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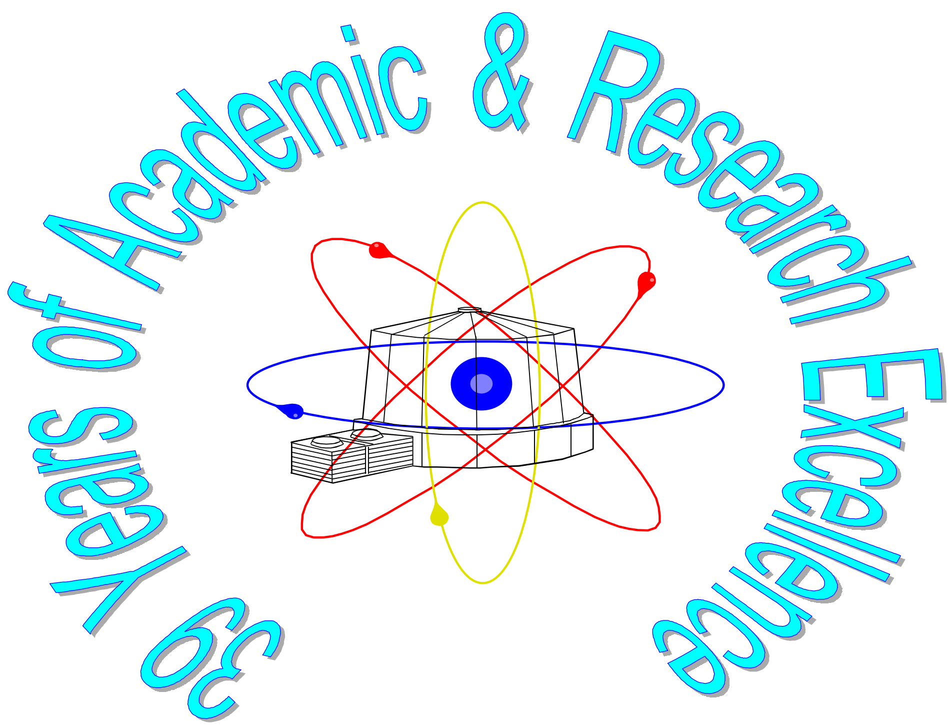
Single Phase Friction Factors for MNR Thermalhydraulic Modelling



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1. Introduction

Accurate single phase friction factor, f, estimates are essential for the prediction of core and pipe flows in the McMaster Nuclear Reactor (MNR). Flows, in turn, are key to good heat transfer coefficient estimation. In addition, because typical thermalhydraulic simulations call for the repeated estimations of f, it is useful to have an accepted, fast, robust and accurate computer code function routine that is publicly available. It is surprising that such a routine is not currently available in the public domain.

This report documents a review of selected literature to determine the appropriate correlations to use for MNR safety analysis. It was found that the standard Moody diagram (which is based on the transcendental Colebrook equation) is sufficiently accurate for pipe flow but problematic to program, whereas simple approximations do not account for pipe wall roughness. Herein, a very simple C-code function is offered that returns an accurate estimate of f as a function of Reynolds Number, Re, and relative roughness ε/D in a non-iterative fashion. In addition, a simple correction factor for channel flow is offered.

2. Background

The friction factor, f, is an artifact of definition, arising from the experimental observation that the pressure drop in a segment of pipe for fully turbulent flow is proportional to the velocity squared, i.e.:

$$\Delta P = \left(\frac{f\,L}{D} + k \right) . \frac{1}{2} \rho v^2 \qquad \textbf{(1)}$$

Where f is the friction factor, L is the pipe length, D is the pipe diameter, k is the form loss factor (to account for bends, entrance and exit losses, valves, orifices, etc.), ρ is the fluid density and v is the fluid velocity. The friction factor, f, is constant with respect to velocity under those conditions of high turbulence. At lower flow, however, f was found to be a function of velocity and pipe wall roughness. Moody [Moody 1944] provided a useful representation of the work done up to that time and his celebrated Moody Chart, reproduced as figure 1, remains today as the most useful expression for engineering applications. Moody took the seminal work of Colebrook [Colebrook 1938-1939], who stated that

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}} \right) \qquad \textbf{(2)}$$

and plotted f as a function of Reynolds Number, Re and relative roughness, ε/D. Re is defined in the usual fashion as $\rho D v/\mu$, where ρ is the fluid density, D is the pipe diameter, v is the average velocity and μ is the viscosity.

At Re below approximately 2000, the flow is laminar and the relationship between pressure drop and velocity can be derived strictly from first principles. Pressure drop turns out to be proportional to velocity, not velocity squared when the flow is laminar. It should be no surprise to find that f (defined to accommodate the turbulent case where pressure drop is proportional to velocity squared) is no longer a constant. Indeed, as discussed in a later section, we find that

$$f = \frac{64}{Re} \tag{3}$$

and is thus independent of roughness. The reader should be cautioned that f is sometimes defined such that f = 16 / Re. Herein, that definition is avoided.

As velocity is increased, turbulence begins to form and the friction factor increases rapidly. This flow zone is called the critical zone as shown in figure 1. This can lead to unstable flow since the friction factor can vary widely and non-monotonically in a narrow Re range. This zone is avoided in designed systems whenever possible.

Beyond Re of 3000 or 4000, the flow becomes decidedly turbulent. A transition region is defined as shown in figure 1 wherein f is a function of boundary layer related losses and turbulent inertial losses. As Re is increased still further, f is dominated by turbulent losses. The transition region is longer for smoother pipes.

By way of an explanation that is rooted in phenomenology (following Moody), the laminar boundary layer at the walls becomes thinner in turbulent flow as velocity is increased. At some velocity, the layer becomes so thin, compared to surface irregularities, that "the laminar flow is broken up into turbulence" [Moody 1944]. Thus, at a sufficiently high Re, the flow is fully turbulent, inertial losses dominate and f can be determined from [Karman 1930]

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon/D}{3.7}\right) \tag{4}$$

For smooth pipes, it was known that f is given by [Moody 1944]

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{2.51}{Re\sqrt{f}}\right) \tag{5}$$

which is a transcendental function that requires an iterative solution. Colebrook proposed that the transition zone be modeled as a combination of the two functions:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right) \tag{6}$$

as stated earlier. In fact, because the roughness term tends to dominate at high Re, the function works well even in the fully turbulent zone. Moody concluded that the Colebrook expression (6) was more than adequate to give f for any Re > 2000. Certainly the accuracy of the expression was well within the experimental error (about ± 5% for smooth pipes and ± 10% for rough pipes). No better formulation has been found to date. Indeed, none is needed given that even the error in experimental data for fully developed flow in pipes is swamped in real life applications by the uncertainties in pipe size (tolerances), form losses (bends, exits, entrances, valves, orifices, etc.), undeveloped flow, transients and thermal events.

3. Function development

The Moody diagram is very convenient for the visual lookup of f given Re and ε/D. However, the transcendental nature of the Colebrook expression is not conducive to simulation codes as iteration is required. This is time consuming when millions of function calls to the friction factor routine occur, as is the case for a typical simulation. In addition, the expression is problematic to program since convergence is dependent on a reasonable initial guess of f. This is not insurmountable of course but the complication should be avoided if possible. For nuclear applications, the pipes are invariably smooth or close to smooth and for that case, several expressions have been developed to approximate the Colebrook expression. Common expressions are

Blasius [Bird 1960]

$$f = \frac{4 \mathrm{x} 0.0791}{Re^{0.25}}, \qquad 2100 < Re < 10^5 \tag{7}$$

McAdams [Rust 1979]

$$f = \frac{0.184}{Re^{0.2}}, \qquad 3 * 10^4 < Re < 2 * 10^6 \tag{8}$$

Chemical Engineer's Handbook [Rust 1979]

$$f = 0.0056 + \frac{0.5}{Re^{0.32}}, \qquad 4*10^3 < Re < 3*10^6 \tag{9}$$

A check on their accuracy was performed and it was found that Blasius and McAdams gave an f that was indistinguishable from Colebrook in their range of applicability and gave significant errors when used outside their range. The Handbook expression performed very well for the entire range of Re from 2000 on up as shown in figure 2. There is a slight overestimate at very high Re, making the expression close to the nearly smooth case of ε/D of 0.000005. The drawback of these simple and easy to program expressions is that they do not account for roughness.

To address these limitations, a non-iterative approximation to Colebrook that accounts for roughness is sought. Looking at the Colebrook expression, we see that the complicating term is 2.31 / (Re f). We need a reasonable estimate of f to substitute into the expression <u>only</u> when ε/D is small (i.e., smooth or close to smooth pipes). When the pipe is rough, the 2.31 / (Re f) term is small compared to the ε/D term, especially at higher Re. But we have a reasonable estimate of f from the Handbook expression, equation 9. Further, the Handbook expression gives a good first order estimate for moderately rough pipes at low Re. With these observations in mind, we modify the Colebrook expression to:

$$\frac{1}{\sqrt{f}} = -2 \log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f_{Handbook}}} \right) \tag{10}$$

making the direct calculation of f possible. Using a standard spreadsheet program, this expression was evaluated for the range of Re and roughness covered by the Moody diagram and compared to the f as determined by the Colebrook expression. The <u>worst</u> error was found to be 0.28% at Re = 2000 and ε/D = 0.005. Typical errors for lower roughness were 0.02% or less (see table 1). These errors are insignificant compared to the 5 or 10% errors in the fundamental data (generated under controlled conditions for fully developed flow) or the additional errors of typical real life applications.

4. Turbulent Flow in MNR

The Colebrook expression is generally applicable to circular pipes and hence can be directly applied to the MNR heat transport system. At typical coolant flows of 100 kg/s, coolant velocities in the main piping are over 2 m/s, giving a Re about 2 x 10^6. The pipes are very smooth but the roughness has not been measured. As shown on figure 2, the MNR coolant flow is well into the turbulent range under normal operating conditions. Flow would have to drop some 3 orders of magnitude before the critical zone or laminar flow would be encountered in the pipes. At that point the coolant would be effectively stagnant for friction and heat transfer calculation purposes.

The core geometry is thin rectangular channels with aspect ratios (width to thickness) of the order of 10:1 to 20:1, depending on the fuel assembly type. The equivalent diameter, D, for use in the expression for Re is defined as 4 x flow area / wetted perimeter. [IAEA 1980] suggests that the Blasius and McAdams approximate expressions can be used for thin rectangular channels, which, by inference, implies that the Colebrook expression is applicable. [Esselman 1958] performed pressure drop tests for MNR type channels for Re up to 5 * 10^5 and relative roughness from smooth to 0.0036. The results reported agreed with the Colebrook expression almost precisely. However, rectangular geometries, according to [Idelchik 1986, pg 65 and 85] have correction factors ranging from 1.1 for a thickness / width ratio of 0 (i.e. infinite parallel plates) to 1.0 for a thickness / width ratio of 1 (i.e. square). Idel'chik gives a table of correction factors for various aspect ratios as follows:

thickness to width, t/w	turbulent correction factor, $k_{correction}$
0.00	1.10
0.10	1.08
0.20	1.06
0.40	1.04
0.60	1.02
0.80	1.01
1.00	1.00

A simple quadratic fit yields:

$$k_{correction} = 1.097-0.177*\left(\frac{t}{w}\right)+0.083*\left(\frac{t}{w}\right)^2 \qquad \textbf{(11)}$$

with an error less than 0.5 %, which is far less than the inherent error in the measured data. The friction factor for rectangular channels is thus

$$f_{rect} = k_{correction} * f_{pipe} \qquad \textbf{(12)}$$

To be conservative, it is recommended that the correction factor be applied to channel flow.

Typical core flows are 0.7 to 1.0 m/s giving a Re of about 16 x 10^4. As shown on figure 2, the MNR coolant flow is well into the turbulent range under normal operating conditions. Flow would have to drop an order of magnitude before the critical zone or laminar flow would be encountered. At that point the coolant would be effectively stagnant for friction and heat transfer calculation purposes. The laminar friction factor is discussed next.

5. Laminar Flow in Pipes and Channels

For steady, incompressible, laminar pipe flow of Newtonian fluids, the momentum equation yields a balance between the driving force (pressure gradient) and shear stress resistance:

$$\frac{\partial P}{\partial z} = \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r} \right) \quad \textbf{(13)}$$

where z is the direction of flow and r is the radius. Since the pressure gradient in the z direction is not a function of r, we can integrate twice directly to yield:

$$v_z(r) = -\frac{R^2}{4\mu}\frac{\partial P}{\partial z}\left(1 - \frac{r^2}{R^2} \right) \quad \textbf{(14)}$$

Averaging over the radius gives the cross sectional averaged flow as:

$$\langle v_z \rangle = -\frac{R^2}{8\mu}\frac{\partial P}{\partial z} \quad \textbf{(15)}$$

For a finite flow length, we can replace the pressure gradient (having a negative slope) with the pressure drop (a positive number) over the pipe length:

$$\langle v_z \rangle = \frac{R^2}{8\mu}\frac{\Delta P}{L} \quad \textbf{(16)}$$

If we substitute this average axial flow for <u>one</u> of the velocities of the defining equation of f:

$$\Delta P = \left(\frac{f\,L}{D} \right).\frac{1}{2}\rho v^2 \quad \textbf{(17)}$$

we find:

$$f = \frac{16D\mu}{\rho v R^2} = \frac{64\mu}{\rho v D} = \frac{64}{Re} \quad \textbf{(18)}$$

Thus f is inversely proportional to velocity and independent of roughness for laminar flow.

For flow between infinite parallel plates, the integration in Cartesian coordinates yields:

$$f = \frac{96}{Re} \quad \textbf{(19)}$$

Thus f for laminar flow for infinite parallel plates is 1.5 times that of circular pipes. Rectangular geometries, according to [Idelchik 1986, pg 65 and 85] have correction factors ranging from 1.5 for a thickness / width ratio of 0 (i.e. infinite parallel plates) to 0.89 for a thickness / width ratio of 1 (i.e. square). Idel'chik gives a table of correction factors for various aspect ratios as follows:

thickness to width, t/w	laminar correction factor, $k_{correction}$
0.00	1.50
0.10	1.34
0.20	1.20
0.40	1.02
0.60	0.94
0.80	0.90
1.00	0.89

A simple cubic fit yields:

$$k_{correction} = 1.503\text{-}1.894{*}\left(\frac{t}{w}\right)+2.034{*}\left(\frac{t}{w}\right)^2\text{-}0.755{*}\left(\frac{t}{w}\right)^3 \qquad \textbf{(20)}$$

with an error less than 0.75 %, which is far less than the inherent error in the measured data.

6. Conclusion

The modified Colebrook expression is non-iterative, making it simple to program, robust and fast. The slight error introduced over the classical Colebrook expression is insignificant. It is concluded that the modified expression can be used in practical engineering calculations for pipe flow in place of the classical expression at no loss and significant gain. It is applicable for MNR heat transport conditions. The expression appears applicable for channel flow as well although the Idel'chik correction factor sould be applied to be conservative.

7. References

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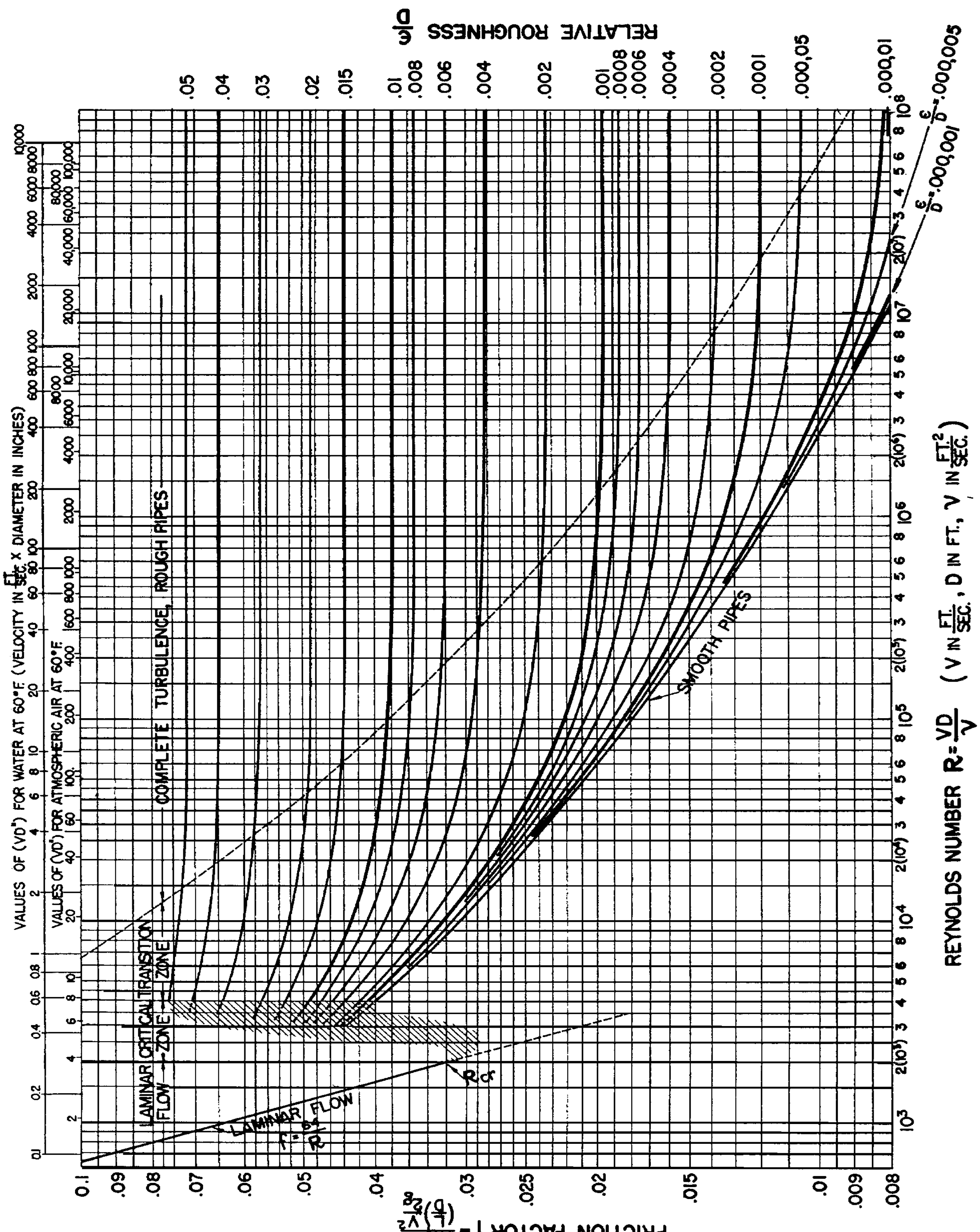

Figure 1 Friction factor versus Reynolds Number [source: Moody 1944].

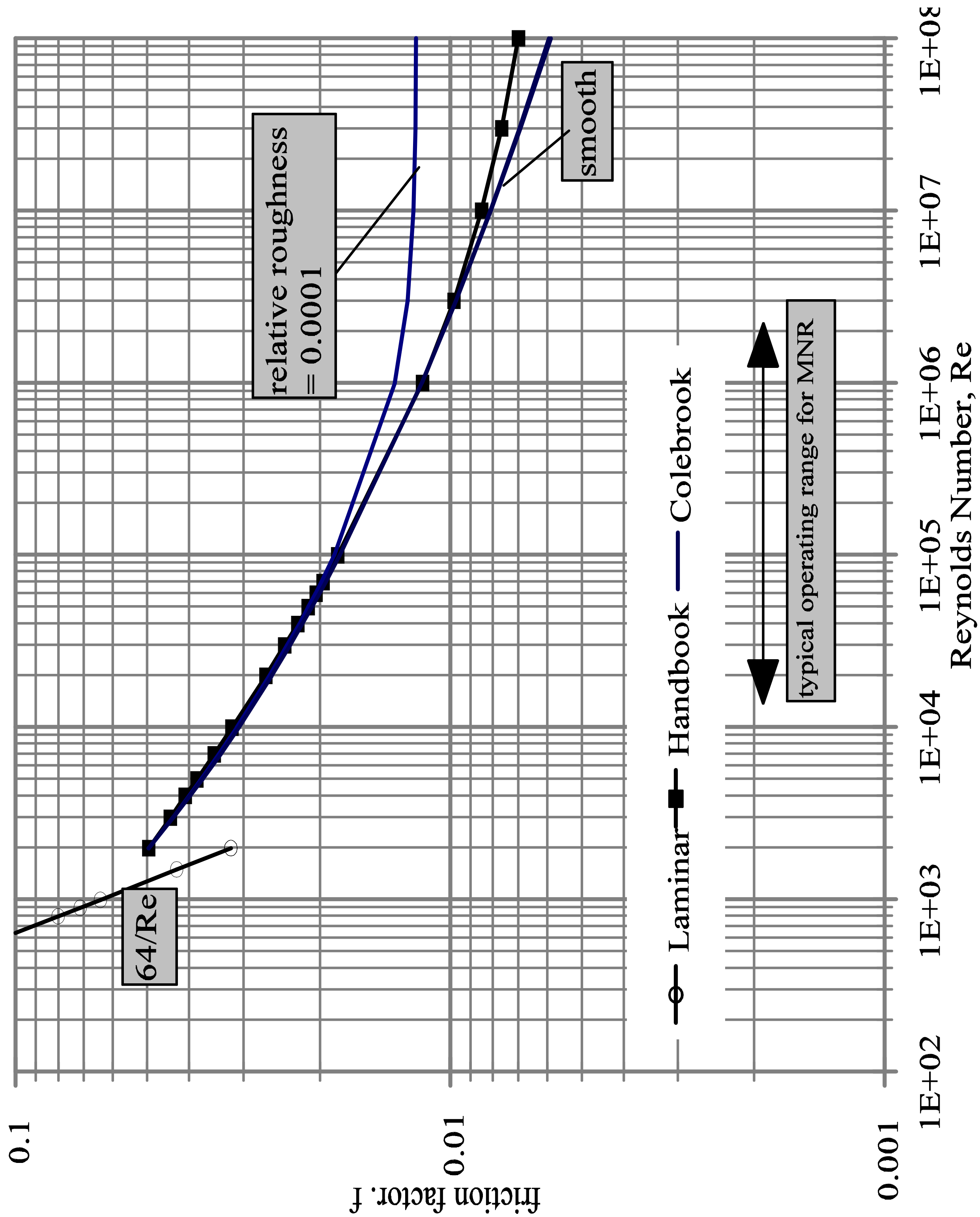

Figure 2 A comparison of the Handbook approximation to the Colebrook expression

	Relative Roughness, ε/D							
Reynolds No	0	0	0	0.001	0.004	0.02	0.04	0.05
2e+03	-0.0013	-0.0011	0.0002	0.0130	0.0508	0.1752	0.2553	0.2829
3e+03	-0.0117	-0.0116	-0.0101	0.0041	0.0434	0.1528	0.2113	0.2301
4e+03	-0.0150	-0.0148	-0.0132	0.0021	0.0421	0.1390	0.1838	0.1975
5e+03	-0.0159	-0.0157	-0.0140	0.0021	0.0425	0.1287	0.1641	0.1747
7e+03	-0.0158	-0.0156	-0.0136	0.0039	0.0439	0.1135	0.1371	0.1439
1e+04	-0.0143	-0.0141	-0.0119	0.0072	0.0455	0.0977	0.1119	0.1159
2e+04	-0.0101	-0.0098	-0.0070	0.0146	0.0457	0.0692	0.0727	0.0737
3e+04	-0.0076	-0.0072	-0.0039	0.0185	0.0432	0.0549	0.0554	0.0556
4e+04	-0.0060	-0.0055	-0.0019	0.0206	0.0404	0.0460	0.0454	0.0453
5e+04	-0.0048	-0.0043	-0.0004	0.0218	0.0378	0.0399	0.0387	0.0384
6e+04	-0.0039	-0.0034	0.0008	0.0225	0.0355	0.0354	0.0339	0.0335
7e+04	-0.0033	-0.0027	0.0017	0.0228	0.0334	0.0319	0.0302	0.0299
1e+05	-0.0020	-0.0013	0.0037	0.0229	0.0285	0.0248	0.0231	0.0227
1e+06	0.0003	0.0022	0.0090	0.0093	0.0064	0.0041	0.0035	0.0034
3e+06	-0.0009	0.0023	0.0068	0.0043	0.0027	0.0016	0.0014	0.0013
1e+07	-0.0029	0.0022	0.0036	0.0016	0.0010	0.0005	0.0005	0.0004
3e+07	-0.0046	0.0018	0.0016	0.0006	0.0004	0.0002	0.0002	0.0002
1e+08	-0.0063	0.0010	0.0006	0.0002	0.0001	0.0001	0.0001	0.0001

Table 1 Error in the modified Colebrook expression in %, defined as $(f_{eqn\ 10} - f_{eqn\ 6}) / f_{eqn\ 6} * 100\%$.