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Network Calculations

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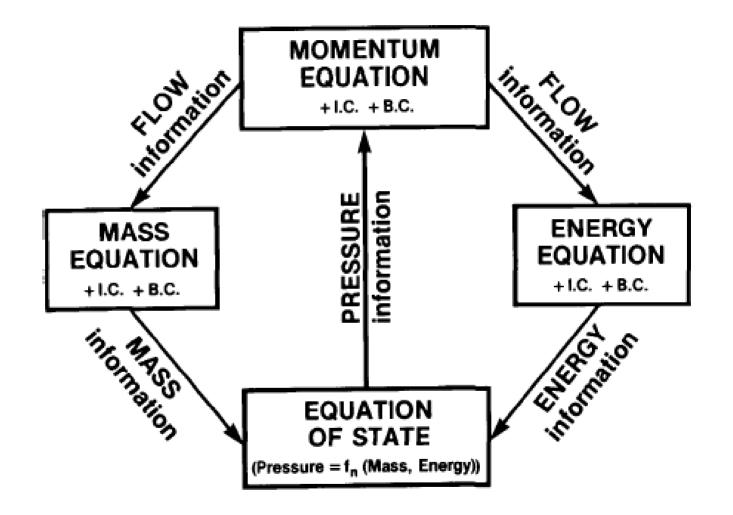
Outline

- Outline of Porsching method
- Development of fully-implicit back-substituted form (FIBS)
- Fully explicit scheme
- Semi-implicit scheme
- Fully implicit scheme
- Programming notes

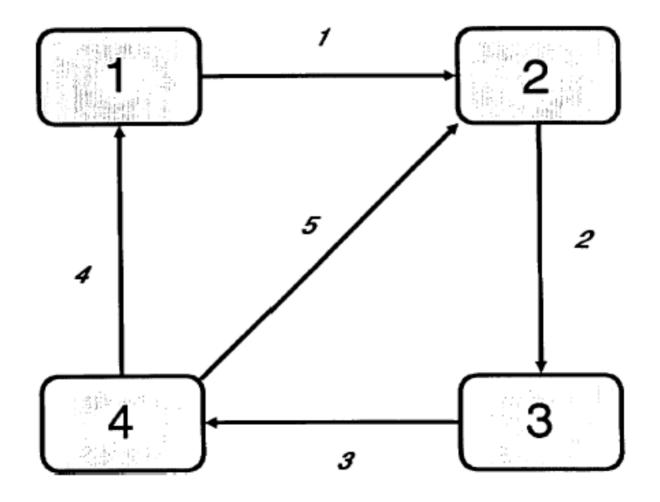
Porsching method

- This algorithm, involving the Jacobian (derivative of the system state matrix), is used originally in the computer code FLASH-4 and subsequently in the OPG code SOPHT, and evolved into forms used in RETRAN
- The strength of Porsching's approach lies in its recognition of flow as the most important dependent parameter and, hence, its fully implicit treatment of flow
- Based on system state matrix which contains all the system dynamics in terms of the dependent parameters of mass, energy and flow. Back substitution finally gives a matrix rate equation in terms of the system flow (the unknown) and the system derivatives.
- Porsching form is identical to the "Rate" form and is a subset of the fully implicit back-substituted form and is easily derived from it
- Some codes, but not all use this method

Thermal-hydraulic System Simulation Equations



Sample Thermal-hydraulic Network



Fully-implicit back-substituted form (FIBS)

 $\dot{\mathbf{u}} = \mathbf{f}(\mathbf{t}, \mathbf{u})$ $\mathbf{u} = \mathbf{u}(\mathbf{M}_i, \mathbf{H}_i, \mathbf{W}_j)$

 $\dot{\mathbf{u}} = \mathbf{f}^{\mathbf{t}} + \Delta \mathbf{t} \mathbf{J} \dot{\mathbf{u}}$ J is the Jacobian of the system of equations

 $\Delta \mathbf{u} = \Delta \mathbf{t} \mathbf{f}^{\mathbf{t}} + \Delta \mathbf{t} \mathbf{J} \Delta \mathbf{u}$

 $[\mathbf{I} - \Delta \mathbf{t} \ \mathbf{J}] \Delta \mathbf{u} = \Delta \mathbf{t} \ \mathbf{f}^{\mathbf{t}}$

$$\frac{\partial W}{\partial t} = \frac{A}{L} \left[\left(P_{IN} - P_{OUT} \right) - \left(\frac{f \cdot L}{D} + k \right) \cdot \frac{W^2}{2g_c \cdot \rho \cdot A^2} \right] - A \cdot \rho \cdot g / g_c \cdot \sin(\theta)$$

$$\frac{dW_{j}}{dt} = \frac{A_{j}}{L_{j}} \left(P_{u} + S_{WP} \Delta P_{u} - P_{d} - S_{WP} \Delta P_{d} \right) + k_{j} \left(W_{j} + S_{WW} \Delta W_{j} \right)^{2} + b_{wj}$$
$$= \frac{\Delta W_{j}}{\Delta t}$$

$$b_{wj} = (A_j/L_j) (h_j \rho_j g + \Delta P_{pump})$$

$$\frac{dM_i}{dt} = \sum_{j \forall d} \left(W_j + S_{MW} \Delta W_j \right) - \sum_{j \forall u} \left(W_j + S_{MW} \Delta W_j \right) \approx \frac{\Delta M_i}{\Delta t}$$

$$\frac{dH_{i}}{dt} = \sum_{j \forall d} \left(W_{j} + S_{HW} \Delta W_{j} \right) \frac{\left(H_{j} + S_{HH} \Delta H_{j}\right)}{\left(M_{j} + S_{HM} \Delta M_{j}\right)} - \sum_{j \forall u} \left(W_{j} + S_{HW} \Delta W_{j}\right) \frac{\left(H_{j} + S_{HH} \Delta H_{j}\right)}{\left(M_{j} + S_{HM} \Delta M_{j}\right)} + Q_{i}$$

$$= \sum_{j \forall d} \left(\frac{W_{j}H_{j}}{M_{j}} + \frac{S_{HW}H_{j}}{M_{j}} \Delta W_{j} + \frac{S_{HH}W_{j}}{M_{j}} \Delta H_{j} - \frac{S_{HM} W_{j} H_{j}}{M_{j}^{2}} \Delta M_{j} \right)$$

$$- \sum_{j \forall u} \left(\frac{W_{j}H_{j}}{M_{j}} + \frac{S_{HW}H_{j}}{M_{j}} \Delta W_{j} + \frac{S_{HH}W_{j}}{M_{j}} \Delta H_{j} - \frac{S_{HM} W_{j} H_{j}}{M_{j}^{2}} \Delta M_{j} \right) + Q_{i}$$

$$\approx \frac{\Delta H_{i}}{\Delta t}$$

$$\Delta P_{i} = \frac{\partial P_{i}}{\partial M_{i}} \Delta M_{i} + \frac{\partial P_{i}}{\partial H_{i}} \Delta H_{i} + \frac{\partial P_{i}}{\partial V_{i}} \Delta V_{i}$$
$$\frac{\Delta P_{i}}{\Delta t} = C_{1i} \frac{\Delta M_{i}}{\Delta t} + C_{2i} \frac{\Delta H_{i}}{\Delta t} \qquad \text{For constant volume}$$

- The system unknowns to be solved for are ΔW , ΔM , ΔH and ΔP
- The mass equation is simple and is used to eliminate ΔM in terms of ΔW. Flow is chosen as the prime variable since it is the main actor in thermal-hydraulic systems.
- The enthalpy equation poses a problem as it is too complex to permit a simple substitution; Porsching surmounts this by setting $S_{HH} = S_{HM} = 0$, ie making the solution explicit in specific enthalpy.

$$\Delta \mathbf{M} = \Delta t \mathbf{A}^{\mathbf{MW}} [\mathbf{W}^{\mathsf{t}} + \mathbf{S}_{\mathbf{MW}} \Delta \mathbf{W}]$$

$$\mathbf{A}^{\mathbf{MW}} = \begin{pmatrix} -1 & 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 \end{pmatrix} \qquad \text{nodes}$$

$$\begin{split} \Delta \mathbf{W} &= \Delta t \Big\{ \mathbf{A}^{\mathbf{WP}} \Big[\mathbf{P}^{t} + \mathbf{S}_{\mathbf{WP}} \Delta \mathbf{P} \Big] + \mathbf{A}^{\mathbf{WW}} \Big[\mathbf{W}^{t} + 2\mathbf{S}_{\mathbf{WW}} \Delta \mathbf{W} \Big] + \mathbf{B}^{\mathbf{W}} \Big\} \\ \mathbf{A}^{\mathbf{WW}} &= \begin{pmatrix} -\mathbf{k}_{1} | \mathbf{W}_{1} | & & \\ -\mathbf{k} | \mathbf{W}_{2} | & 0 \\ & 0 & -\mathbf{k}_{5} | \mathbf{W}_{5} | \end{pmatrix} \qquad \mathbf{B}^{\mathbf{W}} = \begin{pmatrix} \mathbf{A}_{1} / \mathbf{L}_{1} (\mathbf{h}_{1} \mathbf{\rho}_{1} \mathbf{g} + \Delta \mathbf{P}_{pumpl}) \\ \mathbf{A}_{2} / \mathbf{L}_{2} (\mathbf{h}_{2} \mathbf{\rho}_{1} \mathbf{g} + \Delta \mathbf{P}_{pumpl}) \\ & \vdots \end{pmatrix} \\ \mathbf{A}^{\mathbf{WP}} &= \begin{pmatrix} \mathbf{A}_{1} / \mathbf{L}_{1} & -\mathbf{A}_{1} / \mathbf{L}_{1} & 0 & 0 \\ 0 & \mathbf{A}_{2} / \mathbf{L}_{2} & -\mathbf{A}_{2} / \mathbf{L}_{2} & 0 \\ 0 & 0 & \mathbf{A}_{3} / \mathbf{L}_{3} & -\mathbf{A}_{3} / \mathbf{L}_{3} \\ -\mathbf{A}_{4} / \mathbf{L}_{4} & 0 & 0 & \mathbf{A}_{4} / \mathbf{L}_{4} \\ 0 & -\mathbf{A}_{5} / \mathbf{L}_{5} & 0 & \mathbf{A}_{5} / \mathbf{L}_{5} \end{pmatrix} \end{split}$$

$$\begin{split} \Delta \mathbf{H} &= \Delta t \left(\mathbf{A}^{HW} \left[\mathbf{W}^{t} + \mathbf{S}_{HW} \Delta \mathbf{W} \right] + \mathbf{S}_{HH} \mathbf{A}^{HH^{*}} \Delta \mathbf{H}^{*} - \mathbf{S}_{HM} \mathbf{A}^{HM^{*}} \Delta \mathbf{M}^{*} + \mathbf{B}^{H} \right) \\ \Delta \mathbf{H}^{*} &= \begin{pmatrix} \Delta H_{1} \\ \Delta H_{2} \\ \Delta H_{3} \\ \Delta H_{4} \\ \Delta H_{4} \end{pmatrix} , \quad \Delta \mathbf{M}^{*} &= \begin{pmatrix} \Delta M_{1} \\ \Delta M_{2} \\ \Delta M_{3} \\ \Delta M_{4} \\ \Delta M_{4} \end{pmatrix} \\ \mathbf{A}^{HW} &= \begin{pmatrix} -H_{1}/M_{1} & 0 & 0 & +H_{4}/M_{4} & 0 \\ H_{1}/M_{1} & -H_{2}/M_{2} & 0 & 0 & H_{4}/M_{4} \\ 0 & H_{2}/M_{2} & -H_{3}/M_{3} & 0 & 0 \\ 0 & 0 & H_{3}/M_{3} & -H_{4}/M_{4} & -H_{4}/M_{4} \end{pmatrix} \end{split}$$

$$\mathbf{A}^{\mathrm{HH}*} = \begin{pmatrix} -W_1/M_1 & 0 & 0 & +W_4/M_4 & 0 \\ W_1/M_1 & -W_2/M_2 & 0 & & +W_5/M_4 \\ 0 & W_2/M_2 & -W_3/M_3 & 0 & 0 \\ 0 & 0 & W_3/M_3 & -W_4/M_4 & -W_5/M_4 \end{pmatrix}$$

$$\mathbf{A}^{\mathrm{HM}} = \begin{pmatrix} -W_{1}H_{1}/M_{1}^{2} & 0 & 0 & W_{4}H_{4}/M_{4}^{2} & 0 \\ W_{1}H_{1}/M_{1}^{2} & -W_{2}H_{2}/M_{2}^{2} & 0 & 0 & W_{5}H_{4}/M_{4}^{2} \\ 0 & W_{2}H_{2}/M_{2} & -W_{3}H_{3}/M_{3}^{2} & 0 & 0 \\ 0 & 0 & W_{3}H_{3}/M_{3}^{2} & -W_{4}H_{4}/M_{4}^{2} & -W_{5}H_{4}/M_{4}^{2} \end{pmatrix}$$

$\Delta \mathbf{H}^* = \mathbf{I}^{\mathbf{L}\mathbf{N}} \Delta \mathbf{H}$

$$I^{\text{LN}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \downarrow$$

 $\mathbf{A}^{\mathrm{HH}*} \Delta \mathbf{H}^* = \mathbf{A}^{\mathrm{HH}*} \mathbf{I}^{\mathrm{LN}} \Delta \mathbf{H}$ $= \mathbf{A}^{\mathrm{HH}} \Delta \mathbf{H}$ $\mathbf{A}^{\mathrm{HM}*} \Delta \mathbf{M}^* = \mathbf{A}^{\mathrm{HM}*} \mathbf{I}^{\mathrm{LN}} \Delta \mathbf{M}$ $= \mathbf{A}^{\mathrm{HM}} \Delta \mathbf{M}.$

 $\Delta \mathbf{H} = \Delta t \ \left\{ \mathbf{A}^{HW} \left(\mathbf{W}^t + S_{HW} \Delta \mathbf{W} \right) + S_{HH} \mathbf{A}^{HH} \Delta \mathbf{H} - S_{HM} \mathbf{A}^{HM} \Delta \mathbf{M} + \mathbf{B}^{H} \right\}$

$$\Delta \mathbf{H} = \Delta \mathbf{t} \{ \mathbf{A}^{HW} \left(\mathbf{W} + \mathbf{S}_{HW} \Delta \mathbf{W} \right) + \mathbf{S}_{HH} \mathbf{A}^{HH} \Delta \mathbf{H} - \Delta \mathbf{t} \mathbf{S}_{HM} \mathbf{A}^{HM} \mathbf{A}^{MW} \left(\mathbf{W}^{t} + \mathbf{W}^{MW} \Delta \mathbf{W} \right) + \mathbf{B}^{H} \}$$

 $\Delta \mathbf{H} = \Delta t [\mathbf{I} - \Delta t \ S_{HH} \ \mathbf{A}^{HH}]^{-1} \ \{ \mathbf{A}^{HW} \ (\mathbf{W}^t + S_{HW} \ \Delta \mathbf{W}) - \Delta t \ S_{HM} \ \mathbf{A}^{HM} \ \mathbf{A}^{MW} (\mathbf{W}^t + S_{MW} \ \Delta \mathbf{W}) + \mathbf{B}^{H} \}$

 $\Delta \mathbf{P} = \mathbf{C}_1 \,\Delta \mathbf{M} + \mathbf{C}_2 \,\Delta \mathbf{H},$

$$\mathbf{C}_{1} = \begin{pmatrix} \mathbf{C}_{11} & & & \\ & \mathbf{C}_{12} & \mathbf{0} & \\ & & \mathbf{C}_{13} & \\ & & & \mathbf{C}_{14} \end{pmatrix}$$

 $\Delta \mathbf{P} = -\Delta t \mathbf{C}_1 \mathbf{A}^{\text{MW}} (\mathbf{W}^{\text{t}} + S_{\text{MW}} \Delta \mathbf{W}) + \Delta t \mathbf{C}_2 [\mathbf{I} - \Delta t S_{\text{HH}} \mathbf{A}^{\text{HH}}]^{-1} [\mathbf{A}^{\text{HW}} (\mathbf{W}^{\text{t}} + S_{\text{HW}} \Delta \mathbf{W})]$ $-\Delta t S_{HM} A^{HM} A^{MW} (W^{t} + S_{MW} \Delta W) + B^{H}]$ $\equiv \Delta t \mathbf{A}^{\mathbf{PW1}} \mathbf{W}^{\mathsf{t}} + \Delta t \mathbf{A}^{\mathbf{PW2}} \Delta \mathbf{W} + \Delta t \mathbf{B}^{\mathbf{P}}$ $\mathbf{A}^{PW1} = \mathbf{C}_1 \mathbf{A}^{MW} + \mathbf{C}_2 \left[\mathbf{I} - \Delta t \mathbf{S}_{HH} \mathbf{A}^{HH} \right]^{-1} \left[\mathbf{A}^{HW} - \Delta t \mathbf{S}_{HM} \mathbf{A}^{HM} \mathbf{A}^{MW} \right]$ $\mathbf{A}^{PW2} = \mathbf{S}_{MW} \mathbf{C}_1 \mathbf{A}^{MW} + \mathbf{C}_2 [\mathbf{I} - \Delta t \mathbf{S}_{HH} \mathbf{A}^{HH}]^{-1} [\mathbf{S}_{HW} \mathbf{A}^{HW} - \Delta t \mathbf{S}_{HM} \mathbf{S}_{MW} \mathbf{A}^{HM} \mathbf{A}^{MW}]$ $\mathbf{B}^{\mathbf{P}} = \mathbf{C}_2 [\mathbf{I} - \Delta t \mathbf{S}_{\mathbf{H}\mathbf{H}} \mathbf{A}^{\mathbf{H}\mathbf{H}}]^{-1} \mathbf{B}^{\mathbf{H}}$ $\Delta \mathbf{W} = \Delta t \left\{ \mathbf{A}^{WP} \left[\mathbf{P}^{t} + \Delta t \ S_{WP} \left(\mathbf{A}^{PW1} \ \mathbf{W}^{t} + \mathbf{A}^{PW2} \ \Delta \mathbf{W} + \mathbf{B}^{P} \right) \right] + \mathbf{A}^{WW} \left[W^{t} + 2S_{WW} \ \mathbf{A}^{WW} \ \Delta \mathbf{W} \right] + \mathbf{B}^{W} \right\}$ $[\mathbf{I} - \Delta t(2 \mathbf{S}_{WW} \mathbf{A}^{WW} + \Delta t \mathbf{S}_{WP} \mathbf{A}^{WP} \mathbf{A}^{PW2})] \Delta \mathbf{W}$ $= \Delta t \{ [\mathbf{A}^{WW} + \Delta t S_{WP} \mathbf{A}^{WP} \mathbf{A}^{PW1}] \mathbf{W}^{t} + \mathbf{B}^{W} + \mathbf{A}^{WP} [\mathbf{P}^{t} + \Delta t S_{WP} \mathbf{B}^{P}] \}$

FIBS – Final Set of Equations

$A \Delta W = B$

$$\begin{split} \mathbf{A}^{PW1} &= \mathbf{C}_1 \ \mathbf{A}^{MW} + \mathbf{C}_2 \left[\mathbf{I} - \Delta t \ S_{HH} \ \mathbf{A}^{HH} \right]^{-1} \left[\mathbf{A}^{HW} - \Delta t \ S_{HM} \ \mathbf{A}^{HM} \ \mathbf{A}^{MW} \right] \\ \mathbf{A}^{PW2} &= S_{MW} \ \mathbf{C}_1 \ \mathbf{A}^{MW} + \mathbf{C}_2 \left[\mathbf{I} - \Delta t \ S_{HH} \ \mathbf{A}^{HH} \right]^{-1} \left[S_{HW} \ \mathbf{A}^{HW} - \Delta t \ S_{HM} \ S_{MW} \ \mathbf{A}^{HM} \ \mathbf{A}^{MW} \right] \\ \mathbf{B}^{P} &= \mathbf{C}_2 \left[\mathbf{I} - \Delta t \ S_{HH} \ \mathbf{A}^{HH} \right]^{-1} \mathbf{B}^{H} \end{split}$$

$$\begin{split} \left[\mathbf{I} - \Delta t (2 \ \mathbf{S}_{WW} \mathbf{A}^{WW} + \Delta t \ \mathbf{S}_{WP} \ \mathbf{A}^{WP} \ \mathbf{A}^{PW2})\right] \Delta \mathbf{W} \\ &= \Delta t \left\{ \left[\mathbf{A}^{WW} + \Delta t \ \mathbf{S}_{WP} \ \mathbf{A}^{WP} \ \mathbf{A}^{PW1} \right] \mathbf{W}^{t} + \mathbf{B}^{W} + \mathbf{A}^{WP} \left[\mathbf{P}^{t} + \Delta t \ \mathbf{S}_{WP} \ \mathbf{B}^{P} \right] \right\} \end{split}$$

$$\begin{split} \Delta \mathbf{M} &= \Delta t \ \mathbf{A}^{MW} \left[\mathbf{W}^{t} + S_{MW} \ \Delta \mathbf{W} \right] \\ \Delta \mathbf{H} &= \Delta t \ \left\{ \ \mathbf{A}^{HW} \left(\mathbf{W}^{t} + S_{HW} \ \Delta \mathbf{W} \right) + S_{HH} \ \mathbf{A}^{HH} \ \Delta \mathbf{H} - S_{HM} \ \mathbf{A}^{HM} \ \Delta \mathbf{M} + \mathbf{B}^{H} \ \right\} \\ \Delta \mathbf{P} &= \mathbf{C}_{1} \ \Delta \mathbf{M} + \mathbf{C}_{2} \ \Delta \mathbf{H} \end{split}$$

$$A^{PW1} = C_1 A^{MW} + C_2 A^{HW}$$

$$A^{PW2} = 0$$

$$B^p = C_2 B^H$$

$$\therefore \Delta W = \Delta t \{ A^{WW} W^t + B^W + A^{WP} P^t \}$$

$$\Delta M = \Delta t A^{MW} W^t$$

$$\Delta H = \Delta t \{ A^{HW} W^t + B^H \}$$

$$\Delta P = C_1 \Delta M + C_2 \Delta H,$$

Fully Explicit Scheme (S=0)

Semi Implicit Scheme (S_{HH} , S_{HM} = 0)

$$\begin{split} \mathbf{A}^{\mathbf{PW1}} &= \mathbf{C}_{1} \ \mathbf{A}^{\mathbf{MW}} + \mathbf{C}_{2} \ \mathbf{A}^{\mathbf{HW}} \\ \mathbf{A}^{\mathbf{PW2}} &= \mathbf{C}_{1} \ \mathbf{A}^{\mathbf{MW}} + \mathbf{C}_{2} \ \mathbf{A}^{\mathbf{HW}} \\ \mathbf{B}^{\mathbf{P}} &= \mathbf{C}_{2} \ \mathbf{B}^{\mathbf{H}} \\ \begin{bmatrix} \mathbf{I} - \Delta t(2 \ \mathbf{A}^{\mathbf{WW}} + \Delta t \ \mathbf{A}^{\mathbf{WP}} \ \mathbf{A}^{\mathbf{PW2}}) \end{bmatrix} \Delta \mathbf{W} \\ &= \Delta t \left\{ \ \begin{bmatrix} \mathbf{A}^{\mathbf{WW}} + \Delta t \ \mathbf{A}^{\mathbf{WP}} \ \mathbf{A}^{\mathbf{PW1}} \end{bmatrix} \mathbf{W}^{t} + \mathbf{B}^{\mathbf{W}} + \mathbf{A}^{\mathbf{WP}} \ \begin{bmatrix} \mathbf{P}^{t} + \Delta t \ \mathbf{B}^{\mathbf{P}} \end{bmatrix} \right\} \\ \Delta \mathbf{M} &= \Delta t \ \left\{ \ \mathbf{A}^{\mathbf{MW}} \ \begin{bmatrix} \mathbf{W}^{t} + \Delta \mathbf{W} \end{bmatrix} \\ \Delta \mathbf{H} &= \Delta t \ \left\{ \ \mathbf{A}^{\mathbf{HW}} \ (\mathbf{W}^{t} + \Delta \mathbf{W}) + \mathbf{B}^{\mathbf{H}} \right\} \\ \Delta \mathbf{P} &= \mathbf{C}_{1} \ \Delta \mathbf{M} + \mathbf{C}_{2} \ \Delta \mathbf{H} \end{split}$$

$$\mathbf{A}^{PW1} = \mathbf{C}_{1} \mathbf{A}^{MW} + \mathbf{C}_{2} [\mathbf{I} - \Delta t \mathbf{A}^{HH}]^{-1} [\mathbf{A}^{HW} - \Delta t \mathbf{A}^{HM} \mathbf{A}^{MW}]$$

$$\mathbf{A}^{PW2} = \mathbf{C}_{1} \mathbf{A}^{MW} + \mathbf{C}_{2} [\mathbf{I} - \Delta t \mathbf{A}^{HH}]^{-1} [\mathbf{A}^{HW} - \Delta t \mathbf{A}^{HM} \mathbf{A}^{MW}]$$

$$\mathbf{B}^{P} = \mathbf{C}_{2} [\mathbf{I} - \Delta t \mathbf{A}^{HH}]^{-1} \mathbf{B}^{H}$$

$$[\mathbf{I} - \Delta t(2 \mathbf{A}^{WW} + \Delta t \mathbf{A}^{WP} \mathbf{A}^{PW2})] \Delta W$$

$$= \Delta t \{ [\mathbf{A}^{WW} + \Delta t \mathbf{A}^{WP} \mathbf{A}^{PW1}] \mathbf{W}^{t} + \mathbf{B}^{W} + \mathbf{A}^{WP} [\mathbf{P}^{t} + \Delta t \mathbf{B}^{P}] \}$$

$$\Delta \mathbf{M} = \Delta t \mathbf{A} \mathbf{M}^{W} [\mathbf{W}^{t} + \Delta \mathbf{W}]$$

$$\Delta \mathbf{H} = \Delta t \{ \mathbf{A}^{HW} (\mathbf{W}^{t} + \Delta \mathbf{W}) + \mathbf{A}^{HH} \Delta \mathbf{H} - \mathbf{A}^{HM} \Delta \mathbf{M} + \mathbf{B}^{H} \}$$

$$\Delta \mathbf{P} = \mathbf{C}_{1} \Delta \mathbf{M} + \mathbf{C}_{2} \Delta \mathbf{H}$$

Fully Implicit Scheme

Programming Notes

- System geometry is contained in \mathbf{A}^{MW}
 - All other matrices are derived from this matrix and node/link properties
- The fully-implicit method is more complicated than the semiimplicit method
 - it requires the addition and multiplication of more matrices as well as a matrix inversion, especially when a large number of nodes is required
 - In one case study, for 9 nodes and links, the cost is a 50% increase in iteration time. But this becomes a 250% increase as one approaches the 36 node/link case.

Programming Notes

- Usually the matrices contain mostly zeros and, in the case of a circular loop, may be diagonally dominant in nature (i.e. non-zero elements occupy one, two or three stripes through the matrix).
- If the multiplication of two large matrices is desired, say NxN in dimension, the time to carry out the operation (N³ multiplications and N³ additions) can be very significant. However, it is possible to reduce the number of individual operations without losing the generality of the method.
- Suffice it to say that, in general, the semi-implicit method has a Courant limit on the maximum time step that can be taken in order to ensure stability. The fully-implicit method does not have this limitation.
- As the Courant time step limit is determined by the nodal residence time, the time step limit is dependent on the node sizes and the flows through the nodes.

Programming Notes

- Usually the matrices contain mostly zeros and, in the case of a circular loop, may be diagonally dominant in nature (i.e. non-zero elements occupy one, two or three stripes through the matrix).
- For example, for a 9 node case, the semi-implicit method required 0.10 seconds per iteration and required 2 iterations to meet the report time of 1.0 seconds. The fully-implicit method meet the report time in one iteration which took 0.14 seconds. At 36 nodes however, the semi-implicit method took 2 x 0.71 seconds while the fully-implicit method took 2.12 seconds.
- Clearly, one method is not superior to the other in all cases.
- Pressure determination involves the use of property derivatives. To avoid the numerical problems associated with discontinuities, smooth functions for properties, and other thermal-hydraulic correlations must be used

Questions?