Final exam December 2000 - Solution

1.

a) Neutron current: $\underline{J}$: net flow of neutron flux thru a surface.

Neutron flux: neutron density, n, times velocity

Neutron density: the local density of neutrons $n(\underline{r}, E, \Omega, t)$ or some integral, ie $n(r, E, t)$.

Neutron fluence: $\Phi(t) = \int_0^{t'} \phi(t)\, t' \, dt'$

b) $\left(\frac{\pi}{a}\right)^2 = B_g^2 > B_m^2 = \left(\frac{\nu\Sigma_f - \Sigma_a}{D}\right)$ subcritical

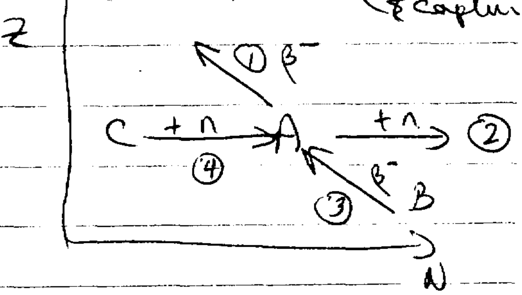
$B_g^2 = B_m^2$ critical

$B_g^2 < B_m^2$ supercritical

c)

$$\frac{dN_A}{dt} = \overset{\textcircled{1}}{-\lambda_A N_A} - \overset{\textcircled{2}}{\left[\sigma_a^A \phi\right] N_A} + \overset{\textcircled{3}}{\lambda_B N_B} + \overset{\textcircled{4}}{\sigma_\gamma^C \phi N_C}$$

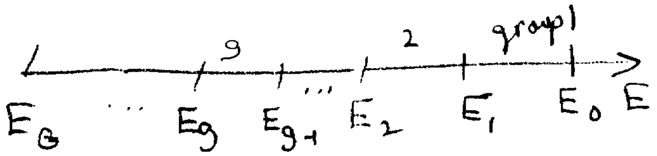
(decay) (neutron & capture) (decay of parent) capture trans- mutation



2. Variables for flux + precursor eqn's.

ϕ_g - neutron flux in energy group g ($g = 1, 2 \ldots G$)

v_g = neutron velocity in group g



t = time

D = diffusion coeff. [cm]

Σ = cross section [cm^{-1}] (subscripts
a = absorption
s = scattering
f = fission)

$\Sigma_{sg'g}$ = Σ_s from group $g' \to g$

χ_g = fission partition function, χ_g^c refers to delayed prec.

β_i = delayed fraction ($\beta = \sum_{i=1}^{N} \beta_i$, N = # of delayed groups)

ν = # of neutrons produced per fission

S_g^{ext} = external source of neutrons.

C_i = delayed precursors.

λ_i = decay const. for C_i

Term by term

all w.r.t. group g

flux: Φ_g : rate of change of neutron density in group g = net diffusion − absorption − scattering out + scattering in + prompt fission + delayed fission

delayed precursors C_i : rate of change of precursor i = decay + fission source.

Variables for poison eqn's

I = iodine concentration
X = Xenon concentration
γ = production fraction
λ = decay constants
σ = microscopic crossections

Term by term

Iodine: rate of change of I concentration = fission source − decay

Xenon: rate of change of Xe concentration = fission source + I decay − Xe decay − burnoff

Fuel

Variables: N_f = concentration of fuel isotope

Term by term

rate of change of fuel concentration = absorption rate (or interaction rate) $\sim \Sigma_a \phi \sim \sigma_a N \phi$

3. Two-group approximation:

a) FAST $$\frac{1}{v_1}\frac{\partial \phi_1}{\partial t} = \nabla\cdot D_1 \nabla\phi_1 - \Sigma_{a_1}\phi_1 - \Sigma_{s_1}\phi_1 + \Sigma_{s_{11}}\phi_1 + \Sigma_{s_{21}}\phi_2$$
$$+ \chi_1(1-\beta)\,\nu_1\Sigma_{f_1}\phi_1 + \nu_2\Sigma_{f_2}\phi_2$$
$$+ \chi_1^c \sum_{i=1}^{N} \lambda_i C_i + S_1^{ext}$$

Assume $\chi_1 = 1$, $\chi_1^c = 1$ (fast group big enough to span
$\chi_2 = 0$, $\chi_2^c = 0$ "birth energies")
$\Sigma_{s_{21}} = 0$ (no upscatter)
$S_{1,2}^{ext} = 0$ (typically)

$$\therefore \frac{1}{v_1}\frac{\partial \phi_1}{\partial t} = \nabla\cdot D_1\nabla\phi_1 - \Sigma_{r_1}\phi_1 + (1-\beta)\Big[\overbrace{\nu_1\Sigma_{f_1}\phi_1}^{\leftarrow\ \text{small } (\sim 3\%)} + \nu_2\Sigma_{f_2}\phi_2\Big]$$
$$+ \sum \lambda_i C_i$$

(where $\Sigma_{r_1} \equiv \Sigma_{a_1} + \Sigma_{s_1} - \Sigma_{s_{11}}$)

For the <u>Thermal</u> group,

$$\frac{1}{v_2}\frac{\partial \phi_2}{\partial t} = \nabla\cdot D_2\nabla\phi_2 - \Sigma_{a_2}\phi_2 - \Sigma_{s_2}\phi_2 + \Sigma_{s_{12}}\phi_1 + \Sigma_{s_{22}}\phi_2$$
$$\equiv \nabla\cdot D_2\nabla\phi_2 - \Sigma_{r_2}\phi_2 + \Sigma_{s_{12}}\phi_1$$

$\uparrow$ ($\equiv \Sigma_{a_2} + \Sigma_{s_2} - \Sigma_{s_{22}}$)

Where $$\nabla\cdot D_g\nabla\phi_g = \frac{\partial}{\partial x} D_g \frac{\partial\phi}{\partial x} + \frac{\partial}{\partial y} D_g \frac{\partial\phi}{\partial y} + \frac{\partial}{\partial z} D_g \frac{\partial\phi}{\partial z}$$

<u>Precursors</u>

$$\frac{\partial C_i}{\partial t} - \lambda_i C_i + \beta_i\left[\nu_1\Sigma_{f_1}\phi_1 + \nu_2\Sigma_{f_2}\phi_2\right]$$

b) In the steady state, $\frac{\partial \phi_1}{\partial t} = \frac{\partial \phi_2}{\partial t} = \frac{\partial C_i}{\partial t} = 0$

Thus, from the precursor equation:

$$\lambda_i C_i = \beta_i [\nu_1 \Sigma_{f_1} \phi_1 + \nu_2 \Sigma_{f_2} \phi_2]$$

$$\therefore \sum_i \lambda_i C_i = \beta [\quad " \quad]$$

∴ fast flux eqn. becomes:

$$0 = \nabla \cdot D_1 \nabla \phi_1 - \Sigma_{r_1} \phi_1 + [\nu_1 \Sigma_{f_1} \phi_1 + \nu_2 \Sigma_{f_2} \phi_2]$$

Thermal eqn is same as before, except $\frac{\partial \phi_2}{\partial t} = 0$

As you might expect, the delayed precursors play no part in the steady state solution in the 2-group approximation.

4. $\frac{1}{v}\frac{\partial \phi}{\partial t} = (\nu \Sigma_f - \Sigma_a)\,\phi + S$, $\phi(0) = 0$

a) At t close to startup, ϕ is low, therefore

$(\nu \Sigma_f - \Sigma_a)\,\phi \ll S$

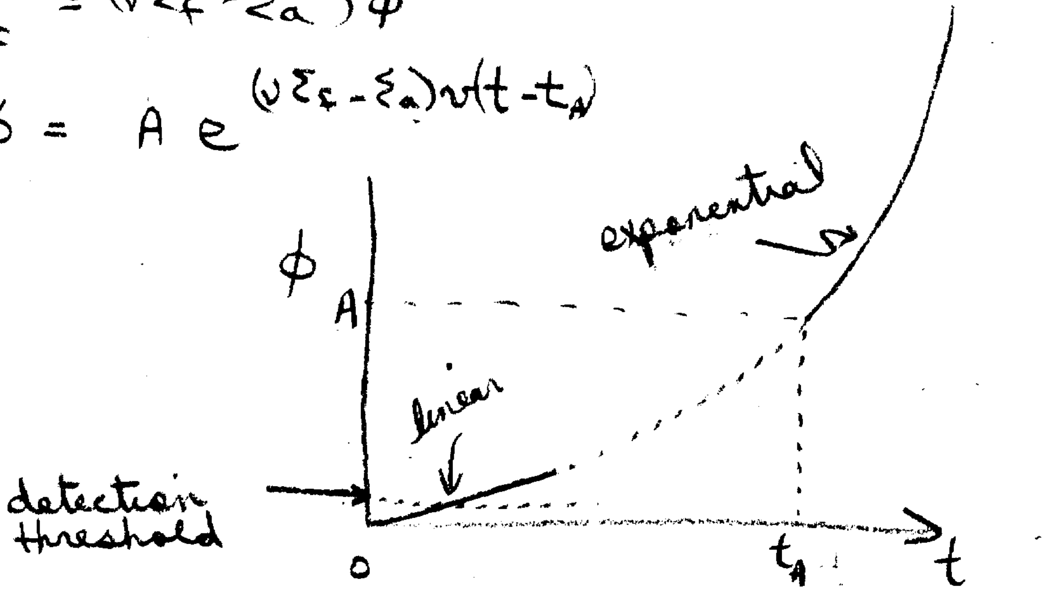
$\therefore \frac{1}{v}\frac{\partial \phi}{\partial t} \simeq S \Rightarrow \phi = vSt$

↑ linear in time

b) as $\phi \uparrow$, eventually, $(\nu \Sigma_f - \Sigma_a)\,\phi \gg S$,

$\therefore \frac{1}{v}\frac{\partial \phi}{\partial t} \simeq (\nu \Sigma_f - \Sigma_a)\,\phi$

$\Rightarrow \phi = A\,e^{(\nu \Sigma_f - \Sigma_a) v (t - t_A)}$



c) If $S = S_0\,\delta(t)$, then sol'n like (b) where A = small and $t_A = 0$

$\phi = \phi(0)\,e^{(\nu \Sigma_f - \Sigma_a) v t}$ when $\phi(0)$ is small.

d) Given same flux detection threshold, it is unwise to have an exponential flux growth below the detection range. Using a source, you can get up to the detectable range with a sub-critical core.

#5 $L^2 = D/\Sigma_a$, $D = \dfrac{1}{3(\Sigma_t - \mu_0 \Sigma_s)}$

For H_2O:

$\mu_0 = 0.676$

$\Sigma_t = 3.45 + 0.022 = 3.47$

$$D = \frac{1}{3(3.47 - 0.676 \times 3.45} = 0.29296$$

$\therefore L^2 = 0.293/0.022 = 13.3165$

$\therefore \underline{L = 3.649}$ (H_2O)

For D_2O:

$\mu_0 = 0.884$

$\Sigma_t = 0.449$

$$\therefore D = \frac{1}{3(0.449 - .884 \times 0.449)} = 6.40$$

$\therefore L^2 = 6.4/3.3\times10^{-5} = 182{,}855$

$\therefore \underline{L \sim 427.61}$ (D_2O)

For 2 reactors, same size but ① = H_2O & ② = D_2O then $B = B_g^2$ = same and both reactors are critical.

$$\therefore \quad K = 1 = \left.\frac{\nu\Sigma_f/\Sigma_a}{1+L^2B^2}\right|_{①} = \left.\frac{\nu\Sigma_f/\Sigma_a}{1+L^2B^2}\right|_{②}$$

It is constructive to consider two cases –

Ⓐ Small reactor

Ⓑ Large reactor

Case A <u>small reactor</u>

For small reactors, B^2 is large (greater curvature - ie $B^2 \sim \left(\frac{\pi}{\text{size}}\right)^2$).

Thus, the smaller the <u>critical</u> reactor, the more the denominator $(1+L^2B^2)$ is dominated by L^2B^2, to give:

$$K \sim \left.\frac{\nu\Sigma_f/\Sigma_a}{L^2\cancel{B^2}}\right|_{①} = \left.\frac{\nu\Sigma_f/\Sigma_a}{L^2\cancel{B^2}}\right|_{②}$$

$$\Rightarrow \left.\frac{\nu\Sigma_f}{D}\right|_{①} = \left.\frac{\nu\Sigma_f}{D}\right|_{②}$$

$D_{①} = 0.293$, $D_{②} = 6.40$

$\therefore$ a <u>critical</u> <u>small</u> D_2O based reactor has to have about 20x times more $\nu\Sigma_f$ (ie fuel) to be critical. Leakage $(1+L^2B^2)$ is much larger in a D_2O reactor for the same size (B^2) because L^2 is so much bigger. This is why you don't see small D_2O reactors.

For H_2O, the leakage is better + more than compensates for the poorer absorption characteristics.

There is more to say about this when multigroup effects are considered (ie moderation effects).

Case B <u>large reactor</u>

Simply the reverse of Case A

↓

Here B^2 is small & in the extreme, $L^2 B^2 \ll 1$ for both reactors - ie leakage is not a dominant factor. Now Σ_a effects dominate $\Sigma_a|_{\textcircled{1}} \gg \Sigma_{a_{\textcircled{2}}}$ ∴ you need less fuel to make a large D_2O reactor go critical than a large H_2O one.

Again, this ignores moderation arguements, which will be addressed in multigroup theory.

6. Inhour eqn: $\rho = \frac{\omega l}{1+\omega l} + \frac{1}{1+\omega l} \sum_i \frac{\omega \beta_i}{\omega + \lambda_i}$

a) If ρ small, then the transient induced will be very slow, ie the time constant, T will be very large + ω ($= 1/T$) will be very small.

$$\therefore \rho \simeq \omega l + \sum_i \omega \beta_i/\lambda_i \doteq \omega \Big[\underbrace{l + \sum_i \beta_i/\lambda_i}_{\equiv \langle l \rangle}\Big]$$

= average lifetime

Now, $l \sim 10^{-4}$ to 10^{-5} seconds, so $l \ll \sum_i \beta_i/\lambda_i$

$\therefore$ for small ρ, $\langle l \rangle$ dominated by precursors, thus $T = \frac{1}{\omega} \simeq \frac{\langle l \rangle}{\rho}$ is also dominated by the precursors, not l.

b) If ρ large, T is small (fast transient) and ω is large.

$$\therefore \rho \simeq \frac{\omega l}{1+\omega l} + \frac{1}{1+\omega l} \sum_i \frac{\cancel{\omega} \beta_i}{\cancel{\omega}} \simeq \frac{\omega l + \beta}{1+\omega l}$$

$$\simeq \frac{\omega l}{1+\omega l} \text{ since } \omega l \gg \beta.$$

$$\therefore \omega l \simeq \frac{\rho}{1-\rho} \Rightarrow T \sim \frac{l(1-\rho)}{\rho}$$

Thus the transient response in the large ρ case is dominated by l, not the precursors.

c. First a word about the two reactor designs.

- CANDU is a natural U fuel, D2O moderated type reactor. The PWR is an enriched U fuel, H2O moderated type reactor.
- The D2O moderator of CANDU means that the core is larger in physical size than a PWR (see the solution to question 5). Since the coolant (heavy and light water in these cases) must be pressurized (to about 100 atmospheres) to prevent too much boiling and to raise thermodynamic efficiency, pressure boundaries must be provided for the coolant. Pressure boundary wall thickness goes up in proportion to vessel diameter (a simple hoop stress calculation shows this), and for CANDU, the wall thickness would be prohibitive if a pressure vessel were used to surround the entire core. Hence pressure tubes are used in CANDU. Even if there was no hoop stress problem, the use of natural fuel dictates that the core be refuelled frequently. So on-line refuelling is required.
- The PWR has a smaller core and enriched fuel, so it can get away with a pressure vessel and batch refuelling, say once a year. The tight H2O core of the PWR means that the reactivity coefficient for void is negative, meaning that the core tends to shut itself down when the power goes up. Just as well, too, because the control rods must penetrate the pressure vessel, making them subject to sudden ejection should the rod housing fail. The rods must have a high worth because they have to compensate for all the extra fuel that must be added to permit a reasonable time between shutdowns that are required to refuel.
- CANDU, on the other hand, does not have, or need to have, a large reactivity inventory since it uses natural U fuel and can refuel continuously. So the control rods have far less worth than a PWR and the maximum reactivity insertion due to inadvertent rod withdrawal will be lower in magnitude, slower in occurring and less likely to occur than in the PWR. However, the void coefficient is positive in the D2O design. The CANDU also has an neutron lifetime 10 times longer than the PWR.
- So, to answer the question,
    - for normal transients, in which the inserted reactivities are small, the time constants are dominated by the delayed precurors. So one design is as good as the other - control is not a problem.
    - But for accident transients, the reactivity insertions can be large. The PWR is prone to much larger insertions in a shorter time. But the reactor tends to shut itself down. Of course, the flip side to that is that the PWR resists being shutdown! **And, here's the point of this question, *the shorter neutron lifetimes make the resulting transients* more rapid since it is the neutron lifetime that dictates the transient response times when insertions are greater than prompt critical.**
    - **The CANDU, on the other hand, has accident scenarios characterized by smaller reactivity insertions that occur less quickly, making them easier to manage and the runaway is slower since the neutron lifetime is 10 time longer in CANDU than in PWR.** The positive reactivity coefficient increases the instability but makes it easier to detect a problem (the flux rate change is an easy signal to detect). In the PWR case, the negative feedback tends to mask the event that we need to detect. To compensate for the positive void coefficient, CANDU uses a second shutdown system as a backup. The PWR has only one system.
    - Both systems have unique characteristics that result from their design. Both have been engineered to be safe enough. In, short, safety coverage is more a function of system behaviour that it is of any single design characteristic.

7. a)

The finite difference equations are easily derived directly from the governing equations.

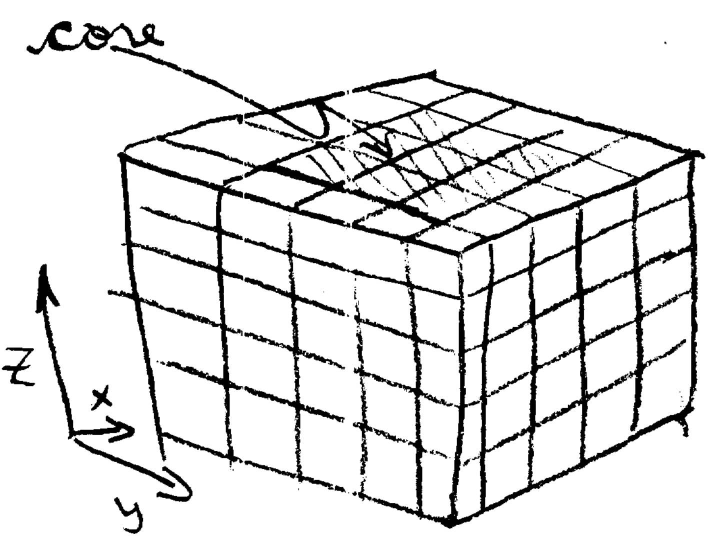


Illustration of reactor grid. There will be a finer division than indicated, perhaps 20 × 20 × 20.

plus similar terms for other faces

Replace $\nabla \cdot D_g \nabla \phi_g$ with:

$$\frac{\partial}{\partial x} D_g \frac{\partial \phi_g}{\partial x} + \frac{\partial}{\partial y} D_g \frac{\partial \phi_g}{\partial y} + \frac{\partial}{\partial z} D_g \frac{\partial \phi_g}{\partial z} \Rightarrow \frac{\frac{D_{EF}(\phi_E - \phi_P)}{\Delta_E} - \frac{D_{WF}(\phi_P - \phi_W)}{\Delta_W}}{\Delta_{EW}}$$

Replace $\frac{\partial Y}{\partial t}$ with $\frac{Y^{t+\Delta t} - Y^t}{\Delta t}$ where $Y = \phi_g, C_i, X, I$ or N_f

The rest of the terms are evaluated as is.

So we have:

$$\frac{\phi_g^{t+\Delta t} - \phi_g^{t}}{\Delta t} = V_g \left\{ \text{Leakage terms as above } - \Sigma_{ag}\phi_g - \Sigma_{sg}\phi_g + \sum_{g'=1}^{G_1} \Sigma_{sg'g}\,\phi_{g'} + X_g(1-\beta) \ldots\ldots \text{ etc.} \right\}$$

If we use a simple explicit scheme, we just evaluate

$$\phi_g^{t+\Delta t} = \phi_g^{t} + \Delta t\, V_g \{ \ldots \}$$

Same for C_i, X, I + N_f.

If we use a semi-implicit scheme, we evaluate the R.H.S. terms at $t+\Delta t$ if possible, gather up $\phi_g^{t+\Delta t}$ terms & solve.

In this manner, we march forward in time. We'll need some scheme to control the flux, typically. We can artifically introduce the fudge factor k in the fission source term ⇒ $\frac{\chi_g(1-\beta)}{k}\sum_{g'=1}^{G} \nu_{g'}\Sigma_{fg'}\phi_{g'}$

and adjust k as follows:

$$k^{t+\Delta t} = \frac{k^t \int_V \phi^{t+\Delta t}\, dV}{\int_V \phi^t\, dV}$$

or introduce a controller:

$$k^{t+\Delta t} = k^t + a\Big(\underset{\substack{\uparrow \\ \text{'measured'} \\ \text{ie calculated}}}{\phi_m^{t+\Delta t}} - \underset{\substack{\nwarrow \\ \text{set point}}}{\phi_{sp}}\Big) + b\,\frac{\phi_m^{t+\Delta t} - \phi_m^{t}}{\Delta t}$$

as we did in assignment #4.

"k" can be applied to the absorption terms to simulate real control rod movement if you prefer. It depends on the intent of the simulation.

Boundary Conditions are flux = 0 at extrapolated lengths. Initial conditions are simply the starting values for ϕ_g, C_i, etc.

b) The speed of response of the equation set is as follows:

ϕ_g ~ μsec → msec.

C_i ~ seconds → minutes

X, I ~ minutes → hours

N_F ~ hours → days

Typical Δt's would be ⤴ .

If you need to calculate poison transients, for instance, why bother to try and track ϕ_g & C_i since we can just assume a given flux or power. Also fuel inventory won't change much in a few hours. So set the neutron velocities to some slow speed, say $v_g = 1$ cm/sec, and set $\lambda_i = 1\ \text{sec}^{-1}$ so that you can use a $\Delta t \sim 1$ second and it won't be costly to sweep through all your equations in a time frame relevant to changes in poisons.

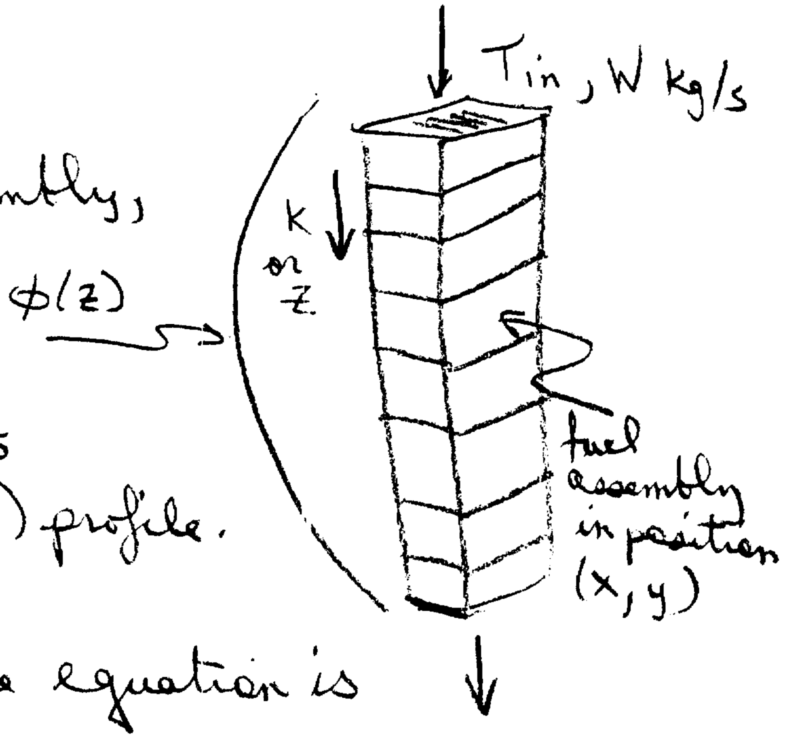
You can apply a fudge factor, F, to each equation, ie

$$F_g \frac{1}{v_g} \frac{\partial \phi_g}{\partial t} = - - - -$$

and simply adjust F_g as needed to "speed up" or "slow down" phenomena you are not interested in.

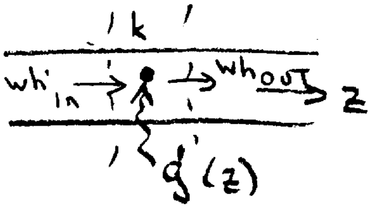
8.

For each fuel assembly, we have the inlet temperature, T_{in}, the flow rate, W kg/s and a flux (ie power) profile.



The enthalpy balance equation is

$$\rho c\frac{\partial T}{\partial t} = W\frac{\partial h}{\partial z} + g'(z) \Rightarrow \rho c\frac{\partial T}{\partial t} = W\rho c\frac{\partial T}{\partial z} + g'(z)$$



heat flux per assembly per unit axial length, proportional to flux, ϕ

$\equiv \gamma\,\phi(x,y,z)$

Thus,

$$\frac{T_k^{t+\Delta t} - T_k^{t}}{\Delta t} = W\frac{\left(T_{k-1}^{t} - T_k^{t}\right)}{\Delta z} + \gamma\,\phi_k(t)$$

assume complete mixing at each node so $T_{in} = T_{upstream}$ + $T_{out} = T_k$

could make this $T_k^{t+\Delta t}$

Solve for $T_k^{t+\Delta t}$ at each time step. Sweep on k.

— end —