UNENE Graduate Course
Reactor Thermal-Hydraulics Design and Analysis

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Rate Form of Equation of State

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Outline

- The Rate Form
- Numerical Investigation – Simple Case
- Numerical Investigation – Practical Case

The Rate Form

- Nature of conservation equations – rate equations
- Nature of equation of state – algebraic equation
  - Although the properties of mass, momentum, and energy are traced and solved as function of time and space, the corresponding local pressure is a pure function of the local state of the fluid
- The rate form of the equation of state is more appropriate for system analysis
  - Fits better with the conservation equations
    - Complete set of 4 equations with 4 variables (pressure, flow, energy and mass)
    - Allows for numerical solution (calculation) of pressure without iteration
    - A numerical scheme is easily formulated and coded

The Rate Form – Simple Case

- System matrix equation has the following form

$$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{A}\mathbf{u} + \mathbf{b}$$

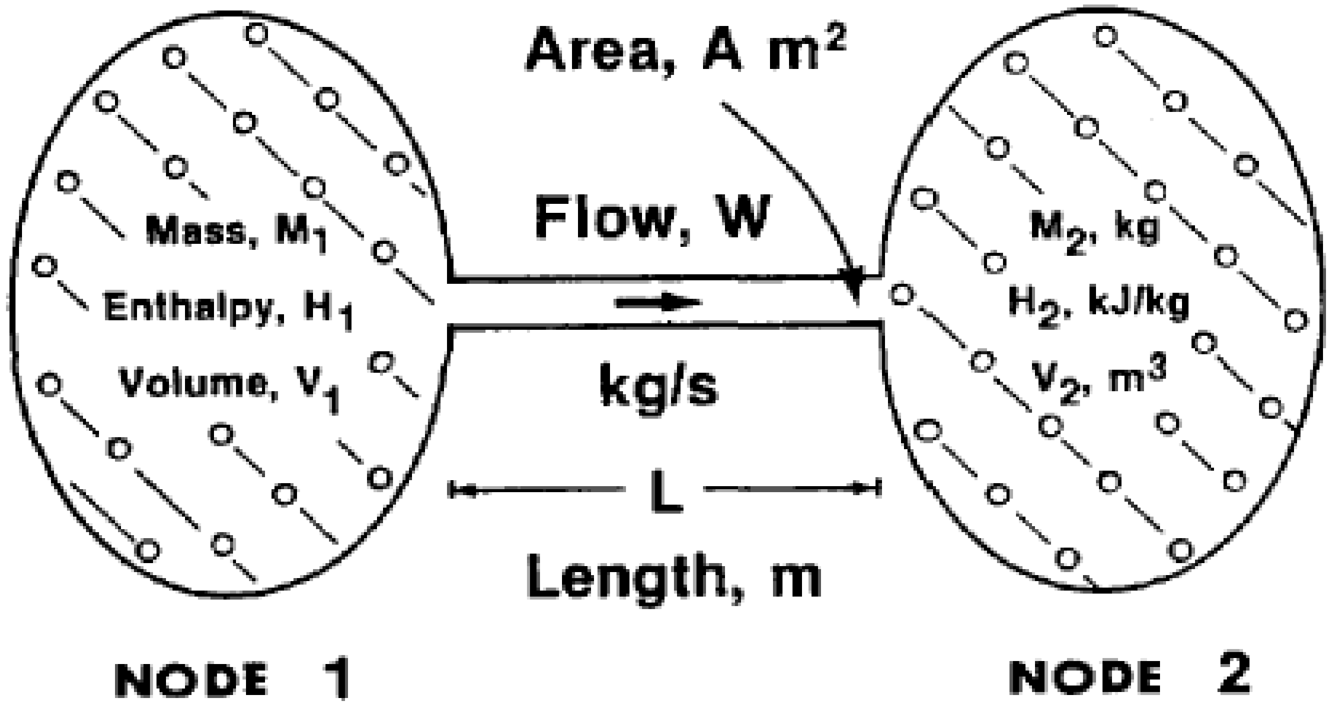
where for a 2-node, 1-link system

Normal method $u = \{M_1, H_1, W, M_2, H_2\}$

Rate method $u = \{M_1, H_1, P_1\ W, M_2, H_{2,}\ P_2\}$

$$\mathbf{u}^{t+\Delta t} = \mathbf{u}^{t} + \Delta t[\mathbf{A}\mathbf{u} + \mathbf{b}]$$

The Rate Form – Simple Case

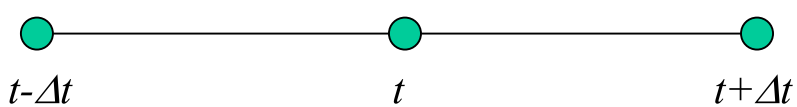


The Rate Form – Simple Case – Normal Method

- By the normal method, pressure is obtained at the new time step, $t+\Delta t$, by iteration of the equation of state using the values of the mass and enthalpy at the new time step

$$P^{t+\Delta t} = \text{fn}(\rho^{t+\Delta t}, h^{t+\Delta t})$$

- The starting guess for pressure is the value at time '*t*'
- Feedback in the iteration scheme is generated by using an older value of pressure $P^{t-\Delta t}$

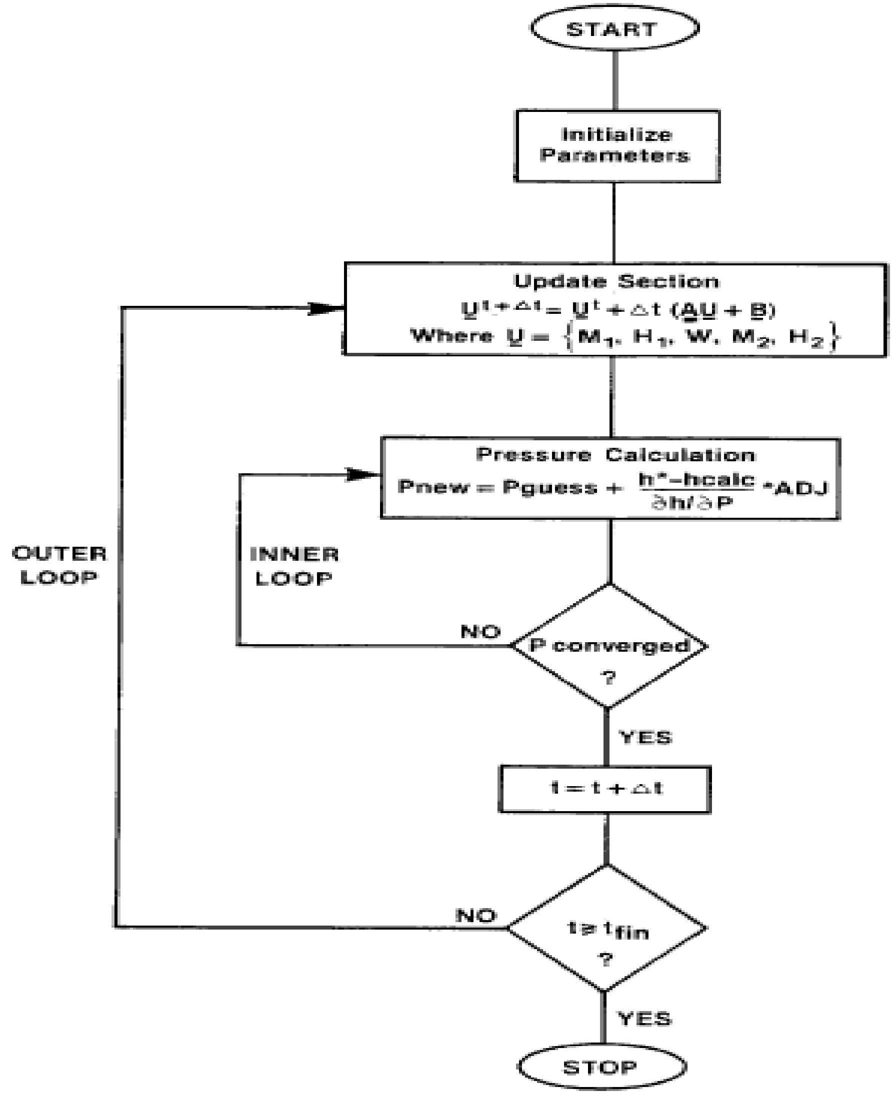


The Rate Form – Simple Case – Normal Method

$$P_{new} = P_{guess} + \frac{h - h_{est}}{\partial h / \partial P} * ADJ$$

- h is a known value of of h at $t+\Delta t$, and h_{est} is the estimated value based on the guessed pressure
- ADJ is an adjustment factor [0, 1], to allow for experimentation with the amount of feedback
- Iteration continues until a convergence criterion P_{err} is satisfied.

The Rate Form – Simple Case – Normal Method



The Rate Form – Simple Case – Rate Method

- By the rate method, the pressure at the new time step, $t+\Delta t$, is obtained directly from the rate equation

$$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{A}\mathbf{u} + \mathbf{b}$$

$$\mathbf{P}_i^{t+\Delta t} = \mathbf{P}_i^{t} + \Delta t[\mathbf{A}\mathbf{u} + \mathbf{b}]_i$$

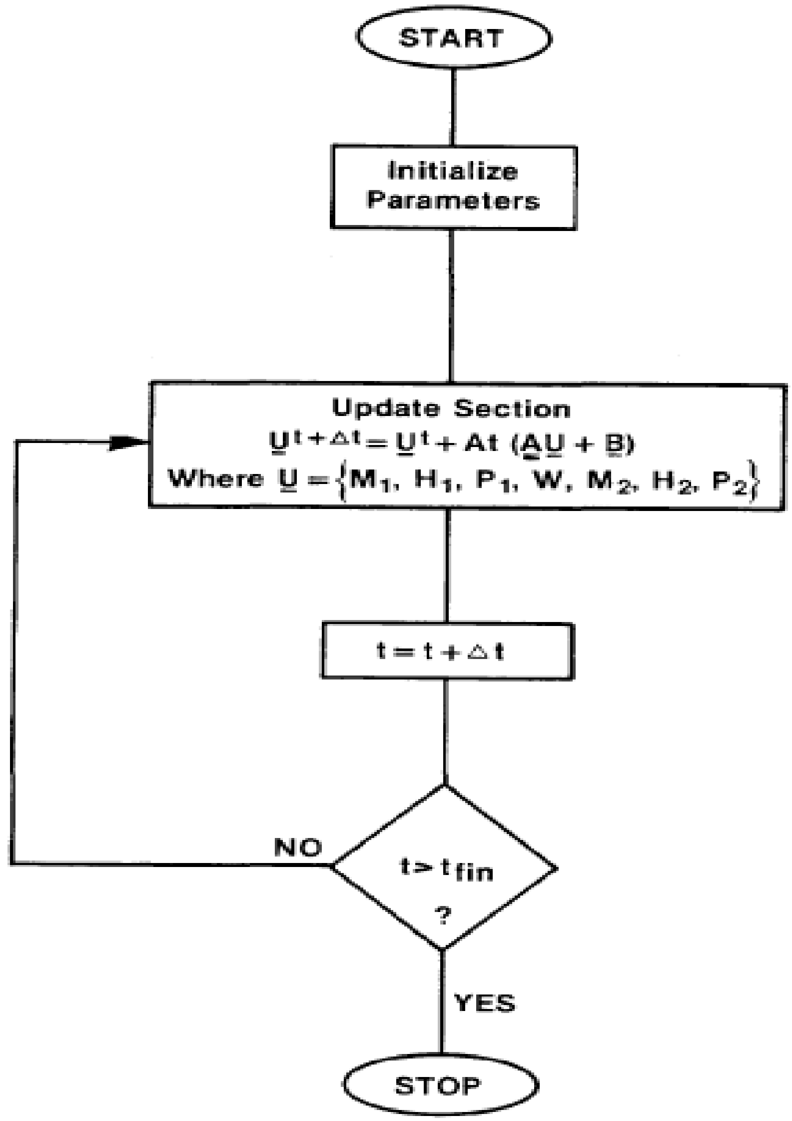
- To cope with the drift of the pressure, a correction term is used (pressure is not a conserved property)

$$\mathbf{P}_i^{t+\Delta t} = \mathbf{P}_i^{t} + \Delta t[\mathbf{A}\mathbf{u} + \mathbf{b}]_i + \frac{h - h_{est}}{\partial h / \partial P} * ADJ$$

The Rate Form – Simple Case – Rate Method

- The main difference between the normal method and the rate method is that during a time step between t and $t+\Delta t$
  - The normal method employs parameters such as density, quality, etc., derived from the pressure at time $t+\Delta t$
  - The rate method employs parameters derived from the pressure and rate of change of pressure at time t.
- The rate method is easier to implement, more robust, and more than 20 times faster under certain conditions
- The rate method has been proven to be consistently better in terms of numerical performance

The Rate Form – Simple Case – Rate Method



The Rate Form – Implicit Numerical Schemes

- Nodal equations for a system of two nodes and one link

$$\frac{dM_1}{dt} = -W \quad \text{and} \quad \frac{dM_2}{dt} = +W$$

$$\frac{dH_1}{dt} = -h_1 W \quad \text{and} \quad \frac{dH_2}{dt} = +h_1 W$$

$$\frac{dP_i}{dt} = \frac{F_1 \frac{dM_i}{dt} + F_2 \frac{dH_i}{dt}}{M_g F_4 + M_f F_5}, \qquad i = 1,2$$

previously derived

The Rate Form – Implicit Numerical Schemes

- Considering the pressure and flow rate equations (after substituting for dM/dt and dH/dt)

$$\frac{dW}{dt} = \frac{A}{L}(P_1 - P_2) - \frac{A}{L}K|W|W$$

$$\frac{dP_1}{dt} = -\chi_1 W \quad \text{and} \quad \frac{dP_2}{dt} = +\chi_2 W$$

$$\chi = \frac{F_1 + hF_2}{M_g F_4 + M_f F_5}$$

The Rate Form – Implicit Numerical Schemes

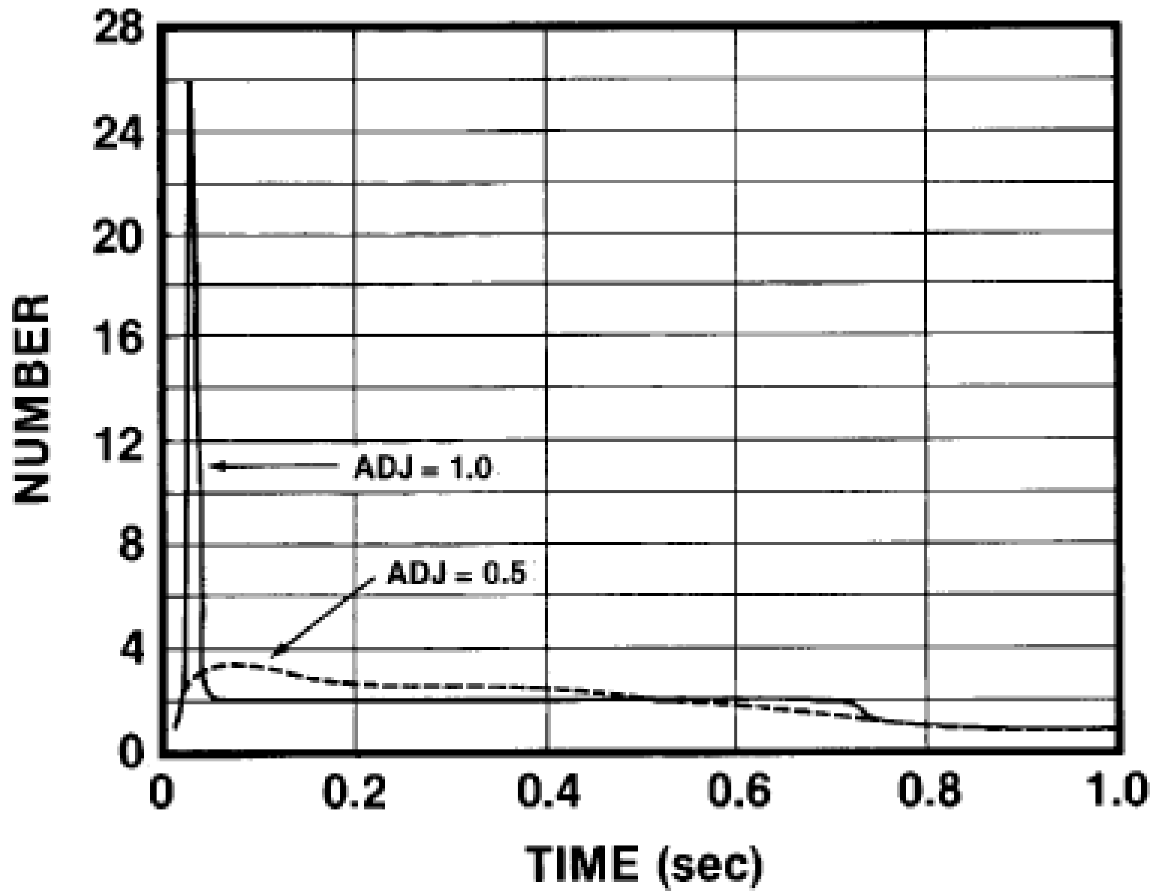


Figure 5.4 Number of iterations per pressure routine call for the normal method with a time step of 0.01 seconds and a pressure error tolerance of 0.001 of full scale (10 mPa).

The Rate Form – Implicit Numerical Schemes

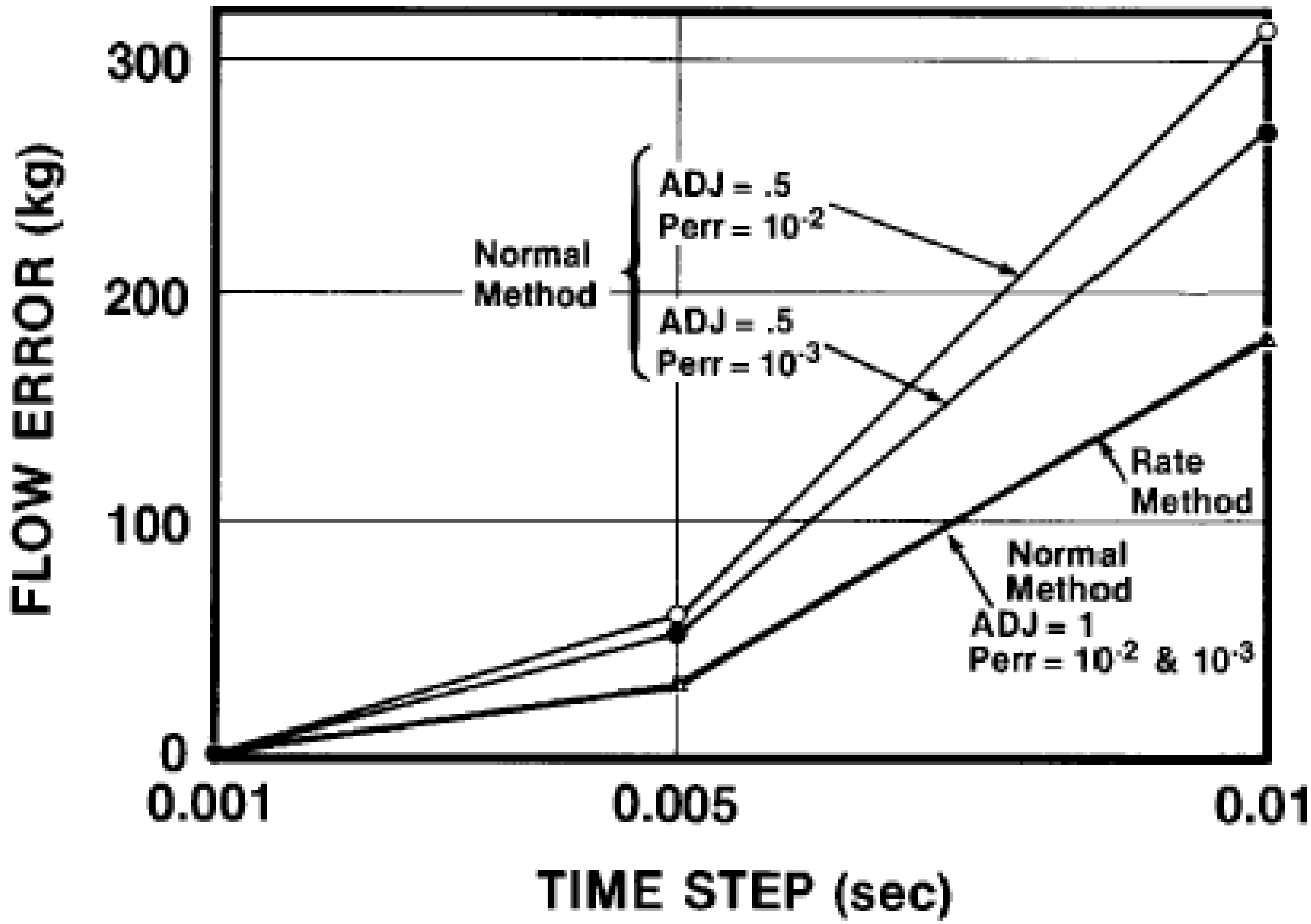


Figure 5.5 Integrated flow error for the rate method and the normal method for various fixed time steps, convergence tolerances and adjustment factors.

The Rate Form – Implicit Numerical Schemes

- Employing a scheme implicit in flow

$$\frac{W^{t+\Delta t} - W^{t}}{\Delta t} = \frac{A}{L}\left(P_1^{t+\Delta t} - P_2^{t+\Delta t}\right) - \frac{A}{L}K\left|W^{t}\right|W^{t+\Delta t}$$

$$\frac{P_i^{t+\Delta t} - P_i^{t}}{\Delta t} = \pm\chi_i W^{t+\Delta t} \rightarrow P_i^{t+\Delta t} - P_i^{t} = \pm\chi_i W^{t+\Delta t}\Delta t$$

- Implicit rate form of the equation of state

$$W^{t+\Delta t} = \left[1 + \frac{A}{L}K\left|W^{t}\right|\Delta t + \frac{A}{L}\left(\chi_1 + \chi_2\right)\Delta t^2\right]^{-1}\left[W^{t} + \frac{A}{L}\left(P_1^{t} - P_2^{t}\right)\Delta t\right]$$

- Implicit normal form of the equation of state

$$W^{t+\Delta t} = \left[1 + \frac{A}{L}K\left|W^{t}\right|\Delta t\right]^{-1}\left[W^{t} + \frac{A}{L}\left(P_1^{t+\Delta t} - P_2^{t+\Delta t}\right)\Delta t\right]$$

The Rate Form – Implicit Numerical Schemes

- Considering the eigenvalues and eigenvectors of the above equations

$$\frac{\partial \mathbf{u}(t)}{\partial t} = \mathbf{A}(\mathbf{u},t)\mathbf{u}(t) \qquad \mathbf{u}(t) = \sum_{\ell=1}^{N} \mathbf{u}_\ell e^{\alpha_\ell t}$$

$\mathbf{u}_\ell$ = eigenvectors
α_ℓ = eigenvalues.

- Explicit formulation

$$\mathbf{u}^{t+\Delta t} = \sum_{\ell=1}^{N} (1+\alpha_\ell \Delta t)\mathbf{u}_\ell$$

- Implicit formulation

$$\mathbf{u}^{t+\Delta t} = \sum_{\ell=1}^{N} \frac{\mathbf{u}_\ell}{(1-\alpha_\ell \Delta t)}$$

The Rate Form – Implicit Numerical Schemes

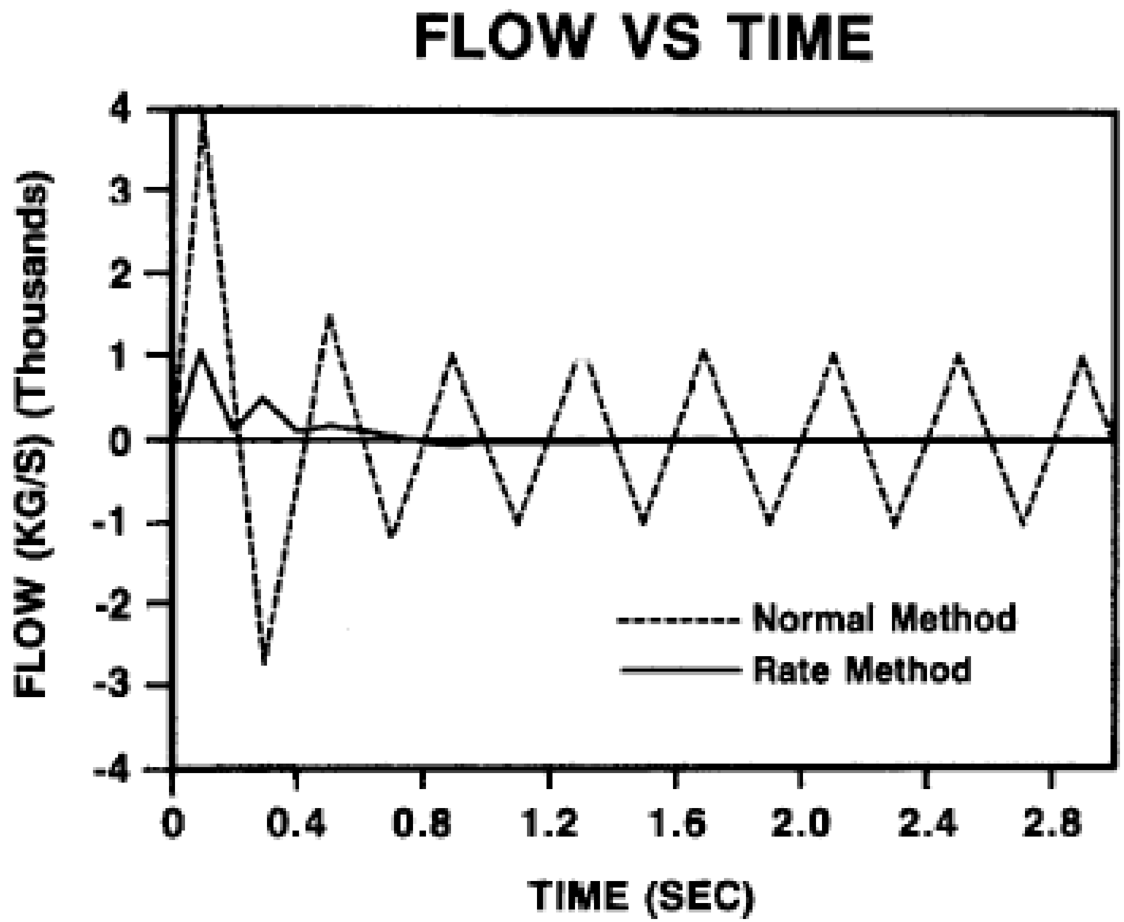
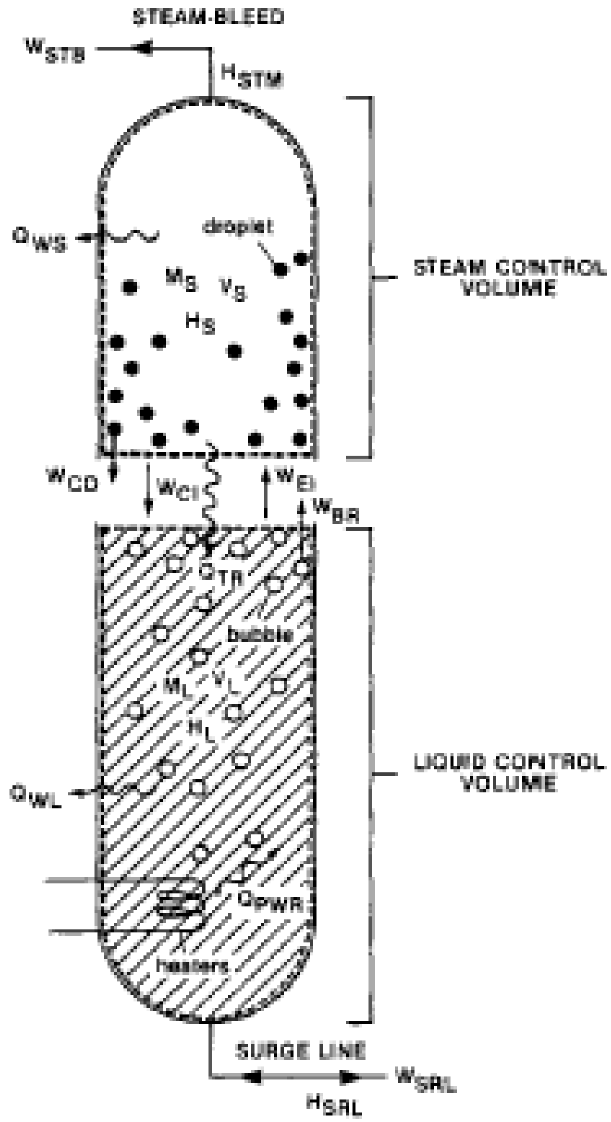


Figure 5.6 Flow vs. time for the implicit forms of the normal and rate method.

The Rate Form – Practical Case



The Rate Form – Practical Case

$$\frac{dM_s}{dt} = -W_{STB} - W_{CD} - W_{CI} + W_{EI} + W_{BR}$$

$$\frac{dM_L}{dt} = W_{SRL} - W_{EI} - W_{BR} + W_{CD} + W_{CI}$$

$$\frac{dH_S}{dt} = -W_{STB}h_{gST} - W_{CD}h_{fST} - W_{CI}h_{gST} + W_{EI}h_{sLQ} + W_{BR}h_{gLQ} - Q_{WS} + Q_{TR} - (1 - \ )[(1 - \ )Q_{COND} + Q_{EVPR}]$$

and

$$\frac{dH_L}{dt} = W_{SRL}h_{SRL} - W_{EI}h_{fLQ} - W_{BR}h_{gLQ} + W_{CI}h_{fST} + W_{CD}h_{fST} - Q_{WL} + Q_{PWR} - Q_{TR} - \ [(1 - \ )Q_{COND} + Q_{EVPR}$$

$$\frac{dV_S}{dt} = -\frac{dV_L}{dt}$$

The Rate Form – Practical Case

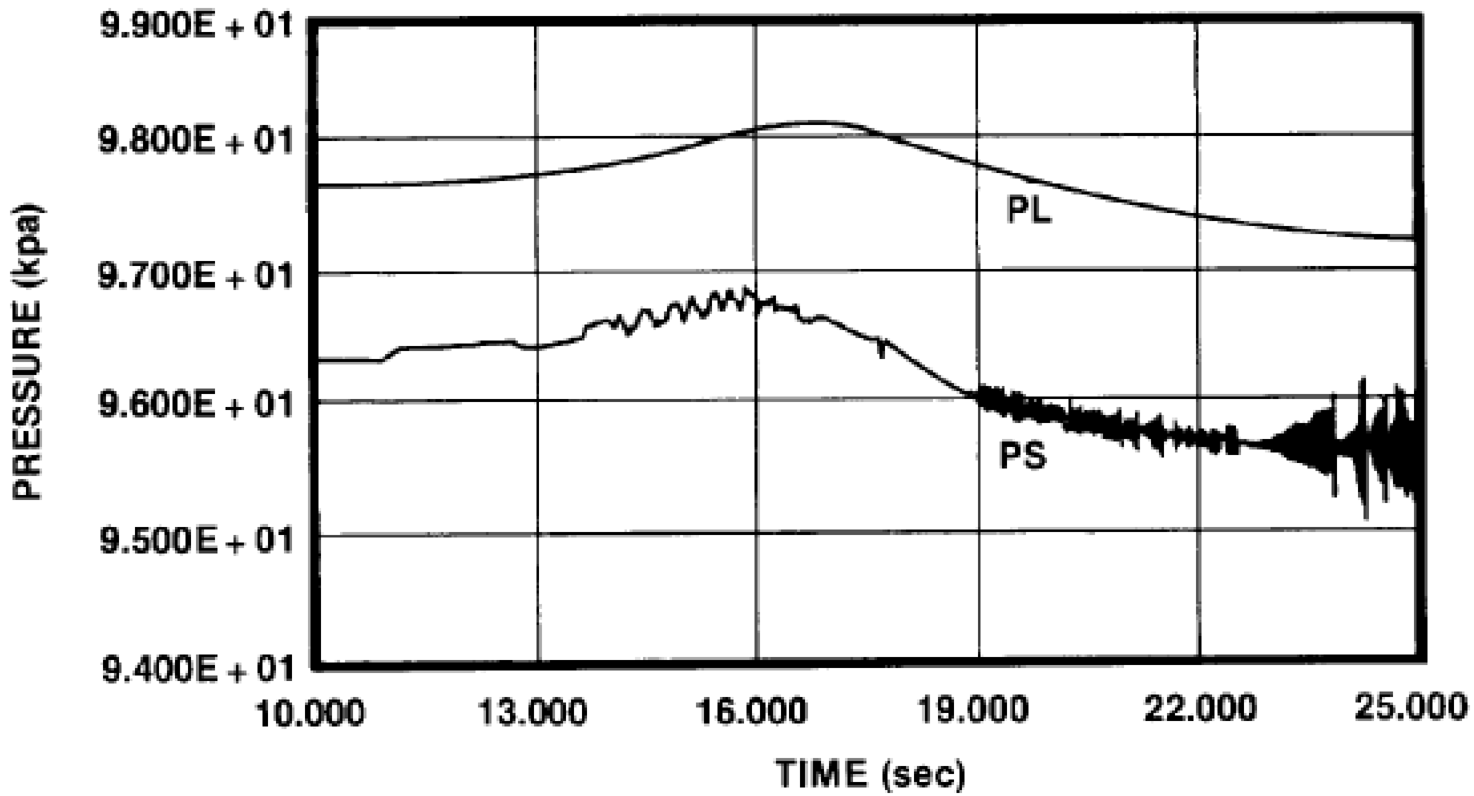


Figure 5.8 Pressurizer's pressure transient for the normal method with error tolerance of 0.2%.

The Rate Form – Practical Case

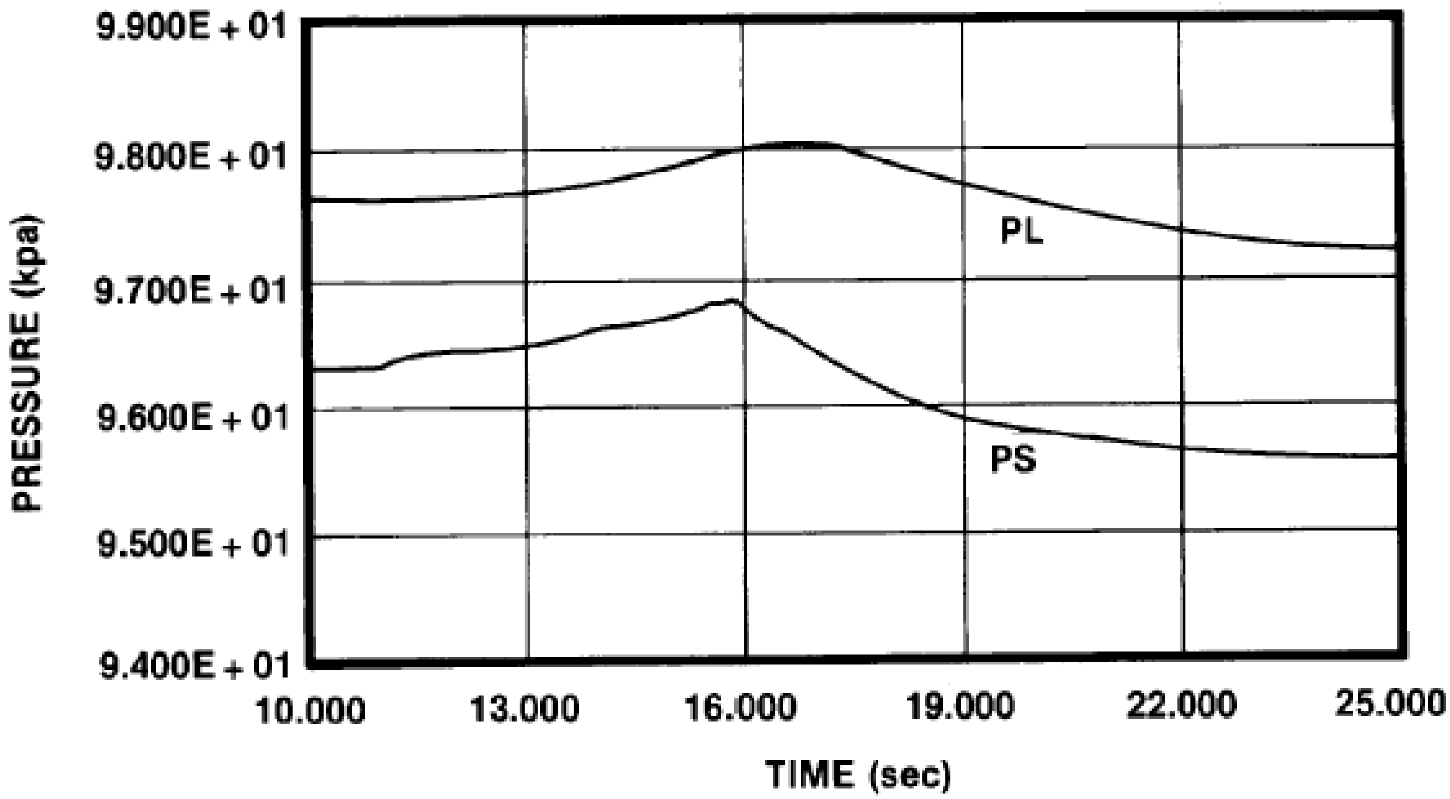


Figure 5.9 Pressurizer's pressure transient for the rate method.

The Rate Form – Practical Case

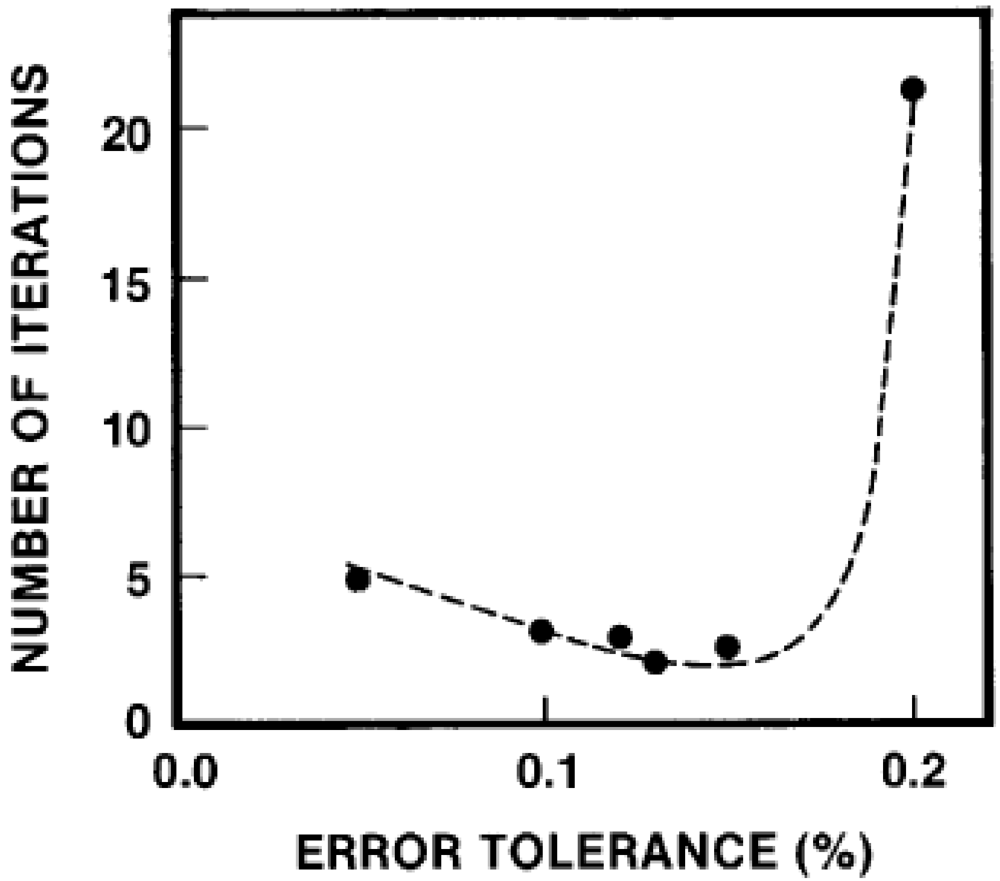


Figure 5.10 Averaged number of iterations per pressure routine call for the normal method in simulating pressurizer problem.

Conclusions

- The rate methods offers the following advantages
  - It permits proper focus on the two main variables, flow and pressure
  - The same form is appropriate for eigenvalue extraction and for numerical simulation
  - The program is easier to implement
  - Programs are more robust
  - The time step control and detection of rapid changes is improved

Questions?