

COOLING TOWER MODEL IN MNR

1 COOLING TOWER MODEL

The cooling tower model used in Liu's study [2] was adapted from the model developed by Eric Weber (1988). One modification is the use of a counterflow effectiveness model based on the work of James Braun (1988) and implemented by John Rice (1991). The reader should consult Braun and Weber for a complete development of the basic model equations. In the process of modifying Weber's tower model, one possible source of error was anticipated. The model used by Weber was developed for a crossflow cooling tower, which was the type of tower considered in Weber's as well as in the current study. However, Braun's effectiveness model was based on a counterflow cooling tower. Therefore a new effectiveness model based on a crossflow tower was attempted for this study. The resulting data showed that the crossflow effectiveness model made no improvement on the existing counterflow method. As seen from the tower model analysis at the end of this chapter, the use of Braun's counterflow method produced very accurate results.

1.1 Effectiveness Cooling Tower Model

To derive the model equations, the following assumptions were made:

- Heat and mass transfer in the direction normal to flows only.
- Negligible heat and mass transfer through tower walls to the environment.
- Negligible heat transfer from the tower fans to the air or water streams.
- Constant water and dry air specific heats (constant $C_{p,a}$ and $C_{p,w}$).
- The mass fraction of water vapor in the air-vapor mixture is approximately equal to the humidity ratio.
- Uniform temperature throughout the water stream at any cross section.
- Uniform cross-sectional area of the tower.

Using a steady-state energy balance, mass balance, and mass diffusion relations on an incremental volume (see Figure 1), Braun and Weber developed the following differential equations:

$$\frac{dT_w}{dV} = \frac{\frac{dh_a}{dV} - C_{p,w}(T_w - T_{ref})\frac{dw_a}{dV}}{\left[\frac{\dot{m}_{w,i}}{\dot{m}_a} - (w_{a,o} - w_a)\right]C_{p,w}} \quad (1)$$

$$\frac{dw_a}{dV} = -\frac{Ntu}{V_T}(w_a - w_{s,w}) \quad (2)$$

$$\frac{dh_a}{dV} = -\frac{Le \cdot Ntu}{V_T}[(h_a - h_{s,w}) + (w_a - w_{s,w})(1/Le - 1)h_{g,w}] \quad (3)$$

where $Le = \frac{h_c}{h_D C_{p,w}}$

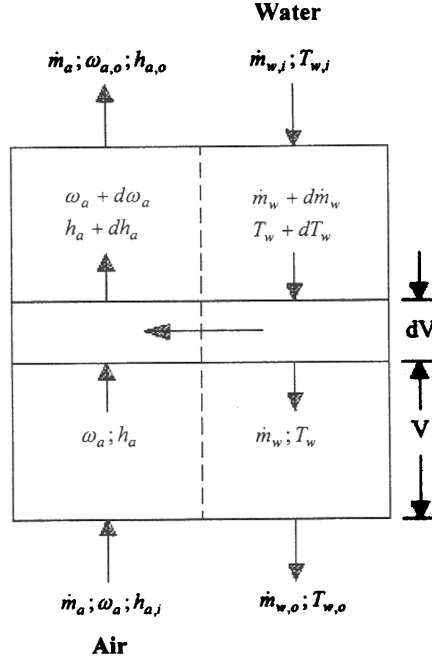


Figure 1: Schematic of a counterflow cooling tower

$$Ntu = \frac{h_D A_V V_T}{\dot{m}_a}$$

Given the inlet conditions, equations (1)-(3) can be solved numerically for the exit conditions of both the air and water streams. However this solution requires numerically integrating the equations over the entire tower volume from air inlet to outlet.

This analysis is simplified considerably by the use of **Merkel's assumptions**. **Merkel neglected the effect of the water loss due to evaporation and set the Lewis number to unity ($Le = 1$)**, which reduces the equations to the following:

$$\frac{dT_w}{dV} = \frac{\dot{m}_a (dh_a/dV)}{\dot{m}_w C_{p,w}} \quad (4)$$

$$\frac{dh_a}{dV} = -\frac{Ntu}{V_T} (h_a - h_{s,w}) \quad (5)$$

where T_w is the water temperature, \dot{m}_a is the mass flow rate of air through the cooling tower, \dot{m}_w is the mass flow rate of water through the tower, h_a is the enthalpy of the moist air per pound of dry air, $h_{s,w}$ is the enthalpy of saturated air at the water volume, dV is a differential volume, V_T is the total volume, and Ntu is the number of transfer units for the cooling tower. As for the condenser, Ntu is the dimensionless parameter used for heat exchanger analysis.

Braun defines a saturation specific heat C_s , as the derivative of the saturated air enthalpy with respect to temperature evaluated at the water temperature. It has the units of specific heat. He rewrites equation (4) in terms of air enthalpies only by C_s :

$$\frac{dh_s}{dV} = \dot{m}_a C_s (dh_a/dV) \quad (6)$$

$$C_s = \left[\frac{dh_s}{dT} \right]_{T=T_w} \quad (7)$$

Braun then states that equations (6) and (7) can be solved analytically for the exit conditions if the saturation enthalpy is linear with respect to temperature (i.e. constant C_s). Figure 2 (Braun, 1988) shown below depicts the variation of the saturation enthalpy with temperature, along with a straight line connecting two typical water inlet and outlet states.

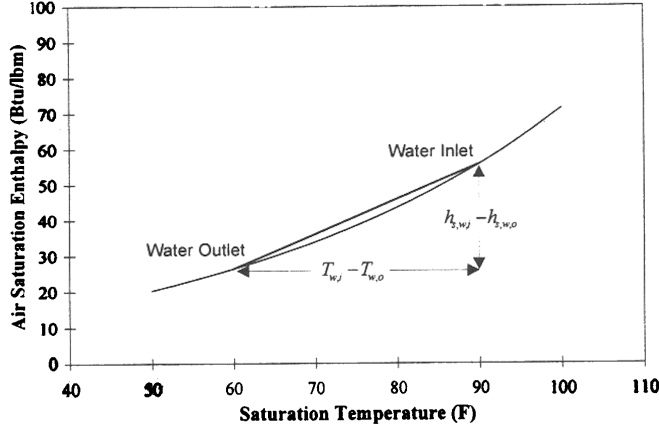


Figure 2: Defining a linearized air saturation enthalpy

It is clear that the air saturation enthalpy does not vary linearly with temperature. However, by choosing an appropriate average slope between the inlet and outlet water conditions, as seen in the Figure 2, an effectiveness relationship can be derived in terms of C_s . This air-side effectiveness, ε_a is defined as the ratio of the actual heat transfer to the maximum possible air-side heat transfer that would occur if the exiting air stream were saturated at the temperature of the incoming water (i.e., $h_{a,o} = h_{s,w,i}$)

The actual heat transferred in terms of this effectiveness is then given as the following:

$$Q = \varepsilon_a \dot{m}_a (h_{s,w,i} - h_{a,i}) \quad (8)$$

where $h_{s,w,i}$ is the enthalpy of saturated air at inlet water conditions, and $h_{a,i}$ is the enthalpy of the incoming air. This is analogous to a dry counterflow heat exchanger, the air-side effectiveness ε_a is evaluated:

$$\varepsilon_a = \frac{1 - \exp[-Ntu(1 - C_r)]}{1 - C_r \exp[-Ntu(1 - C_r)]} \quad (9)$$

where,

$$C_r = \frac{C_{min}}{C_{max}} = \frac{\dot{m}_a C_s}{\dot{m}_{w,i} C_{p,w}} \quad (10)$$

The average value for the saturation specific heat is estimated as the average slope between the inlet and outlet water states:

$$C_s = \frac{h_{s,w,i} - h_{s,w,o}}{T_{w,i} - T_{w,o}} \quad (11)$$

where $h_{s,w,o}$ is the enthalpy of saturated air at exit water conditions. The outlet air enthalpy and water temperature can then be determined from overall energy balances equations

$$h_{a,o} = h_{a,i} + \varepsilon_a (h_{s,w,i} - h_{a,i}) \quad (12)$$

$$T_{w,o} = \frac{\dot{m}_{w,i} (T_{w,i} - T_{ref}) C_{pw} - \dot{m}_a (h_{a,o} - h_{a,i})}{\dot{m}_{w,o} C_{pw}} + T_{ref} \quad (13)$$

In the effectiveness equations used for this study, the water loss in the tower is neglected, so $\dot{m}_{w,i}$ is set equal to $\dot{m}_{w,o}$ (i.e., $\dot{m}_{w,i} = \dot{m}_{w,o}$), producing the following equation

$$T_{w,o} = T_{w,i} - \frac{\dot{m}_a (h_{a,o} - h_{a,i})}{\dot{m}_w C_{pw}} \quad (14)$$

Therefore, the exiting water temperature can be determined by equation (14)

1.2 Correlating Manufacturing Data

The cooling tower model requires correlation with manufacturer data to calculate the individual tower coefficients and verify the model's accuracy. Given the required water and air flow rates, the entering air wet bulb temperature, and the tower water inlet temperature, the manufacturer catalog predicts the approach and the tower water outlet temperature. The tower manufacturer offers towers of different box sizes, each of which has its own dimensions and cooling capacity. The tower modelling process involves the following equation, derived from Lowe and Christie (1961).

$$Ntu = c \left(\frac{\dot{m}_w}{\dot{m}_a} \right)^{1+n} \quad (15)$$

where c and n are empirical constants, or coefficients, specific to a particular tower box size. According to this equation, the data for each box size should correlate as a straight line on a log-log plot Ntu versus the flow rate ratio.

1.3 Calculating Tower Coefficient

Coefficients Calculating the mass flow rate ratio simply requires converting the water flow rate, which is given in gallons per minute, and the air flow rate, which is given in cubic feet per minute, into mass flows.

Calculating the Ntu term requires the iterative solution of equations (8) - (12), (14), and the following heat transfer equation

$$\dot{Q} = \dot{m}_w C_{p,w} (T_{w,i} - T_{w,o}) \quad (16)$$

Figure 3 shows the process of obtaining the tower coefficients for the first tower box size. The data obtained from the manufacturer's catalog are plotted on the chart, and a linear curve fit is applied to the data. The equation of this fitted line is equivalent to equation (15), when the log of both sides of the equation is taken, and is used to obtain the tower coefficients.

This procedure is repeated to produce individual tower exponents and constants for each tower box size. These coefficients are then used for each tower to calculate operating conditions over varying water inlet, air wet bulb, and air dry bulb temperatures and varying water and air flow rates

In Figure 3 the model’s predictions of the tower water outlet temperature over a wide range of operating temperatures and flow rates are compared to results from the manufacturer performance data. As seen from the figure. the correlation of the data is excellent when compared to Figure 3. and so the scatter of the data points in Figure 4 is shown to have little negative effect on the model.

1.4 Crossflow Cooling Tower Model

It should be noted that the cooling tower manufacturer data were obtained for a crossflow cooling tower. Therefore, a crossflow effectiveness model was originally attempted for this study. However, the crossflow model did not produce better results than the original counterflow model, so the decision was made to use the counterflow model in the simulation program.

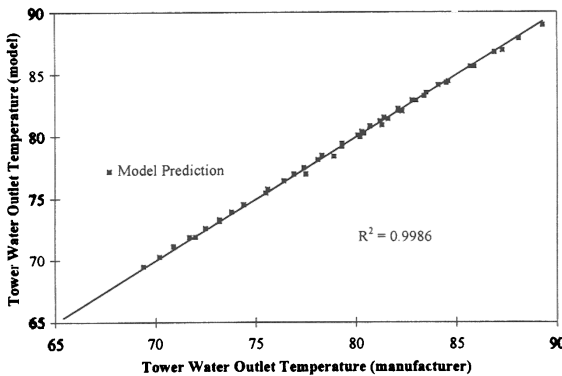
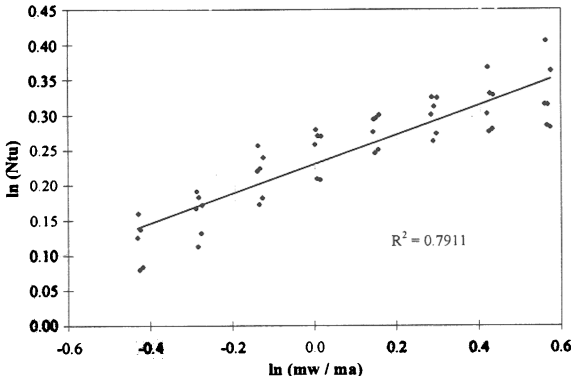


Figure 3: Driving tower box 1 coefficients

Figure 4: Accuracy of the new tower model

2 Application to MNR Cooling Tower

2.1 MNR Cooling Tower

The cooling towers in the McMaster Nuclear Reactor (MNR)[6] are inside a lighted frost fence enclosure with a locked gate. Each of the two towers comprises a concrete basin, a steel support structure, PVC louvres and drift eliminators, decking, upper trays, distribution valves, motor and fan. The active components are two Marley Model NCB2A1 cooling towers with dual speed 575V three-phase, 25 HP motors driving 3.6 metre, nine-blade fans at either 900 or 1800 RPM. A vibration switch shuts down the motor if the blades are unbalanced or if their movement is impeded. The nominal air flow through each tower is $73.9 \text{ m}^3/\text{s}$ ($156,600 \text{ ft}^3/\text{min}$).

Based on the above information from the model (Marley Model NCB2A1 cooling tower), the cooling tower is crossflow type as shown in Figure 5.

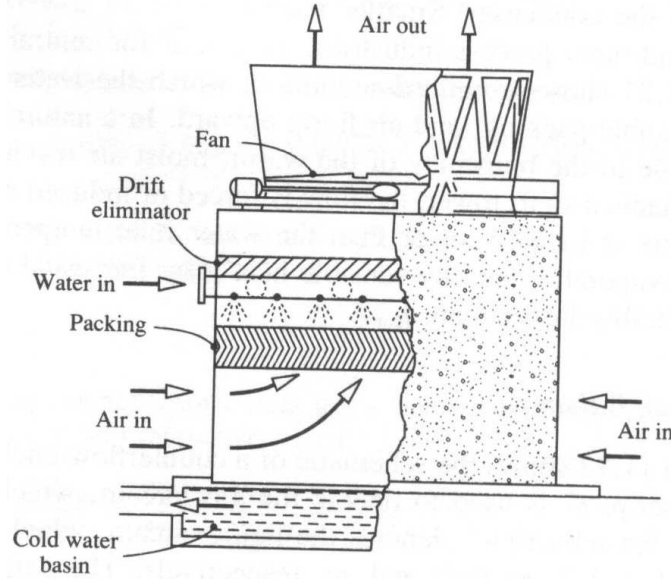


Figure 5: Cooling tower type in MNR

2.2 Numerical Calculation

Air density is slightly changing with temperature. Hence, assuming the air behaves an ideal gas, its density can be assessed in equation (17) as follows [5]. Note that the specific heat capacity of air at the temperature range of interest can be assumed about $1008 \text{ J}/(\text{kg} \cdot \text{K})$ from Table 1.

$$\rho_{air} = \frac{p}{RT} = \frac{101.3 \times 10^3 \text{ N} \cdot \text{m}^{-2}}{287 \text{ J}/(\text{kg} \cdot \text{K})} \left(\frac{1}{T [\text{K}]} \right) = \frac{352.96}{T [\text{K}]} \quad (17)$$

Therefore, air flow rate through the cooling tower can be determined in equation (18).

$$\dot{m}_a = \rho V = \left(\frac{352.96}{T} \right) \left(73.9 \frac{\text{m}^3}{\text{s}} \right) = \frac{2.6084 \cdot 10^4}{T [\text{K}]} \text{ kg s}^{-1} \quad (18)$$

Table 1: Air specific heat

T[K]	$C_p[J/kgK]$
250	1009
260	1009
270	1009
280	1008
290	1007
300	1005

Table 2: Water density

T[°C]	$\rho[kg/m^{-3}]$
0	1000
10	1000
20	998
30	996

On the other hand, the water density is not changing substantially over the temperature range of interest ($0 < T < 30^\circ\text{C}$), which is less than 1%. Therefore, the water density can be set constant as $\rho = 998\text{kg} \cdot \text{m}^{-3}$ at $T=20^\circ\text{C}$ as shown in Table 2.

$$\dot{m}_w = \frac{151L}{s} = \rho_{(T=20^\circ\text{C})}V = \left(998\frac{\text{kg}}{\text{m}^3}\right) \left(151\frac{L}{s} \frac{10^{-3}\text{m}^3}{1L}\right) = 150.7\text{kg}\cdot\text{s}^{-1}$$

$$\dot{m}_w = \frac{150.7\text{kg} \cdot \text{s}^{-1}}{2} = 75.35\text{kg} \cdot \text{s}^{-1} \quad \text{for each cooling tower}$$

In order to determine C_r by equation (10), C_s should be first determined from equation (11).

$$C_s = \frac{h_{s,w,i} - h_{s,w,o}}{T_{w,i} - T_{w,o}} = \frac{C_{p,s,a}T_{w,i} - C_{p,s,a}T_{w,o}}{T_{w,i} - T_{w,o}} = C_{p,s,a} = 3375 \left(\frac{J}{\text{kg} \cdot K}\right)$$

$$C_r = \frac{\dot{m}_a}{\dot{m}_{w,i}C_{p,w}}C_{p,s,a} = \frac{3375}{75.35 \cdot 4186} \frac{2.6084 \cdot 10^4}{T[K]} = \frac{278.63}{T[K]} \quad (19)$$

Note that the specific heat of saturated air at the water conditions is assessed over the temperature of interest as shown in Figure 6.

Since there is no information available about the MNR cooling tower to calculate the Ntu , the operating data were used to assess it as shown in Figure 7, similar to Figure 3. Therefore, Ntu can be given by the simple formula with ration of air and water flow rates as follows:

$$\ln(Ntu) \approx 4.6217 \ln(\dot{m}_w/\dot{m}_a) - 0.3317 \quad (20)$$

$$\varepsilon_a = \frac{1 - \exp[-Ntu(1 - C_r)]}{1 - C_r \exp[-Ntu(1 - C_r)]} \quad (21)$$

$$T_{w,o} = T_{w,i} - \frac{\dot{m}_a \varepsilon_a (h_{s,w,i} - h_{a,i})}{\dot{m}_w C_{pw}} \quad (22)$$

From the above parameters with the known inlet temperature and flow rate of coolant flow and air flow through the cooling tower, the outlet temperature of the coolant and air can be simply determined from equations (19-22).

2.3 Validation

see attached data

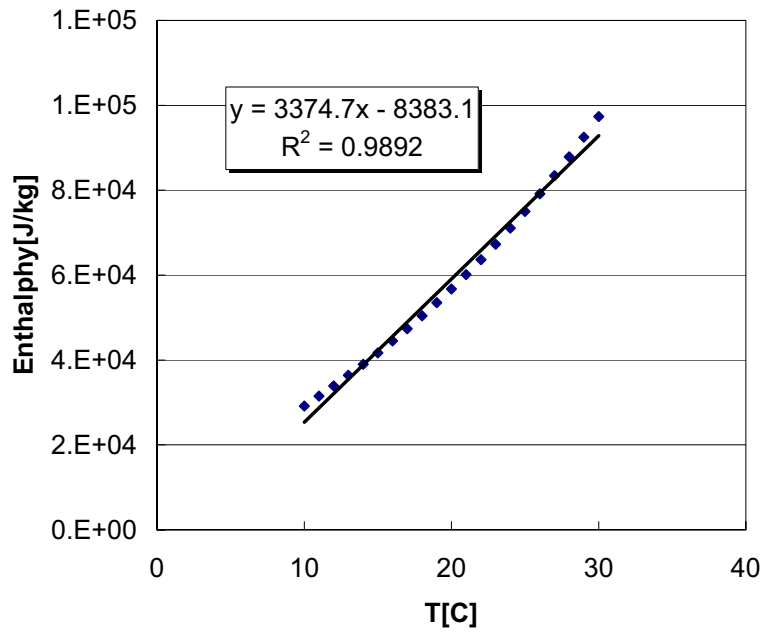


Figure 6: Enthalpy assessment of saturated air (Data from reference[5])

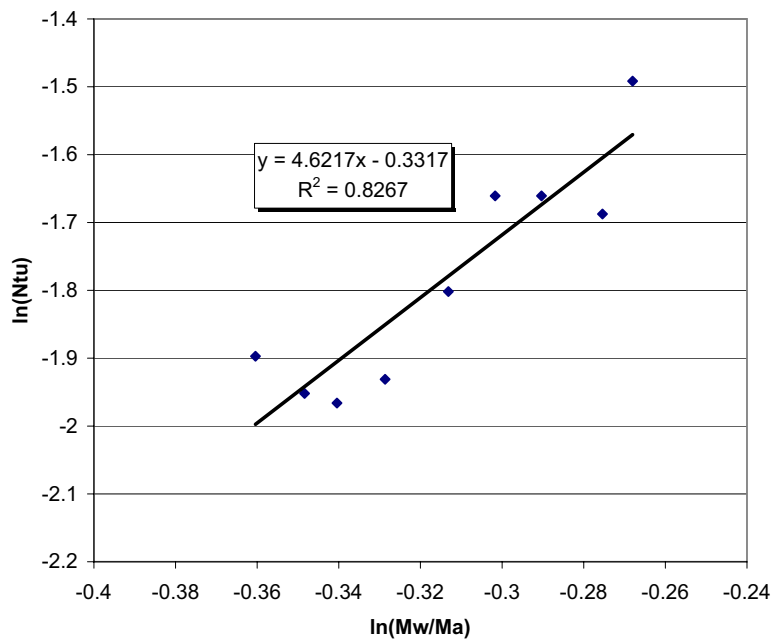


Figure 7: Ntu assessment in MNR cooling tower

References

- [1] Braun, J.E. “Methodologies for the Design and Control of Central Cooling Plants.” Ph.D. Thesis, University of Wisconsin, 1988
- [2] Liu, H.H. “Analysis and Performance Optimization of Commercial Chiller/Cooling Tower Systems.” Masters Thesis, Georgia Institute of Technology, June 1997.
- [3] Lowe, H.J. and D.G. Christie. “Heat Transfer and Pressure Drop Data on Cooling Tower Packings, and Mode Studies of the Resistance of Natural Draft Towers to Air-flow.” ASME Heat Transfer Proceedings, Paper 113, 1961.
- [4] Weber, E.D. “Modeling and Generalized Optimization of Commercial Building Chiller/Cooling Tower Systems.” Masters Thesis, Georgia Institute of Technology, November 1988.
- [5] Mills, A.F. “Heat and Mass Transfer” Richard D. Irwin, Inc., 1995
- [6] MNR SAR, 2002

Cooling tower data from MNR operation during January 2004

note: Tw,i=HX outlet temperature in secondary side

Ta=outside ambient temperature from weather data (obtained from Dave Gilbert)

Tw,i[C]	Ta[C]	Ta[K]	Ma[kg/s]	ln(Mw/Ma)	Ntu	ln(Ntu)	Cr	Ea	Tw,o[C]
18.39	-22	251	103.9	-0.3484	0.142	-1.9519	1.1101	0.12	14.869
19.67	-25	248	105.2	-0.3604	0.15	-1.8971	1.1235	0.13	15.609
17.89	-20	253	103.1	-0.3404	0.14	-1.9661	1.1013	0.12	14.59
17.44	-17	256	101.9	-0.3287	0.145	-1.931	1.0884	0.13	14.265
18.06	-13	260	100.3	-0.3132	0.165	-1.8018	1.0717	0.14	14.647
16.33	-10	263	99.18	-0.3017	0.19	-1.6607	1.0594	0.16	12.987
16.78	-7	266	98.06	-0.2903	0.19	-1.6607	1.0475	0.16	13.544
16.89	-3	270	96.61	-0.2754	0.185	-1.6874	1.032	0.16	13.949
16.89	-1	272	95.9	-0.268	0.225	-1.4917	1.0244	0.18	13.569

Prediction of HX inlet temperature in secondary side at various conditions

Tw,i[C]	Ta[C]	Ta[K]	Ma[kg/s]	ln(Mw/Ma)	Ntu	ln(Ntu)	Cr	Ea	Tw,o[C]
20	10	283	92.17	-0.2284	0.25	-1.3872	0.9846	0.2	16.551
20	15	288	90.57	-0.2109	0.271	-1.3063	0.9675	0.21	16.697
20	18	291	89.64	-0.2005	0.284	-1.2584	0.9575	0.22	16.798
25	15	288	90.57	-0.2109	0.271	-1.3063	0.9675	0.21	20.632
25	15	288	90.57	-0.2109	0.271	-1.3063	0.9675	0.21	20.632
30	15	288	90.57	-0.2109	0.271	-1.3063	0.9675	0.21	24.568
30	20	293	89.02	-0.1937	0.293	-1.2267	0.951	0.23	24.64
35	20	293	89.02	-0.1937	0.293	-1.2267	0.951	0.23	28.525
35	10	283	92.17	-0.2284	0.25	-1.3872	0.9846	0.2	28.51
40	10	283	92.17	-0.2284	0.25	-1.3872	0.9846	0.2	32.496
40	30	303	86.09	-0.1601	0.342	-1.0716	0.9196	0.26	32.432