

UNENE Graduate Course  
Reactor Thermal-Hydraulics Design and  
Analysis  
McMaster University  
Whitby  
March 19-21, April 23-25, May 2, 2004

# Nodalization

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# Outline

- Macro conservation equations
- Node-link diagrams
- Nodal diffusion
- Matrix notation

# Macro Conservation Equations

Mass

$$\frac{\partial M_i}{\partial t} = \sum_j \rho_j \cdot v_j \cdot A_j = \sum W_j$$

Momentum

$$\frac{\partial W}{\partial t} = \frac{A}{L} \left[ (P_{IN} - P_{OUT}) - \left( \frac{f \cdot L}{D} + k \right) \cdot \frac{W^2}{2g_c \cdot \rho \cdot A^2} \right] - A \cdot \rho \cdot g / g_c \cdot \sin(\theta)$$

Energy

$$\frac{\partial H}{\partial t} = \sum W_{IN} \cdot h_{IN} - \sum W_{OUT} \cdot h_{OUT} + Q$$

# Macro Conservation Equations

- The mass and energy equations are averaged; do not capture details within the volume
- Knowing mass and energy, and using the equation of state, gives the pressure
- Flow is driven by the pressure difference between two points
- Flow generates pressure drop
- Momentum equation is applied between points of known pressure; i.e., between volumes
- In the node-link diagram (staggered grid method) volumes are represented by nodes, flow paths are represented by links

# Nodal Diffusion

- In the node - link representation of flow in a pipe, no flow detail is considered as the fluid moves along the pipe
  - Therefore, no diffusion, dispersion, advection, flow profiles or flow regimes are explicitly permitted
  - Not too crude an approximation for the calculation of pressure drops and flows
  - Perfect mixing at the nodes is assumed
- For plug flow in a pipe, flow is transmitted along the pipe relatively undisturbed. If no diffusion or turbulent dispersion existed, a sharp discontinuity in a property would propagate undisturbed.
- A very large number of nodes are needed to transmit a disturbance without appreciable distortion.
- Thus, nodalization creates a form of diffusion in much the same manner as finite difference schemes create numerical diffusion. Attaining convergence in nodalization is, in essence, converging the model to plug flow behaviour.

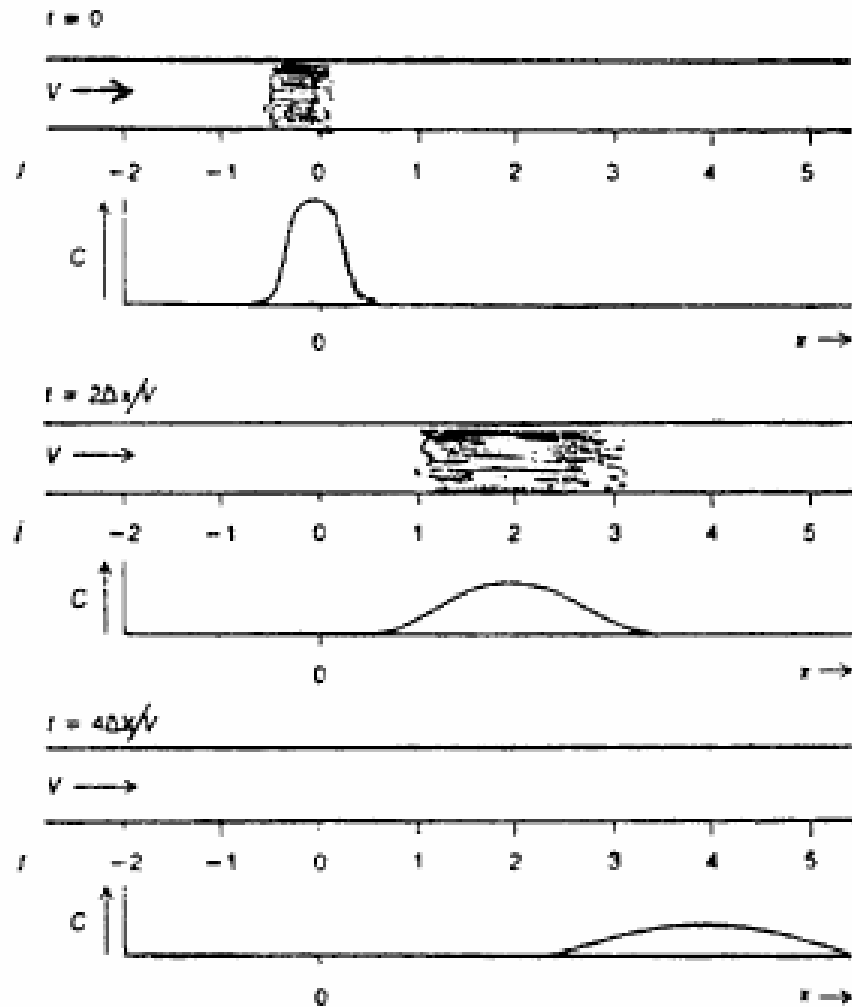
# Nodal Diffusion

$$C_{OUT} = C_{IN} \cdot e^{-\frac{t}{\tau}}$$

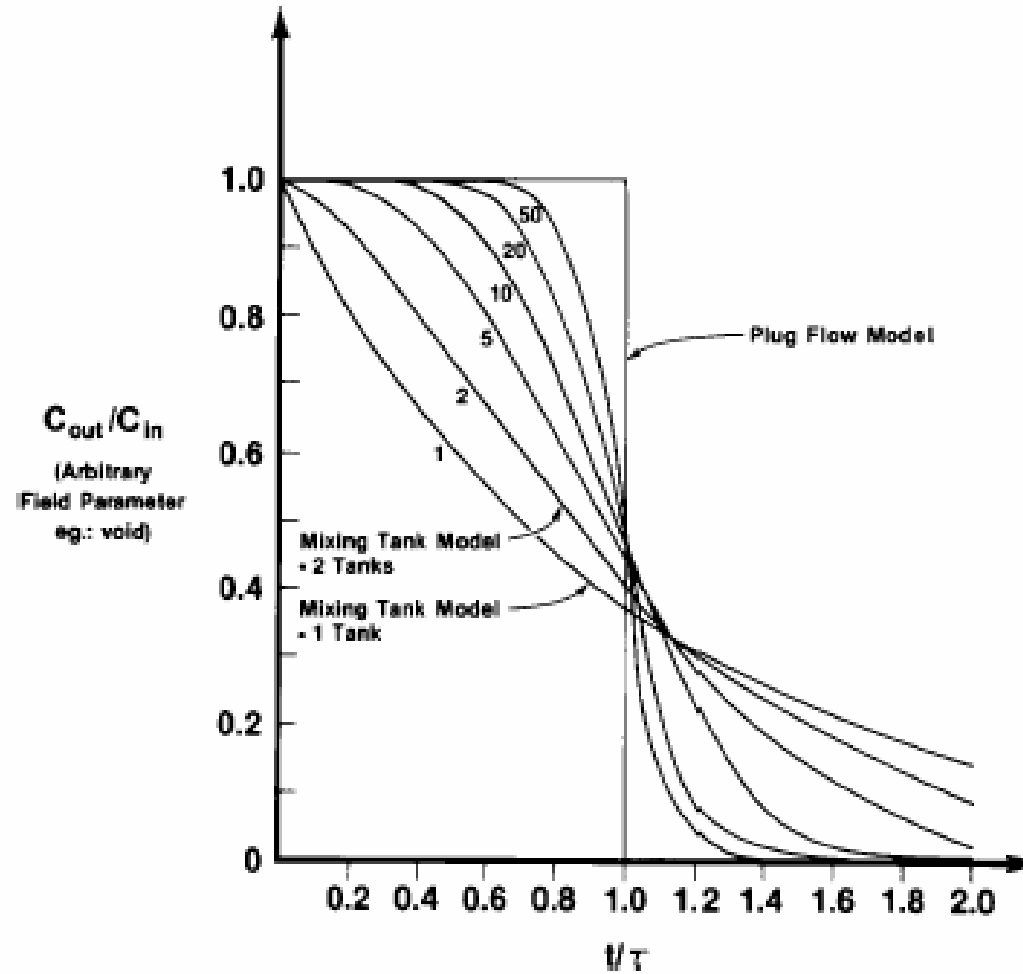
$$C_{OUTNODE2} = C_{INNODE1} \cdot \left(1 + \frac{2t}{\tau}\right) e^{-\frac{2t}{\tau}}$$

$$C_{OUTNODE2} = C_{INNODE1} \cdot e^{-\frac{n \cdot t}{\tau}} \sum_{k=1}^N \left(\frac{n \cdot t}{\tau}\right)^{k-1} \frac{1}{(n-1)!}$$

# Nodal Diffusion

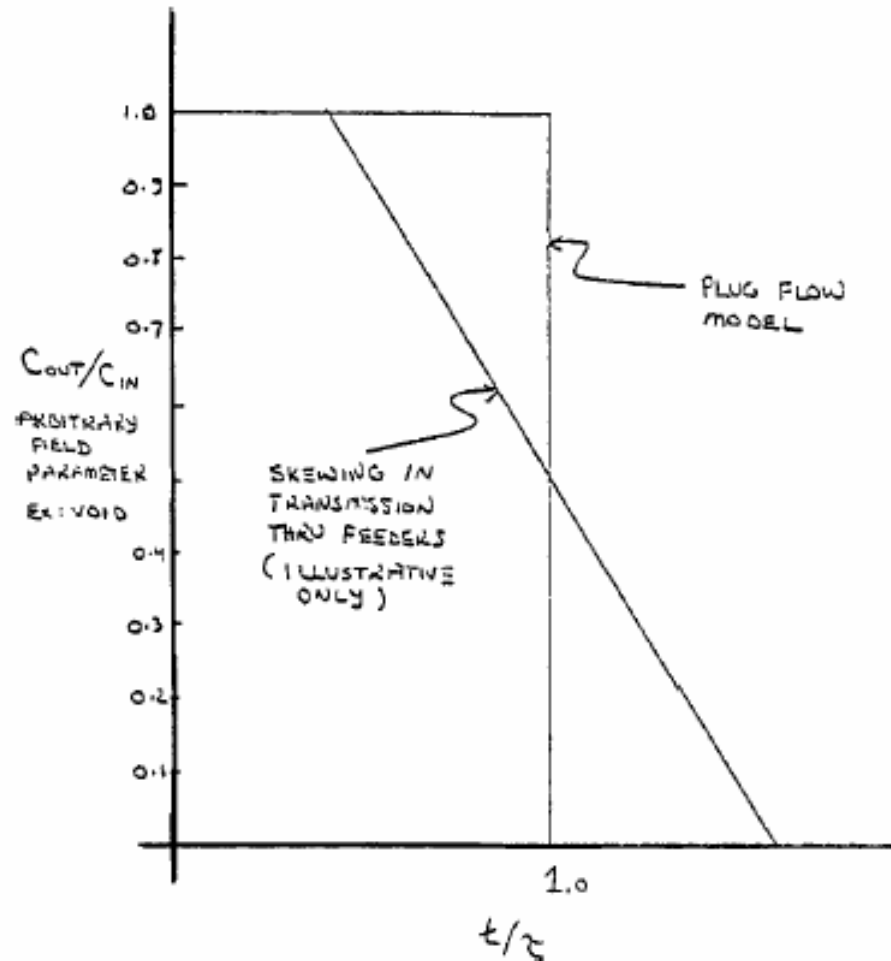


# Nodal Diffusion

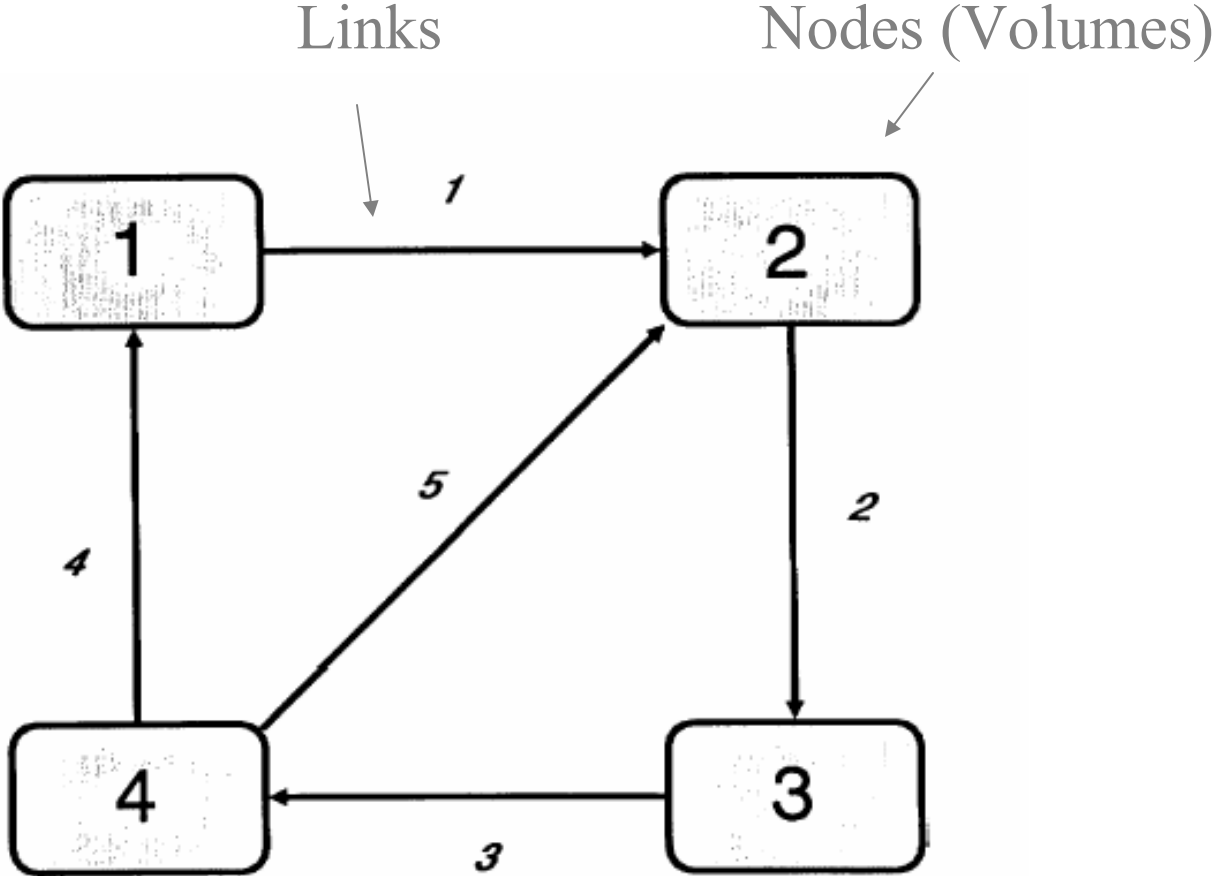




# Nodal Diffusion



# Node-Link Diagrams



# Matrix Form of Equations

$$\frac{dM_1}{dt} = -W_1 + W_4$$

$$\frac{dM_2}{dt} = W_1 - W_2 + W_5$$

$$\frac{dM_3}{dt} = W_2 - W_3$$

$$\frac{dM_4}{dt} = W_3 - W_4 - W_5$$

$$\begin{pmatrix} \dot{M}_1 \\ \dot{M}_2 \\ \dot{M}_3 \\ \dot{M}_4 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 \end{pmatrix} \cdot \begin{pmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \\ W_5 \end{pmatrix} \equiv \dot{m} = A^{MW} \cdot W$$