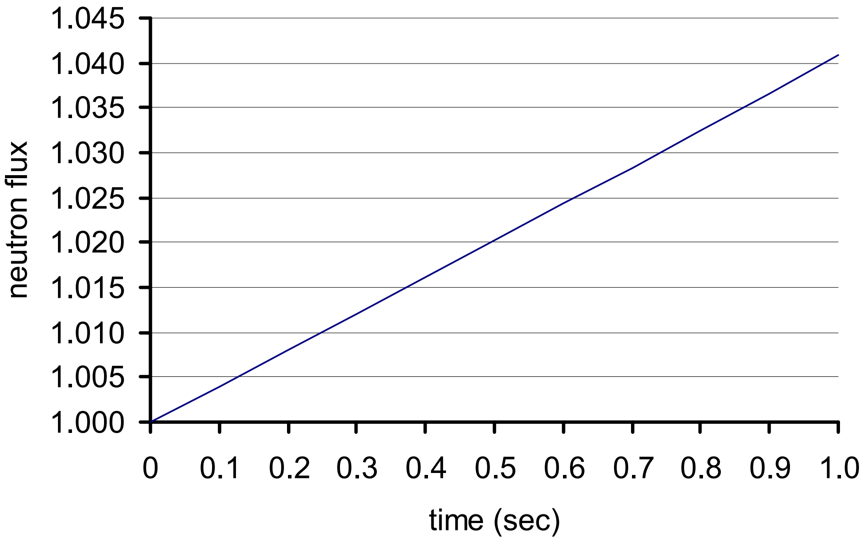
Let ℓ_p be the prompt neutron lifetime, and ℓ_i be the mean lifetime of a delayed neutron in the ith group measured from the instant of fission to the time when the neutron is absorbed, then the average life of all fission neutrons is

$$\ell = (1-\beta)\ell_p + \sum_{i=1}^{6}\beta_i\ell_i \quad \text{and since } \beta << 1, \quad \ell \approx \ell_p + \sum_{i=1}^{6}\beta_i\ell_i \qquad \textit{(3B – 2)}$$

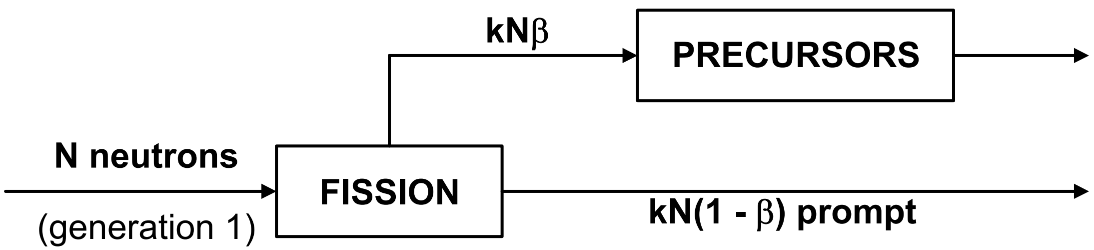
A typical value of ℓ = 0.1 sec is obtained for water moderated reactors, resulting in a period of 25 sec for a 4 mk reactivity change, and a power increase by a factor of 1.04, which is relatively easy to control.

4.1 Prompt and Delayed Neutrons

- **using the average neutron life time as computed in equation (*3B – 2)* results in the type of response, for a 4 mk reactivity increase, as shown on the diagram**
- **while this represents the response of a system that is relatively easy to control, it is not a sufficiently accurate representation of the actual response of a critical reactor, particularly at the time just after the reactivity increase was introduced**
- **a much better, yet still fairly simple approximation of neutron behaviour can be obtained by treating prompt and delayed neutrons as separate entities,**
- **for heavy water moderated reactors, we can use a prompt neutron life time of 0.001 seconds and an average lifetime for all delayed neutrons as 13 seconds**

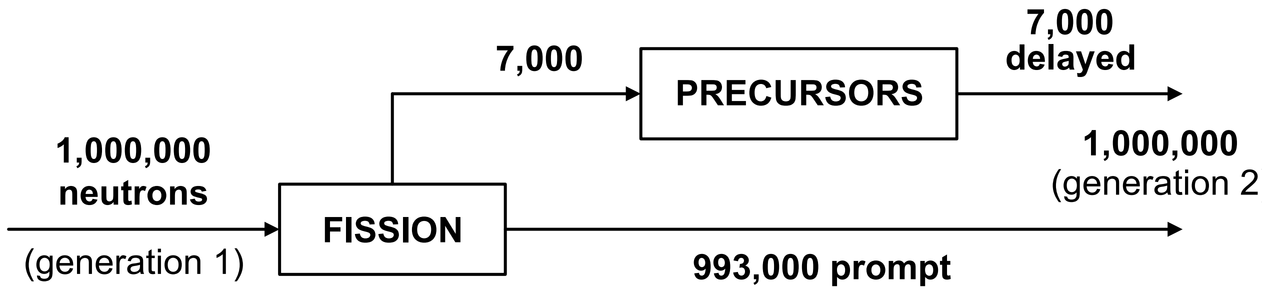


- **the following simple model can be used to illustrate the proposed analysis:**
  - ⇒ **with a multiplication factor of ‘k’ the ‘N’ neutrons of one generation will result in kN neutrons in the next generation**
  - ⇒ **the fraction kNβ give rise to delayed neutron precursors, while kN(1 - β) appear as prompt neutrons; note that the reactor is critical only because in steady state kNβ delayed neutrons are being produced by the precursors**



4.2 Response of Prompt and Delayed Neutrons to a Step Increase in Reactivity

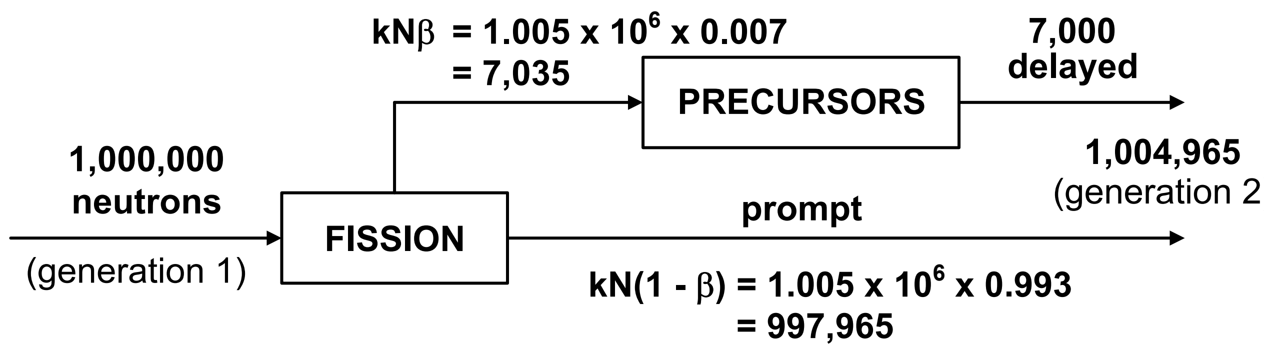
- **consider a critical reactor from the original 1 million in the first generation there will be 993,000 prompt neutrons in the second generation and 7,000 precursors will have been created; criticality is maintained because 7,000 delayed neutrons will also appear from the precursors that were created (on the average) 13 seconds earlier**



- **consider now a 5 mk step increase of reactivity
the total number of neutrons produced by fission will increase to kN = 1,005,000, of which $kN(1 - \beta)$ = 997,965 will appear as prompt neutrons, while $kN\beta$ = 7035 precursors will be created; since the delayed neutrons are being generated from the precursors that were created at an earlier time, before the reactivity increase took place, there will still be 7,000 delayed neutrons generated, as when the reactor was critical; with an average delayed neutron lifetime of 13 seconds, it will several generations before the number of delayed neutrons increases**

in the first five generations the neutron level jumps to:

Gen 1: 1,000,000
Gen 2: 1,004,965
Gen 3: 1,009,920
Gen 4: 1,014,865
Gen 5: 1,019,799
Gen 6: 1,024,724



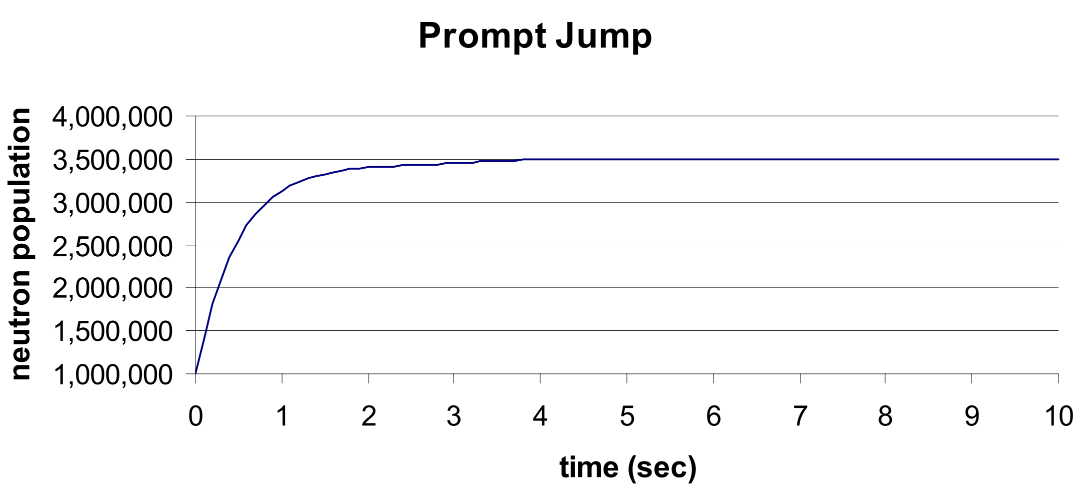
4.3 The Prompt Jump Approximation

- **if we were to assume that the number of delayed neutrons produced by the precursors from prior to the reactivity increase, we obtain a rapid rise in neutron population due to prompt neutrons alone, as shown on the diagram**
- **mathematically it can be shown that the amplitude of the prompt jump is given by:**

$$\frac{\mathbf{N}}{\mathbf{N_0}} = \frac{\beta}{\beta - \delta\mathbf{k}} \qquad \textit{(3B – 3)}$$

for β = 0.007 and δ = 0.005, the prompt jump factor = 3.5

- **following the prompt jump, the rate of increase in the neutron population is much slower, as the effect of the changed concentration of delayed neutron precursors becomes apparent**



4.4 Rate of power increase following the Prompt Jump

- **mathematically it can be shown that the period of the long term neutron flux increase following the prompt jump is given by:**

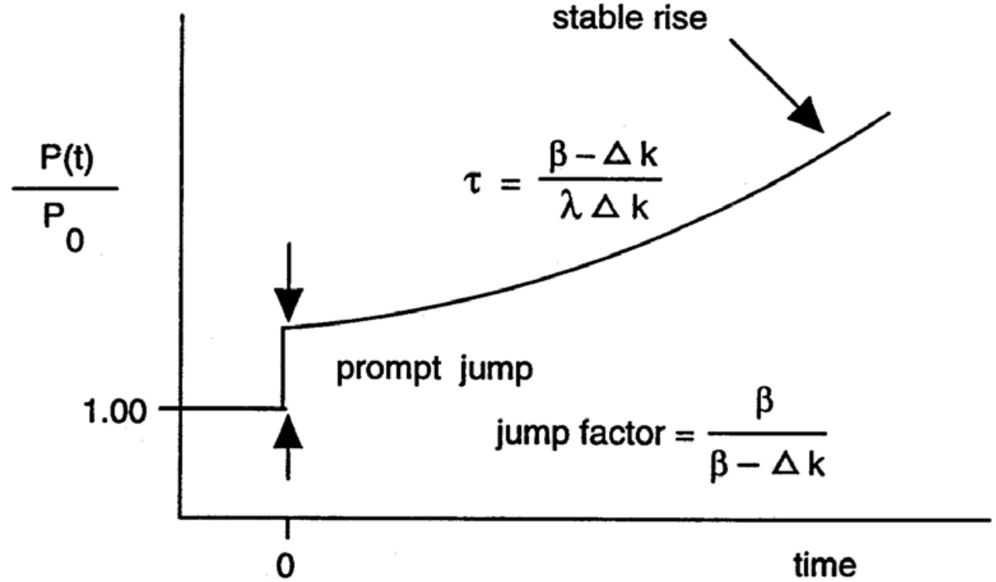
$$\tau = \frac{\beta - \delta\mathbf{k}}{\lambda\, \delta\mathbf{k}} \qquad \textit{(3B – 4)}$$

where $\lambda = \frac{1}{\ell}$ **is the average decay constant of delayed neutron precursors, which is in the order of 0.08 sec^{-1}**

for the example we have been considering, with β = 0.007 and a step reactivity increase of δk = 5 mk, we get

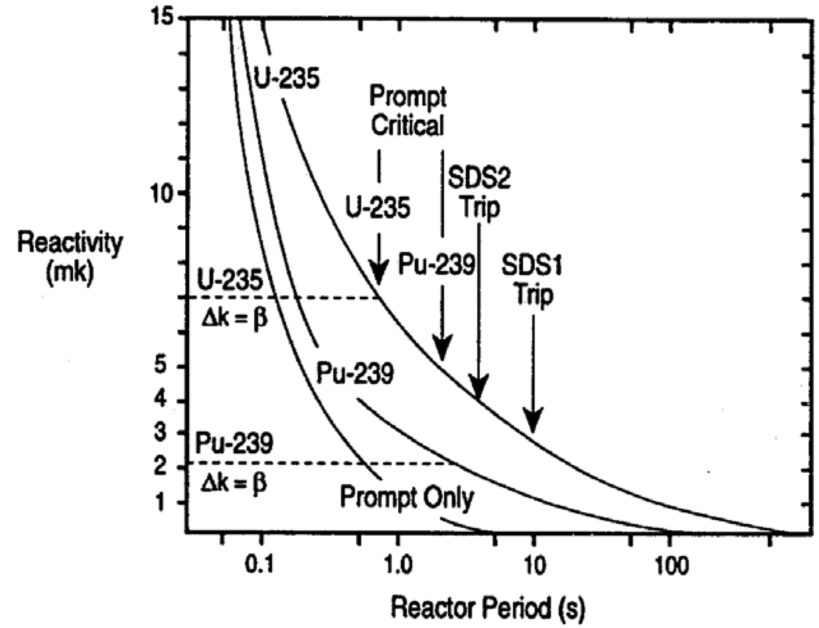
$$\tau = \frac{0.007 - 0.005}{0.08 \times 0.005} = 5 \text{ sec}$$

- **the prompt jump has very important implications for reactor safety, since a sudden injection of a few mk of positive reactivity could cause a dangerous increase in power level output**
- **all reactivity mechanisms must be designed in such a way that they can only insert reactivity at a slow rate, so that the risk of inducing a prompt jump is avoided**



5. PROMPT CRITICAL

- **typical rates of reactivity insertion in power reactors are restricted to values that are far below β, usually to < 0.1 mk/sec**
- **if a reactivity increase δk ≥ β, the reactor would be supercritical on prompt neutrons alone, and power would increase exponentially as derived in section**
- **note the unit of reactivity used in the USA:** $\text{Reactivity in Dollars} = \frac{\delta k}{\beta}$
- **the prompt critical condition must not be reached, or even approached in power reactors, because the shutdown systems are not capable to respond in sufficient time: the design of the reactor, reactivity control mechanisms, safety analysis in cases of loss of coolant, all operating policies and practices must be such that prompt criticality is avoided at all times**
- **the disastrous consequences of prompt criticality, as occurred at the Chernobyl Unit 4 reactor, are well known**
- **note that β changes with the composition of the fuel, for example with the 'age' of the core**
  - ⇒ **for a fresh core of natural uranium fuel, U-235 is the fissile material, and β = 0.007**
  - ⇒ **if we had a core using Pu-239 as the fissile material, prompt criticality would be achieved with a smaller reactivity insertion, since for Pu-239 β = 0.0023**
  - ⇒ **equilibrium CANDU core contains a mixture of U-235 and Pu-239, with an effective value of β = 0.005**



- **from the diagram it can be seen that the reactor trips are set at rates that are slower than prompt critical for either fresh or equilibrium core conditions: SDS1 will drop the shutdown rods if the reactor period reaches 10 sec, and SDS2 will inject liquid poison if the reactor period reached 4 seconds**

6. LARGE NEGATIVE REACTIVITY INSERTIONS – REACTOR TRIPS

- **the equations describing the prompt jump are equally valid when the reactivity insertion is negative**
- **a prompt drop is followed by a stable negative reactor period**
- **from an initial power level of P_0 and a reactivity insertion of –5 mk, and assuming that β = 0.007, equation (*3B – 3*) gives:**

$$\mathbf{P} = \mathbf{P_0}\,\frac{\beta}{\beta - \delta\mathbf{k}} = \mathbf{P_0}\,\frac{\mathbf{0.007}}{\mathbf{0.007} + \mathbf{0.005}} = \mathbf{0.58\ P_0}$$

following the prompt drop the power will decrease with a negative period as given by equation (*3B – 4*)

$$\tau = \frac{\beta - \delta\mathbf{k}}{\lambda\,\delta\mathbf{k}}$$

since this equation is valid for reactivity changes that are small compared with β, consider a case of δk = - 0.2 mk; for a fresh core,

$$\tau = \frac{\mathbf{0.007} + \mathbf{0.0002}}{\mathbf{0.08\ x\ (-0.0002)}} = -\,\mathbf{450\,sec}$$

7. POINT KINETICS EQUATIONS

- **for the purposes of dynamic control system analysis the point reactor kinetics formulations is often acceptable, provided that spatial (and local) effects can be neglected:**

$$\frac{dN(t)}{dt} = \frac{\delta k(t) - \beta}{\ell^*} N(t) + \sum_{i=1}^{6} \lambda_i C_i(t)$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\ell^*} N(t) - \lambda_i C_i(t) \qquad i = 1, 2, \ldots 6$$

where $N(t)$ **= neutron density (also neutron flux, fission rate or neutron power)**
$C_i(t)$ **= effective precursor concentration for group I**
$\delta k(t)$ **= reactivity**
ℓ^* **= mean neutron life time**
β **= total effective delayed neutron fraction** $\left(\beta = \sum_{i=1}^{6} \beta_i \right)$
β_i **= effective delayed neutron fraction of group i**
λ_i **= effective decay constant of group i**
and there is no external neutron source

- **note that for heavy water moderated reactors there are nine additional groups of photoneutrons, produced by D(γ, n) reaction**
- **the point kinetics equations can be solved analytically for a few simple reactivity insertion functions, such as step, ramp and sinusoidal**
- **for more complex reactivity variations numerical methods have to be used**