1. (15 marks)

a. (5 marks)

Boron is a good absorber, ∴ safe to assume negligible scattering (no buildup).

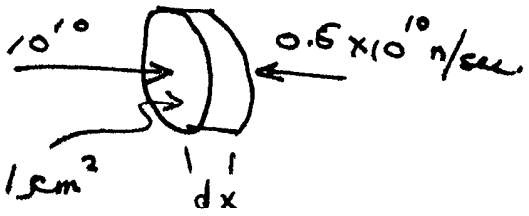
$$\therefore \quad \frac{I(x)}{I(0)} = 0.001 = e^{-\Sigma_a x}$$

$$\therefore \quad x = -\frac{\ln(0.001)}{\Sigma_a} = \frac{+6.91}{103}$$

($\Sigma_a = 103\ cm^{-1}$)

$$= 0.0671\ cm.$$

b. (5 marks)



$$\phi = \frac{\int_V n v \, dV}{\int_V dV}$$

$$= \frac{1.5 \times 10^{10} \times 2.2 \times 10^5}{2.2 \times 10^5}$$

$$= 1.5 \times 10^{10} \ \frac{\text{neutrons}}{cm^2 - sec.}$$

= length swept by neutrons in 1 sec. = 2.2×10^5 cm.
(assume thermal)

$$J = \int_S n \underline{v} \cdot d\underline{S} = (1.0 \times 10^{10} - 0.5 \times 10^{10}) \cdot 1\ cm^2$$

$$= 0.5 \times 10^{10}\ \hat{x}\ \text{neutrons}/cm^2 - sec.$$

(positive x direction)

#2

[10 Marks] Briefly:

a. distinguish between neutron flux and neutron current

(1) $\phi \equiv n v$ (scalar), $\vec{J} \equiv n \vec{v}$ (vector)

b. distinguish between reactivity, ρ, and multiplication factor, k

(1) $k = \frac{\text{production rate}}{\text{loss rate}}$ or $\frac{\#\text{ in generation } n}{\#\text{ in generation } n-1}$, $\rho \equiv \frac{k-1}{k}$

c. describe ε of the four factor formulae

(1) fast fission factor $= \frac{\text{total \# of fissions from fast + thermal}}{\text{\# of fissions from thermal neutrons}}$ (~ 1.03)

d. describe η of the four factor formulae

(1) $\frac{\text{average \# neutrons produced}}{\text{\# neutrons absorbed in fuel}} \simeq \frac{\nu \Sigma_f}{\Sigma_a}$ (~ 2.0)

e. describe f of the four factor formulae

(1) thermal utilization $= \frac{\Sigma_a^{\text{fuel}}}{\Sigma_a^{\text{fuel}} + \Sigma_a^{\text{rest}}}$ (~ 0.7)

f. describe p of the four factor formulae

(1) resonance escape probability (~ 0.9)

g. distinguish between geometric buckling and material buckling

(2) For 1 speed case, $\underbrace{B_g^2 \simeq \left(\frac{\pi}{a}\right)^2}_{\text{geometric buckling}} = \frac{\nu \Sigma_f - \Sigma_a}{D} = \underbrace{B_m^2}_{\text{material buckling}}$ $B_g^2 \simeq B_m^2$ in critical reactor

h. describe fission product poison.

(1) Some fission products, like Xe & Sm, have high absorption cross-sections ∴ act like "poisons" in that they absorb neutrons.

i. describe xenon over-ride.

(1) Xe builds up when a reactor shuts down. If want to start up during this build up period, need to have excess reactivity available.

#3

[10 marks] What is the obvious error in the following expressions? Explain briefly.

a. Steady state one-group neutron balance equation:
$D(r)\nabla^2\phi(r) - \Sigma_a(r)\phi(r) = \nu\Sigma_f(r)\phi(r)$

(2) We have $\frac{1}{v}\frac{\partial\phi}{\partial t} = \nabla\cdot D\nabla\phi - \Sigma_a\phi + \nu\Sigma_f\phi$.
In SS: $-\nabla\cdot D\nabla\phi + \Sigma_a\phi = \nu\Sigma_f\phi$ Also, if $D = f_n(r)$ can't take D outside of $\nabla\cdot(\ )$

b. $\Sigma_{total} < \Sigma_{absorption}$

(1) $\Sigma_{total} = \Sigma_{abs} + \Sigma_{scattering}$ $\therefore \Sigma_{total} \geq \Sigma_{abs}$

c. The gradient of the flux is continuous at an interface

(1) The current is continuous at an interface

d. $\rho = 2$

(1) $\rho \equiv \frac{k-1}{k}$. $0 \leq k \leq \infty \Rightarrow -\infty \leq \rho \leq 1$
ie ρ cannot be > 1.

e. For a reactor operating at constant power, as the fuel is burned up, the flux remains constant over time

(1) Power $\propto \Sigma_f\phi = \sigma_f N_f \phi$. As $N_f \downarrow$, ϕ must $\uparrow$ to keep power constant.

f. I^{135} decays with a half life of 9.17 hours to Xe^{135} which decays with a half life of 6.58 hours

(1) I^{135} decays faster than Xe^{135}
$\uparrow T_{\frac{1}{2}} = 6.58$ hr. $\uparrow T_{\frac{1}{2}} = 9.17$ hr.

g. Neutron current is defined as: $\underline{J} = -D\nabla\phi$

(2) $\underline{J} \approx -D\nabla\phi$. This is an approximation, not a definition. Also $\underline{J}$ is the current density

h. For the same power, the smaller the reactor, the lower the flux.

(1) $P \propto \text{Volume} \times \Sigma_f\phi$
$\therefore$ As Volume $\downarrow$, ϕ must $\uparrow$ for same Power.

#4

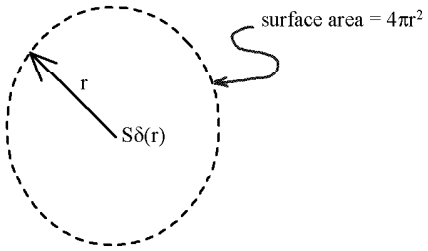
a)

Point Source in an Infinite Medium

For a point source, it is appropriate to work in spherical coordinates:

$$\frac{1}{r^2}\frac{d}{dr}r^2\frac{d\phi}{dr}-\frac{1}{L^2}\phi=S\delta(r)$$

where the source, S, is at r = 0.



Now

$$4\pi r^2 J(r)=S\delta(r)=S$$

$$\therefore \lim_{\delta r\to 0} r^2 J(r)=\frac{S}{4\pi}$$

We define a change of variable

$$\omega=r\phi \Rightarrow \frac{\partial^2\omega}{\partial r^2}-\frac{1}{L^2}\omega=0$$

and, therefore, as before:

$$\omega=Ae^{-r/L}+Ce^{r/L}$$

or

$$\phi=A\frac{e^{-r/L}}{r}+C\frac{e^{r/L}}{r} \quad \text{and} \quad C=0 \text{ as before.}$$

Now,

$$J=-D\frac{d\phi}{dx}=DA\left(\frac{1}{rL}+\frac{1}{r^2}\right)e^{-r/L}$$

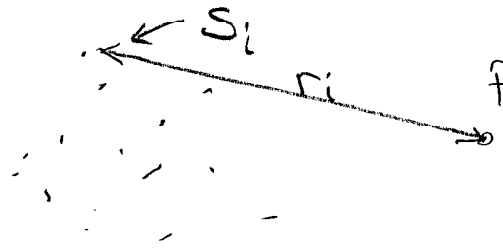
$$\therefore r^2 J\big|_{r=0}=DA\left(\frac{r}{L}+1\right)e^{-r/L}\Big|_{r=0}=DAe^{-0/L}=DA=\frac{S}{4\pi}$$

$$\therefore A=\frac{S}{4\pi D}, \quad \therefore \boxed{\phi=\frac{S}{4\pi Dr}e^{-r/L}}$$

Note: In the above two cases, as in most reactor cases, the flux, ϕ, is proportional to the source strength, S.

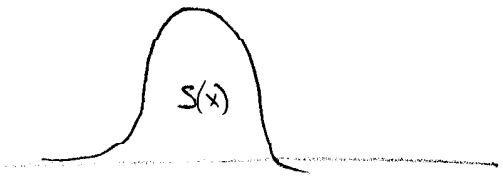
b)

ϕ from a general dist^n of S:



$$\phi(P) = \sum_i \frac{S_i}{4\pi D r_i} e^{-r_i/L}$$

if S_i is discrete point source at r_i (S_i: #/s)



$$\phi(P) = \int_{-\infty}^{\infty} \frac{S(x)}{4\pi D x} e^{-x/L}\, dx$$

where $S(x) \equiv \frac{\#}{cm \cdot s}$

(ie $S(x)dx = \#/s$)

or in general

$$\phi(P) = \int_x \int_y \int_z \frac{S(x,y,z)}{4\pi D r} e^{-r/L} dx\, dy\, dz$$

When r = dist. from point P

$S(x,y,z) = \#/cm^3 \cdot s$

ie $(S(x)dV = \#/s)$

#5

Modelling the Source Term when it is a δ Function

In the equation $-D\frac{\partial^2\phi}{\partial x^2}-\Sigma_a\phi=S$ where $S=S_0\delta(x)$ in the case of a planar source of neutrons and S has units of #/cm^3-sec. Now we know that $\int\delta(x)dx=1$, $\therefore$ units of $\delta(x)=\frac{1}{cm}$, $\therefore S_0\equiv\frac{1}{cm^2\,sec}$.

<u>Watch out</u> when doing numerical solution. Integral through:

$$-\int_{\Delta x}D\frac{\partial^2\phi}{\partial x^2}dx+\int_{\Delta x}\Sigma_a\phi dx=\int_{\Delta x}S_0\underbrace{\delta(x)dx}_{\int=1}=S_0$$

Consider $D\frac{\partial^2\phi}{\partial x^2}$, etc. const over Δx. Thus

$$-D\frac{\partial^2\phi_i}{\partial x^2}\Delta x+\Sigma_a\phi_i\Delta x=S_0 \text{ at the planar source cell}$$

$$= 0 \text{ otherwise.}$$

So, the equation to be numerically simulated is

$$-D\frac{\partial^2\phi_i}{\partial x^2}+\Sigma_a\phi_i=\frac{S_0}{\Delta x}$$

not

$$-D\frac{\partial^2\phi_i}{\partial x^2}+\Sigma_a\phi_i=S$$

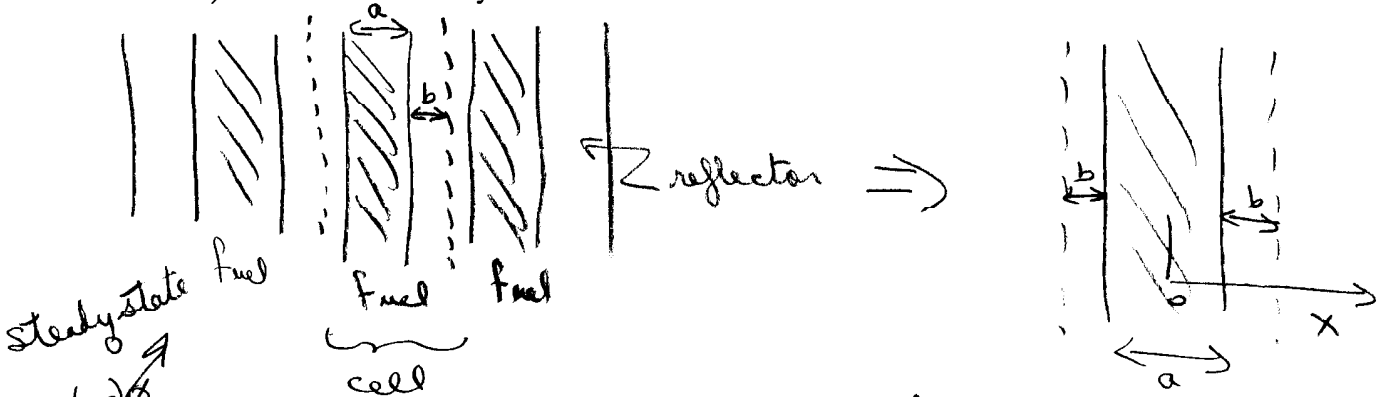
This can be physically interpreted as the planar source term of infinitesimal thickness being spread over a finite thickness cell.

#6

Consider a large one dimensional reactor composed of many replicated identical cells, each containing fuel and moderator. Each cell consists of a central fuel region of thickness "a" surrounded on either side by a reflector of thickness "b". Near the centre of the reactor, we can assume that one cell looks and behaves like its neighbours since the reactor is large. Thus, the flux distribution in each central cell can be calculated independently.

a) What are the governing flux equations for the steady state?

b) What are the boundary conditions?



$$\frac{1}{v}\frac{\partial \phi}{\partial t} = \nabla \cdot D \nabla \phi - \Sigma_a \phi + \nu \Sigma_f \phi \quad \leftarrow \text{general}$$

(a) fuel region: $D_F \frac{\partial^2 \phi_F}{\partial x^2} + (\nu \Sigma_{f_F} - \Sigma_{a_F}) \phi_F = 0$

reflector region: $D_R \frac{\partial^2 \phi_R}{\partial x^2} - \Sigma_{a_R} \phi_R = 0$

(b) Boundary Conditions:

(1) $\left.\frac{\partial \phi_F}{\partial x}\right|_{x=0} = 0$ (symmetry $\therefore$ slope of ϕ at $x=0$ is 0.)

continuity of flux + current at the interface

(2) $\left.\phi_F\right|_{x=a/2} = \left.\phi_R\right|_{x=a/2}$

(3) $\left.J_F\right|_{x=a/2} = \left.J_R\right|_{x=a/2} \Rightarrow D_F \left.\frac{\partial \phi_F}{\partial x}\right|_{x=a/2} = D_R \left.\frac{\partial \phi_R}{\partial x}\right|_{x=a/2}$

(4) Reflection at the cell boundary (ie since one cell look like another, neutrons leaving the cell must have a counterpart entering from the neighbour cell. Hence $\left.J_R\right|_{x=a/2+b} = 0 \Rightarrow \left.\frac{\partial \phi_R}{\partial x}\right|_{x=a/2+b} = 0$.

c) For the numerical solution, $J\Big|_{\pm(a/2+b)} = 0$

This will require a small bit of algebra to gather up terms that lie outside the boundary.

d) Why are the BC's different for the numerical and analytic cases?

Analytically you have 2 equations to solve, each with its own parameters and you have to match up the solutions at the interface - ie you have more boundaries to consider.

Numerically, you just have one equation:

$$0 = \nabla \cdot D(r) \nabla \phi(r) + (\nu \Sigma_f(r) + \Sigma_a(r)) \phi(r).$$

The only "boundaries" are the outside edges.

#7

$(\Sigma_{f1}\phi_1 + \Sigma_{f2}\phi_2)$ $(\sigma_{a_1}^X \phi_1 + \sigma_{a_2}^X \phi_2)$

a) $\frac{\partial X}{\partial t} = \gamma_X \Sigma_f \phi + \lambda_I I - \lambda_X X + \sigma_a^X \phi X$, $\frac{\partial I}{\partial t} = \gamma_I \Sigma_f \phi - \lambda_I I$

where $\phi = \phi_0 \cos Bx$ as usual. $B = \pi/a$.

In steady state: $I = \frac{\gamma_I \Sigma_f \phi_0 \cos Bx}{\lambda_I}$

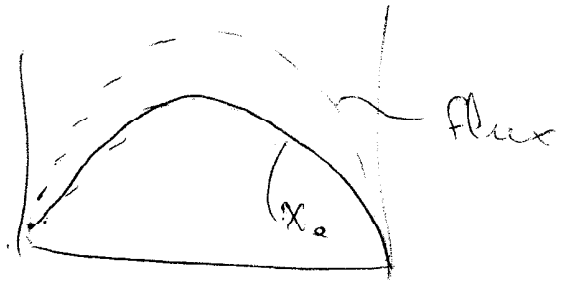
$\therefore$ for X_e: $0 = \gamma_X \Sigma_f \phi + \gamma_I \Sigma_f \phi - \lambda_X X - \sigma_a^X \phi X$

$\therefore$ $\boxed{X = \frac{(\gamma_X + \gamma_I)\Sigma_f \phi_0 \cos Bx}{\lambda_X + \sigma_a^X \phi_0 \cos Bx}}$ mostly ϕ_2

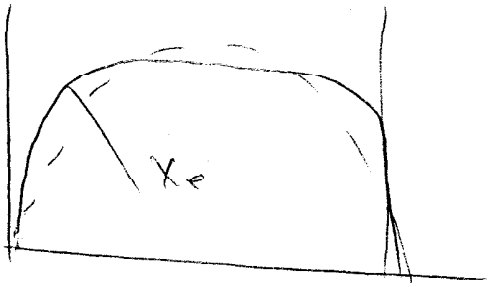
It is not a simple cosine.

b) low ϕ_0

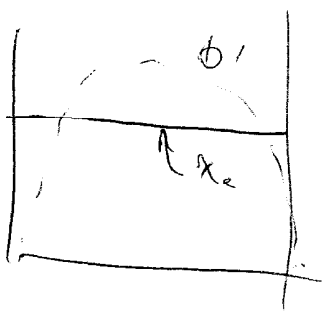
$X = \frac{(\gamma_X + \gamma_I)}{\lambda_X} \Sigma_a \phi_0 \cos Bx$

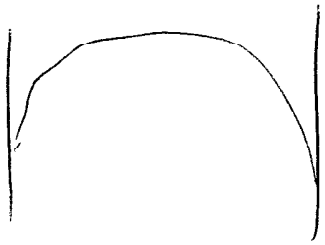


med



high $X = \frac{(\quad)\Sigma_f \phi_0 \cos Bx}{\sigma_a^X \phi_0 \cos Bx}$





$$\frac{1}{v_1}\frac{\partial \phi_1}{\partial t} = 0 = D_1\frac{\partial^2 \phi_1}{\partial x^2} - \Sigma_{a_1}\phi_1 - \Sigma_{s_1}\phi_1 + \nu_2\Sigma_{f_2}\phi_2$$

$$\frac{1}{v_2}\frac{\partial \phi_2}{\partial t} = 0 = D_2\frac{\partial^2 \phi_2}{\partial x^2} - \Sigma_{a_2}\phi_2 + \Sigma_{s_{12}}\phi_1$$

$\Sigma_{f_2}\phi_2$ = fissioning rate which leads to Xe & I.

∴ appropriate $\phi = \phi_2$ (also have minor term $\Sigma_{f_1}\phi_1$)

but spatial distn ~ $\phi_1 \sim \cos\frac{\pi x}{a}$

$\sim \phi_2$.

9 a)

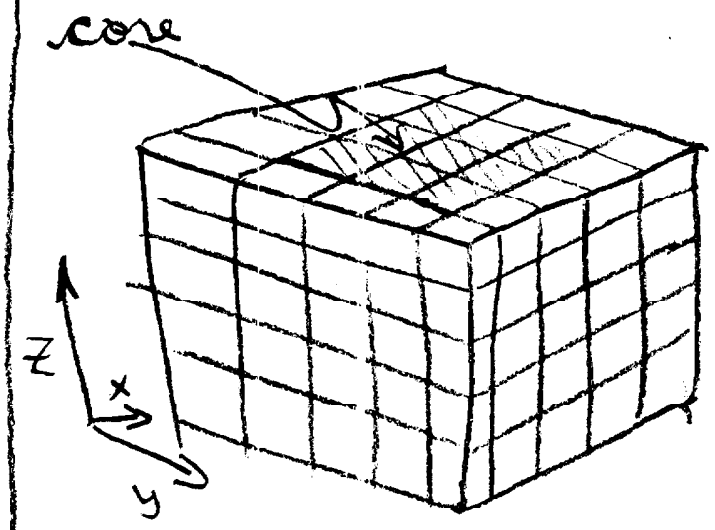


Illustration of reactor grid. There will be a finer division than indicated, perhaps 20 × 20 × 20.

The finite difference equations are easily derived directly from the governing equations.

Replace $\nabla \cdot D_g \nabla \phi_g$ with:

$$\frac{\partial}{\partial x} D_g \frac{\partial \phi_g}{\partial x} + \frac{\partial}{\partial y} D_g \frac{\partial \phi_g}{\partial y} + \frac{\partial}{\partial z} D_g \frac{\partial \phi_g}{\partial z} \Rightarrow \frac{D_{Ef}(\phi_E - \phi_P) - D_{Wf}(\phi_P - \phi_W)}{\Delta_{EW}}$$

Replace $\frac{\partial Y}{\partial t}$ with $\frac{Y^{t+\Delta t} - Y^t}{\Delta t}$ where $Y = \phi_g, C_i, X, I$ or N_f

The rest of the terms are evaluated as is.

So we have:

$$\frac{\phi_g^{t+\Delta t} - \phi_g^t}{\Delta t} = v_g \left\{ \text{Leakage terms as above} - \Sigma_{ag}\phi_g - \Sigma_{sg}\phi_g + \sum_{g'=1}^{G} \Sigma_{sg'g}\phi_{g'} + \chi_g(1-\beta) \ldots \text{etc.} \right\}$$

If we use a simple explicit scheme, we just evaluate

$$\phi_g^{t+\Delta t} = \phi_g^t + \Delta t v_g \{ \ldots \}$$

Same for $C_i, X, I + N_f$.

If we use a semi-implicit scheme, we evaluate the R.H.S. terms at $t+\Delta t$ if possible, gather up $\phi_g^{t+\Delta t}$ terms & solve.

In this manner, we march forward in time.
We'll need some scheme to control the flux, typically.
We can artificially introduce the fudge factor k in the fission source term $\Rightarrow$ $\frac{\chi_g (1-\beta)}{k} \sum_{g'=1}^{G} \nu_{g'} \Sigma_{fg'} \phi_{g'}$

and adjust k as follows:

$$k^{t+\Delta t} = \frac{k^t \int_V \phi^{t+\Delta t} dV}{\int_V \phi^t dV}$$

or introduce a controller:

$$k^{t+\Delta t} = k^t + a\left(\phi_m^{t+\Delta t} - \phi_{sp}\right) + b\,\frac{\phi_m^{t+\Delta t} - \phi_m^t}{\Delta t}$$

($\phi_m^{t+\Delta t}$: 'measured' is calculated; ϕ_{sp}: set point)

as we did in assignment #4.

"k" can be applied to the absorption terms to simulate real control rod movement if you prefer. It depends on the intent of the simulation.

Boundary Conditions are flux = 0 at extrapolated lengths. Initial conditions are simply the starting values for ϕ_g, C_i, etc.

b) The speed of response of the equation set is as follows:

ϕ_g ~ μsec → msec.

C_i ~ seconds → minutes

X, I ~ minutes → hours

N_f ~ hours → days

Typical Δt's would be ↖ .

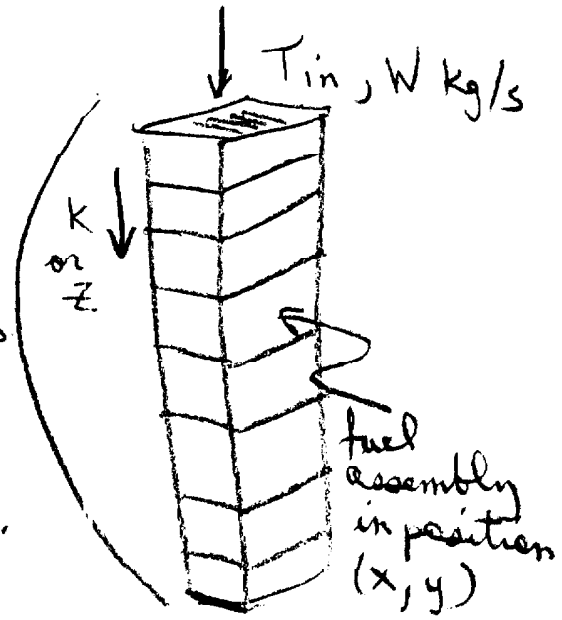
If you need to calculate poison transients, for instance, why bother to try and track ϕ_g & C_i since we can just assume a given flux or power. Also fuel inventory won't change much in a few hours. So set the insertion velocities to some slow speed, say $v_g = 1$ cm/sec, and set $\lambda_i = 1$ sec^{-1} so that you can use a $\Delta t \sim 1$ second and it won't be costly to sweep through all your equations in a time frame relevant to changes in poisons.

You can apply a fudge factor, F_i, to each equation, ie $F_g \frac{1}{v_g} \frac{\partial \phi_g}{\partial t} = \cdots\cdots$

and simply adjust F_g as needed to "speed up" or "slow down" phenomena you are not interested in.

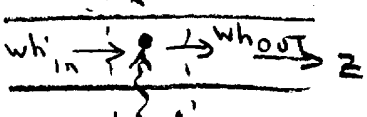
9.

For each fuel assembly, we have the inlet temperature, T_{in}, the flow rate, W kg/s and a flux (ie power) profile.



The enthalpy balance equation is

$$\rho c \frac{\partial T}{\partial t} = w \frac{\partial h}{\partial z} + q'(z) \Rightarrow \rho c \frac{\partial T}{\partial t} = w \rho c \frac{\partial T}{\partial z} + q'(z)$$



↑ heat flux per assembly per unit axial length, proportional to flux, ϕ

$$\equiv \gamma \phi(x, y, z)$$

Thus,

$$\frac{T_k^{t+\Delta t} - T_k^t}{\Delta t} = W \left(\frac{T_{k-1}^t - T_k^t}{\Delta z} \right) + \gamma \phi_k(t)$$

assume complete mixing at each node so $T_{in} = T_{upstream}$ & $T_{out} = T_k$

could make this $T_k^{t+\Delta t}$

Solve for $T_k^{t+\Delta t}$ at each time step. Sweep on k.

— end —