UNENE Graduate Course
Reactor Thermal-Hydraulics Design and Analysis

McMaster University
Whitby
March 19-21, April 23-25, May 2, 2004

Two-Fluid Basic Equations

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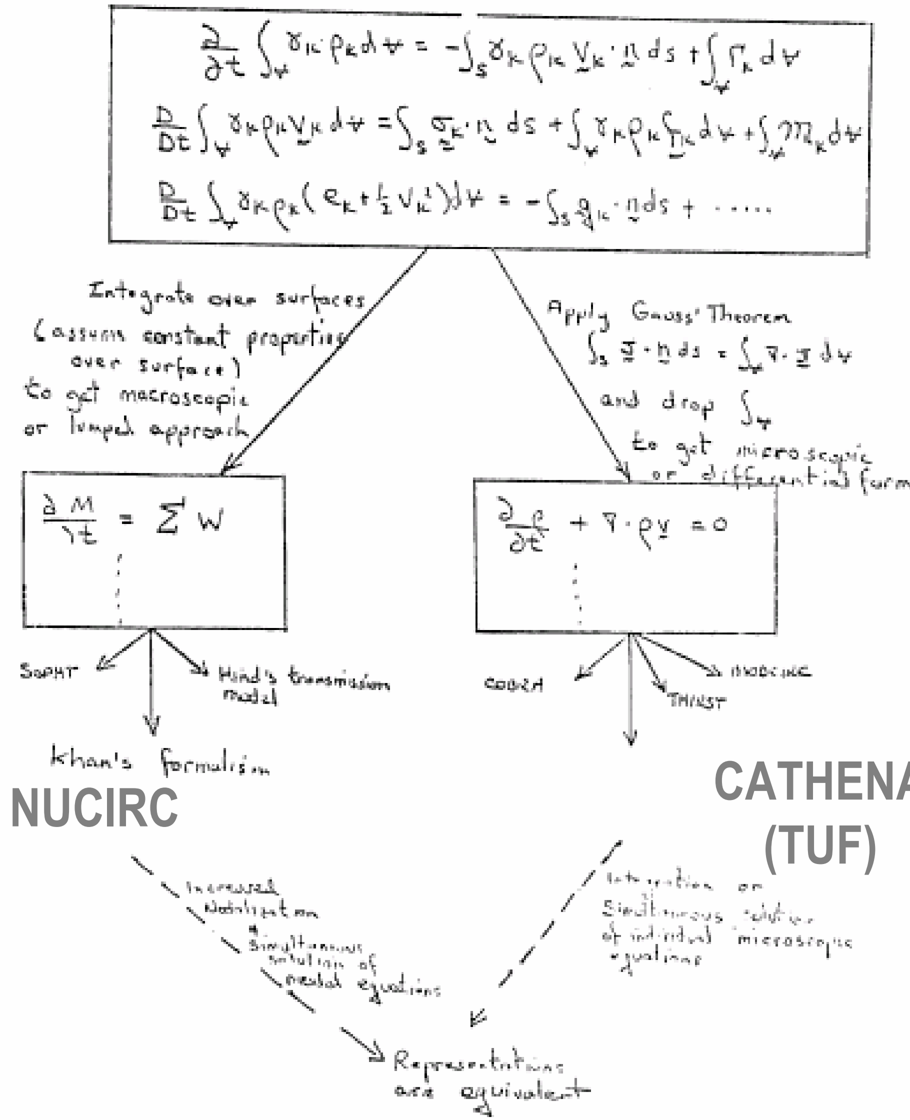
Outline

- Two-Fluid Approach
- Principle of conservation
- Conservation of Mass
- Conservation of Momentum
- Conservation of Energy
- Overview of models

Two-Fluid Model Approach

Integral
(Macro)

Distributed
(Micro)



Principle of Conservation

- What is conservation of a field variable
  - *"What goes in must come out unless it stays there or is generated or lost somehow"*
- Conservation is applied to mass, momentum and energy (heat) transfer
- Definition of derivatives
  - Partial time derivative ($\partial/\partial t$)
    - Observer fixed in space, partial with respect to time, holding x,y,z constant
  - Total time derivative (d/dt)
    - Observer moving independently; must take into account observer velocity ($d/dt = \partial/\partial t + \partial/\partial x \cdot \partial x/\partial t$)
  - Substantial derivative
    - Derivative following the motion; observer moving with the "fluid" ($D/Dt = \partial/\partial t + v_x \cdot \partial/\partial x$)

Conservation Equations

$$\frac{D}{Dt}\iiint \psi \cdot dV = \iiint \Gamma \cdot dV + \iint \vec{S} \cdot \vec{n} \cdot ds$$

Substantial Derivative **Volume Source** **Surface Source**

Integral (macroscopic) form of conservation equations

$$\frac{D}{Dt}\iiint \psi \cdot dV = \iiint \frac{\partial \psi}{\partial t} \cdot dV + \iint \psi \cdot \vec{v} \cdot \vec{n} \cdot ds$$

$$\iiint \frac{\partial \psi}{\partial t} \cdot dV = \iint \psi \cdot \vec{v} \cdot \vec{n} \cdot ds + \iiint \Gamma \cdot dV + \iint \vec{S} \cdot \vec{n} \cdot ds$$

Gauss' Divergence Theorem

$$\iint \vec{A} \cdot \vec{n} \cdot ds = \iiint \nabla \cdot A \cdot dV$$

$$\iiint \frac{\partial \psi}{\partial t} \cdot dV = -\iiint \nabla \cdot \psi v \cdot dV + \iiint \Gamma \cdot dV + \iiint \nabla \cdot S \cdot dV$$

Conservation Equations

$$\frac{\partial \psi}{\partial t} = -\nabla \cdot \psi \cdot v + \Gamma + \nabla \cdot S$$

Storage Flux Volume Source Surface Source

Distributed (microscopic) form of conservation equations

- Conservation is a direct result of experimental observation
- The field variables are continuous within volume V (in time and space)
- Spatial and temporal averaging is used to smooth out variations that are not important to consider

Mass Conservation Equation (Micro)

$$\iiint \frac{\partial}{\partial t}(\gamma_k \cdot \rho_k) dV = -\iint \gamma_k \cdot \rho_k \cdot \vec{v}_k \cdot \vec{n} \cdot ds + \iiint \Gamma_k \cdot dV + \iint \vec{S}_k \cdot \vec{n} \cdot ds$$

$$\iiint \frac{\partial}{\partial t}[(1-\alpha)\rho_1 + \alpha\rho_2)] \cdot dV = -\iint [(1-\alpha)\rho_1 \cdot \vec{v}_1 + \alpha\rho_2 \cdot \vec{v}_2)] \cdot \vec{n} \cdot ds$$
$$+ \iiint (\Gamma_1 + \Gamma_2) \cdot dV + \iint (\vec{S}_1 + \vec{S}_2) \cdot \vec{n} \cdot ds$$

$$\iiint \frac{\partial}{\partial t} dV = -\iint \rho \cdot \vec{v} \cdot \vec{n} \cdot ds$$

Gauss' Divergence Theorem

$$\iint \vec{A} \cdot \vec{n} \cdot ds = \iiint \nabla \cdot \vec{A} \cdot dV$$

$$\frac{\partial}{\partial t}[(1-\alpha) \cdot \rho_1 + \alpha \cdot \rho_2] + \nabla[(1-\alpha) \cdot \rho_1 \cdot v_1 + \alpha \cdot \rho_2 \cdot v_2] = 0$$

Mass Conservation Equation (Macro)

$$-\iint \rho \cdot \vec{v} \cdot \vec{n} \cdot ds = \sum_j \rho_j \cdot v_j \cdot A_j$$

$$\frac{\partial M_i}{\partial t} = \sum_j \rho_j \cdot v_j \cdot A_j = \sum W_j$$

$$\sum_j \rho_j \cdot v_j \cdot A_j = \sum W_j = 0$$

$$v = \frac{W}{\rho \cdot A}$$

Momentum Conservation Equation (Micro)

$$\frac{D}{Dt}\iiint \gamma_k \cdot \rho_k \cdot \vec{v}_k \cdot dV = \iint \vec{\sigma}_k \cdot \vec{n} \cdot ds + \iiint \gamma_k \cdot \rho_k \cdot f_k \cdot dV + \iiint \vec{M}_k \cdot dV$$

$$\iiint \frac{\partial}{\partial t} \cdot \rho \cdot \vec{v} \cdot dV + \iint \rho \cdot \vec{v} \cdot \left(\vec{v} \cdot \vec{n}\right) \cdot ds = \iint \vec{\sigma} \cdot \vec{n} \cdot ds + \iiint \rho \cdot \vec{f} \cdot dV$$

$$\frac{\partial}{\partial t}\left(\rho \cdot \vec{v}\right) + \nabla \cdot \rho \cdot \vec{v} \cdot \vec{v} = \nabla \cdot \vec{\sigma} + \rho \cdot \vec{f}$$

$$\frac{\partial}{\partial t}\left(\rho \cdot \vec{v}\right) + \nabla \cdot \rho \cdot \vec{v} \cdot \vec{v} = -\nabla \cdot P + \nabla \cdot \vec{\tau} + \rho \cdot \vec{f}$$

Momentum Conservation Equation (Macro)

$$V \cdot \frac{\partial \rho \vec{v}}{\partial t} - A_{IN} \cdot \rho_{IN} \cdot v_{IN} \cdot \vec{v}_{IN} + A_{OUT} \cdot \rho_{OUT} \cdot \vec{v}_{OUT} \cdot v_{OUT} = \iint \vec{\sigma}_k \cdot \vec{n} \cdot ds + \iiint \rho \cdot \vec{f} \cdot dV$$

$$V \cdot \frac{\partial \rho \vec{v}}{\partial t} = -\iint P \cdot \vec{I} \cdot \vec{n} \cdot ds + \iiint \left(\nabla \cdot \vec{\tau} + \rho \cdot \vec{f} \right) \cdot dV$$

$$= -A_{OUT} \cdot \vec{P}_{OUT} + A_{IN} \cdot \vec{P}_{IN} - \frac{V \cdot \rho}{L} \left(\frac{f \cdot L}{D} + k \right) \cdot \frac{\vec{v} \cdot |\vec{v}|}{2 g_c} - L \cdot A \cdot \rho \cdot \sin(\theta) \cdot \left(\frac{\vec{g}}{g_c} \right)$$

$$\frac{\partial W}{\partial t} = \frac{A}{L} \left[\left(P_{IN} - P_{IN} \right) - \left(\frac{f \cdot L}{D} + k \right) \frac{W^2}{2 g_c \cdot \rho \cdot A^2} \right] - A \cdot \rho \cdot \frac{g}{g_c} \sin(\theta)$$

$$P_{IN} - P_{OUT} = \rho \cdot \left(\frac{f \cdot L}{D} + k \right) \cdot \frac{V^2}{2 g_c} = \left(\frac{f \cdot L}{D} + k \right) \cdot \frac{W^2}{2 A^2 \cdot \rho \cdot g_c} + \Delta P_{PUMP} + \ldots$$

Energy Conservation Equation (Micro)

$$\frac{D}{Dt}\iiint \gamma_k \cdot \rho_k \cdot \left(c_k + \frac{1}{2} v_k^2 \right) \cdot dV$$
$$= -\iint \vec{q}_k \cdot \vec{n} \cdot ds + \iiint E_k \cdot dV + \iiint \gamma_k \cdot \rho_k \cdot \vec{f}_k \cdot \vec{v}_k \cdot dV + \iint \left(\vec{\sigma}_k \cdot \vec{n} \right) \cdot v_k \cdot ds$$

$$\iiint \frac{\partial}{\partial t}\left[\rho \cdot e + \frac{1}{2}\rho v^2 \right] \cdot dV + \iint \left[\rho \cdot e + \frac{1}{2}\rho v^2 \right] \cdot \vec{v} \cdot \vec{n} \cdot ds =$$
$$= -\iint \vec{q} \cdot \vec{n} \cdot ds + \iiint E \cdot dV + \iiint \rho \cdot \vec{f} \cdot \vec{v} \cdot dV + \iint \left(\vec{\sigma} \cdot \vec{v} \right) \cdot \vec{v} \cdot ds$$

$$\iiint \frac{\partial}{\partial t}\left[\rho \cdot e + \frac{1}{2}\rho v^2 \right] \cdot dV + \iint \rho \cdot e \cdot \vec{v} \cdot \vec{n} \cdot ds = \iiint \nabla \left[\frac{1}{2}\rho \cdot v^2 \cdot \vec{v} \right] \cdot dV$$
$$= -\iint \vec{q} \cdot \vec{n} \cdot ds + \iiint E \cdot dV + \iiint \rho \cdot \vec{f} \cdot \vec{v} \cdot dV + \iiint \nabla \cdot \left(\vec{\sigma} \cdot \vec{v} \right) \cdot dV$$

Energy Conservation Equation (Micro)

$$\iiint \frac{\partial}{\partial t}(\rho \cdot \vec{v}) \cdot \vec{v} \cdot dV + \iiint \vec{v} \cdot (\nabla \cdot \rho \cdot \vec{v} \cdot \vec{v}) \cdot dV = \iiint \vec{v} \cdot (\nabla \cdot \vec{\tau}) \cdot dV$$
$$- \iiint \vec{v} \cdot \nabla P \cdot dV + \iiint \rho \cdot \vec{f} \cdot \vec{v} \cdot dV$$

$$\iiint \frac{\partial}{\partial t}(\rho \cdot e) \cdot dV + \iint \rho \cdot e \cdot \vec{v} \cdot \vec{n} \cdot ds = -\iint \vec{q} \cdot \vec{n} \cdot ds$$
$$+ \iiint E \cdot dV + \iiint \vec{\tau} : \nabla \vec{v} \cdot dV - \iiint P \nabla \cdot \vec{v} \cdot dV$$

$$\frac{\partial}{\partial t}(\rho \cdot e) = -\nabla \cdot q + E$$

$$\rho \cdot C_v \cdot \frac{\partial T}{\partial t} = \nabla \cdot k \cdot \nabla T + E = k \cdot \nabla^2 T + E$$

Energy Conservation Equation (Macro)

$$\iint \rho \cdot e \cdot \vec{v} \cdot \vec{n} \cdot ds = \iint_{A_1} \rho \cdot e \cdot \vec{v} \cdot \vec{n} \cdot ds + \iint_{A_2} \rho \cdot e \cdot \vec{v} \cdot \vec{n} \cdot ds + \ldots$$

$$\iint \rho \cdot e \cdot \vec{v} \cdot \vec{n} \cdot ds = -\sum_{INFLOW} \rho \cdot e \cdot \vec{v} \cdot A_i + \sum_{OUTFLOW} \rho \cdot e \cdot \vec{v} \cdot A = -\sum W_{IN} \cdot e_{IN} + \sum W_{OUT} \cdot e_{OUT}$$

$$-\iint \vec{q} \cdot \vec{n} \cdot ds + \iiint E \cdot dV = Q$$

$$\frac{\partial U}{\partial t} = \sum W_{IN} \cdot e_{IN} - \sum W_{OUT} \cdot e_{OUT} + Q + \iiint \vec{\tau} : \nabla \vec{v} \cdot dV - \iiint P \nabla \cdot \vec{v} \cdot dV$$

$$\iiint \frac{\partial}{\partial t}(\rho \cdot h) \cdot dV + \iint \rho \cdot h \cdot \vec{v} \cdot \vec{n} \cdot ds = -\iint \vec{q} \cdot \vec{n} \cdot ds + \iiint E \cdot dV$$

$$+ \iiint \vec{\tau} : \nabla \vec{v} \cdot dV + \iiint \frac{\partial P}{\partial t} \cdot dV + \iint P \cdot \vec{v} \cdot \vec{n} \cdot ds - \iiint P \nabla \cdot \vec{v} \cdot dV$$

Energy Conservation Equation (Macro)

$$\frac{\partial H}{\partial t} = +\sum W_{IN} \cdot h_{IN} - \sum W_{OUT} \cdot h_{OUT} + Q + \iiint \vec{\tau} : \nabla \vec{v} \cdot dV + \iiint \left(\frac{\partial P}{\partial t} + \vec{v} \cdot \nabla P \right) \cdot dV$$

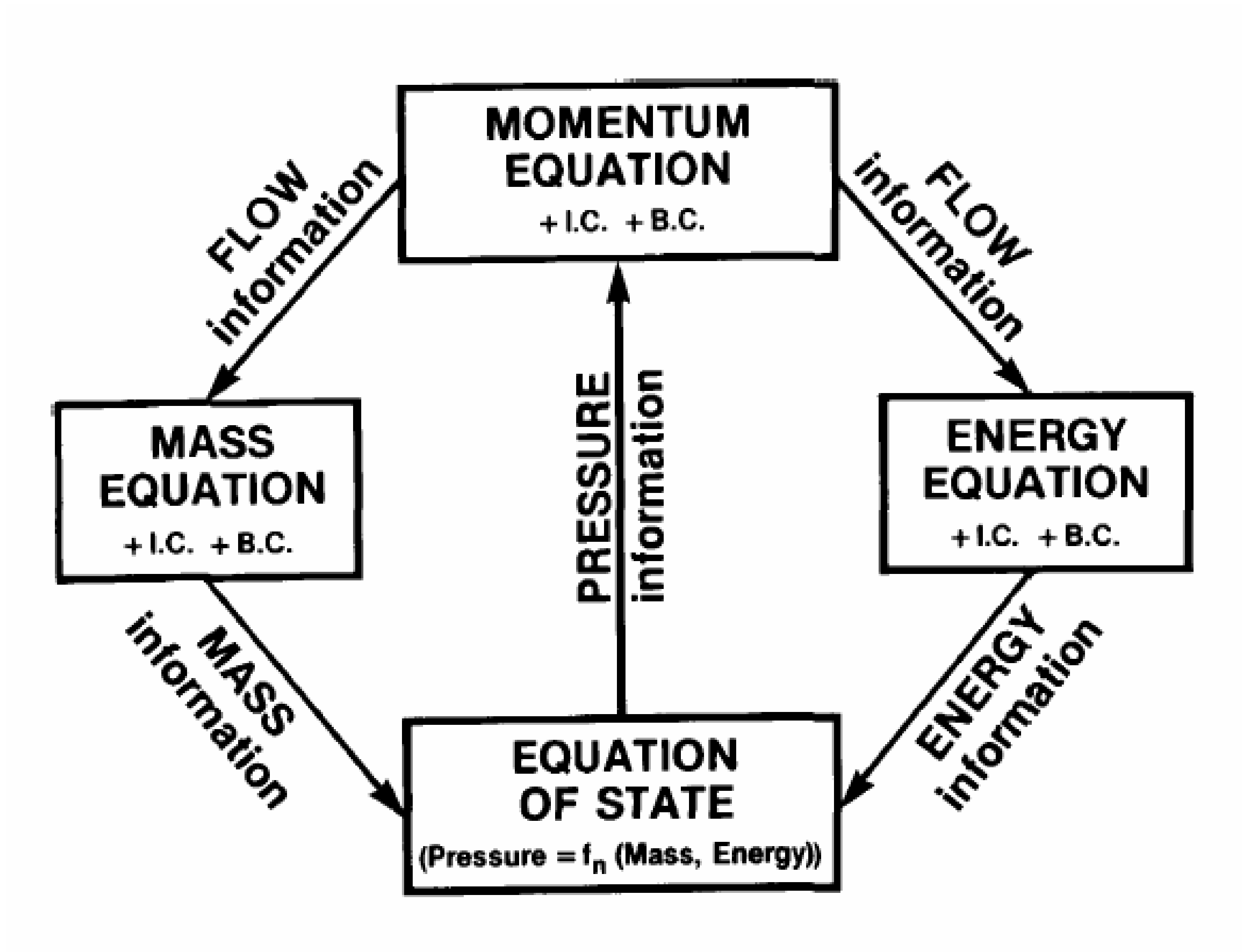
$$Q = \sum W_{OUT} \cdot h_{OUT} - \sum W_{IN} \cdot h_{IN} = W \cdot \left(h_{OUT} - h_{IN} \right) = W \cdot C_p \cdot \left(T_{OUT} - T_{IN} \right)$$

Newtron's Law

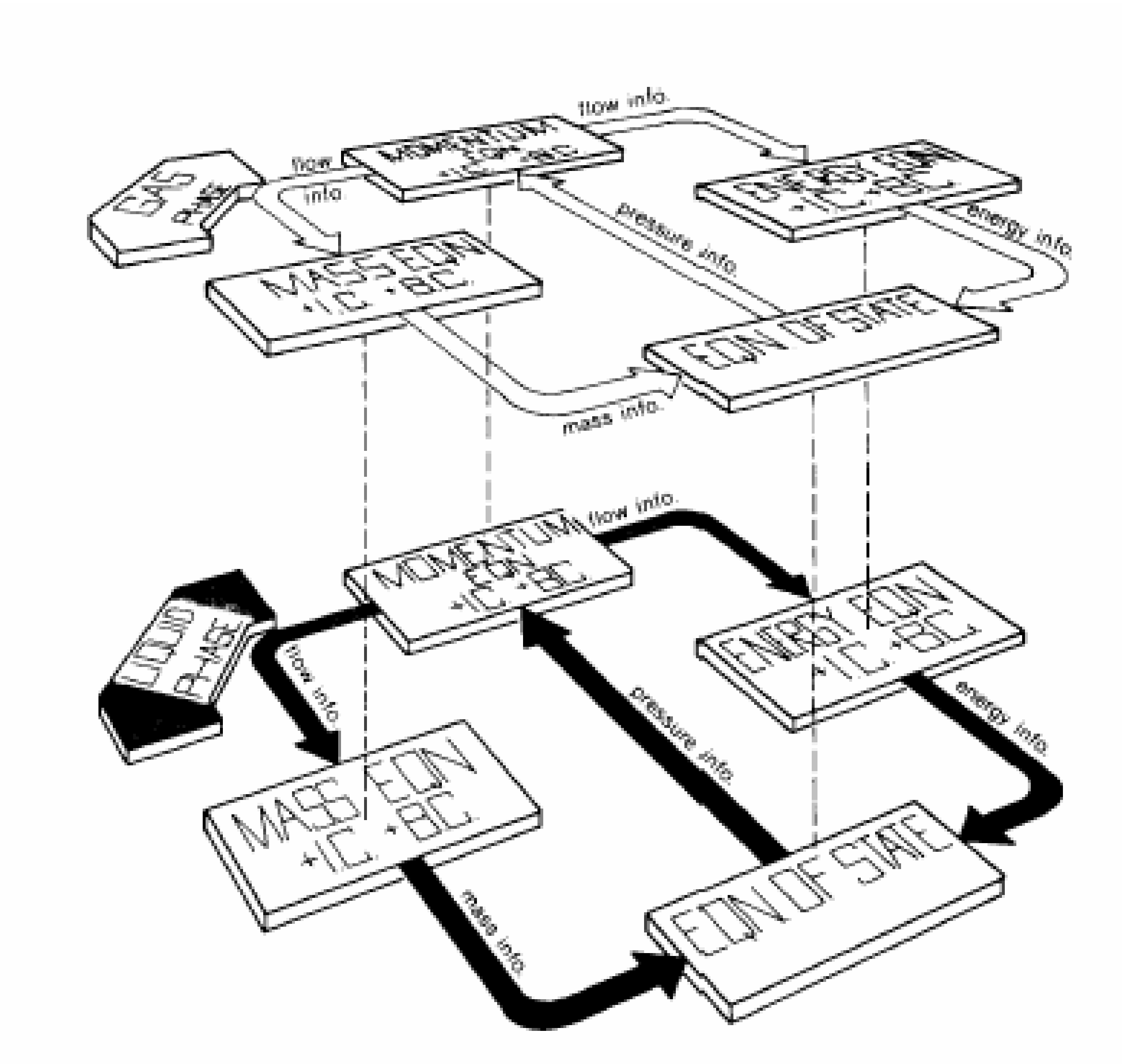
$$\vec{q} \cdot \vec{n} = h_N \cdot \left(T - T_s \right)$$

$$V \cdot \frac{\partial \rho h}{\partial t} - V \cdot \frac{\partial P}{\partial t} \left(\equiv V \cdot \frac{\partial \rho e}{\partial t} \approx V \cdot \rho C_v \frac{\partial T}{\partial t} \right) = \sum W_{IN} \cdot h_{IN} - \sum W_{OUT} \cdot h_{OUT}$$
$$- A \cdot h_N \cdot \left(T - T_s \right) + V \cdot E + \iiint \vec{\tau} : \nabla \vec{v} \cdot dV$$

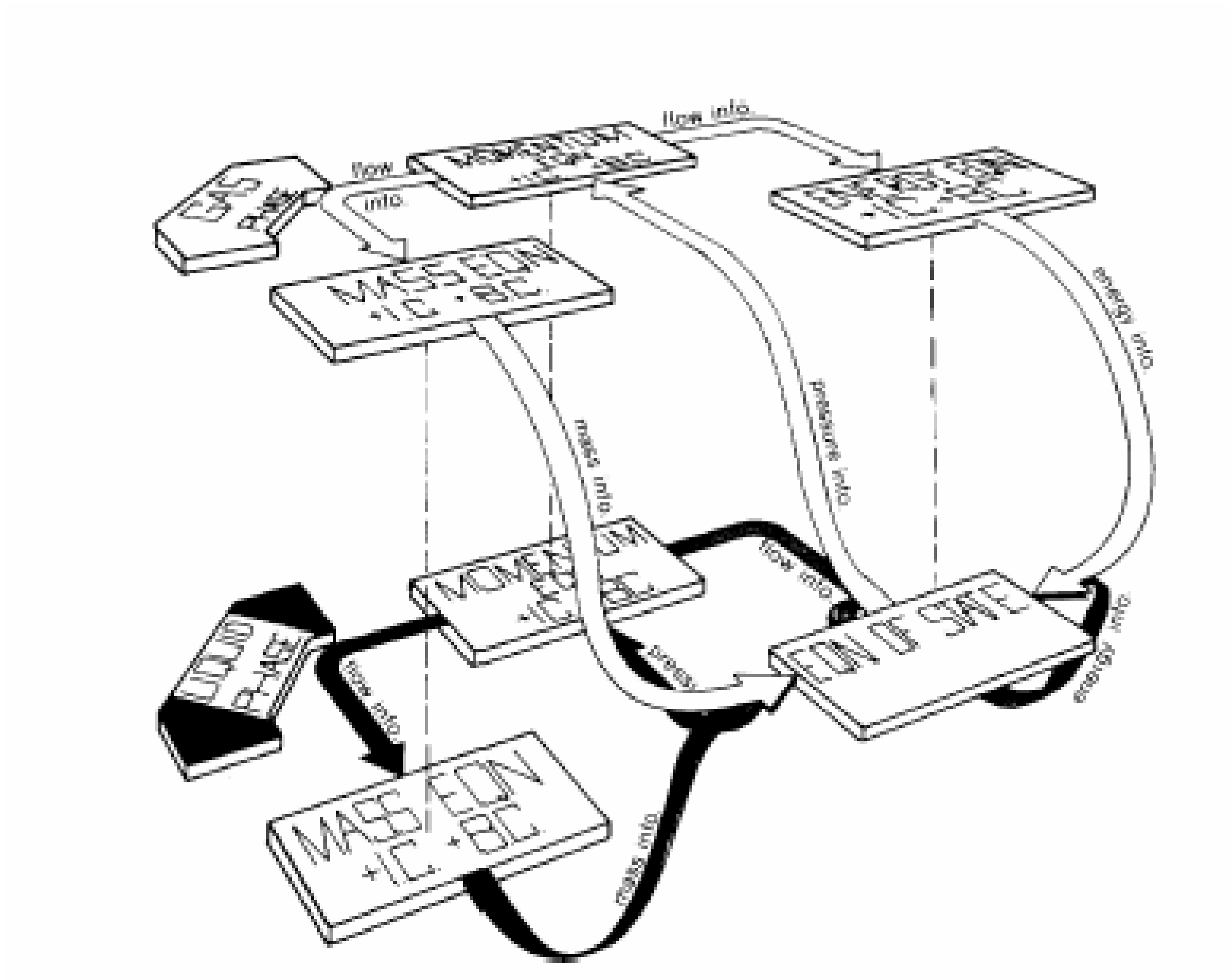
Linkage of Two Fluid Equations



Separate Two Fluid Model



Sombined Two Fluid Model



Questions?