Solution

ENGINEERING PHYSICS 4D3/6D3

DAY CLASS Dr. Wm. Garland
DURATION: 50 minutes
McMASTER UNIVERSITY MIDTERM EXAMINATION November 5, 2003

Special Instructions:
1. Closed Book. All calculators and up to 6 single sided 8 2" by 11" crib sheets are permitted.
2. Do all questions.
3. The value of each question is as indicated. TOTAL Value: 100 marks

THIS EXAMINATION PAPER INCLUDES 3 PAGES AND 4 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

1. [10 marks] A neutron flux exists in an absorbing and scattering medium.
   a. What is the total neutron interaction rate in terms of the local microscopic cross sections, the neutron density, neutron velocity and nuclei density? State the units of each parameter and the overall interaction rate.

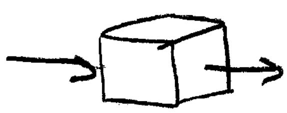
overall interaction rate $= \sigma_T N n v$ (n: neutrons/cm^3, v: cm/s, N: nuclei/cm^3)

$= \#/cm^3\text{-}s$ $\sigma_T = \sigma_a + \sigma_s$ [cm^2]

   b. What is the total neutron interaction rate in terms of the local macroscopic cross sections and neutron flux? State the units of each parameter and the overall interaction rate.

overall interaction rate $= \underbrace{\sigma_T N}_{\Sigma_T \,[cm^{-1}]} \underbrace{n v}_{\phi\, [\#/cm^2\text{-}s]} = \#/cm^3\text{-}s$

2. [10 marks] A cube 10 cm. x 10 cm. x 10cm. has a neutron current of 1.1 x 10^7 n/s coming in through one of its faces and a current of 1.0 x 10^7 n/s going out through another face. The remaining faces have zero current.
   a. What is the rate of change of neutron density per cm^3 in the cube due to this current?



Net current $= (1.1 - 1.0) \times 10^7$ n/s $= 0.1 \times 10^7$ n/s

Volume $= 10^3$ cm^3.

$$\frac{\partial n}{\partial t} = J_{in} - J_{out} = \frac{0.1 \times 10^7}{10^3} = 10^3 \; n/cm^3\text{-}s$$

   b. Assuming thermal neutrons, what is the rate of change of flux?

$v = 2.2 \times 10^5$ cm/s

$$\frac{\partial \phi}{\partial t} = v \frac{\partial n}{\partial t} = 2.2 \times 10^8 \; n/cm^2\text{-}s^2$$

3. [40 marks] Consider a homogeneous slab reactor (ie, one dimension, uniform properties consisting of a mixture of moderator and fuel). There is no external reflector.

a. Starting from the general transient multi-group neutron diffusion equations with delayed precursors, what are the simplified steady state neutron and precursor equations for the 2 group case, assuming no up-scatter and that all neutrons are born in the fast group?

$$\cancelto{0}{\frac{1}{v_g}\frac{\partial \phi_g}{\partial t}} = \nabla\cdot D_g \nabla\phi_g - \Sigma_{a_g}\phi_g - \Sigma_{s_g}\phi_g + \sum_{g'=1}^{G}\Sigma_{s_{g'g}}\phi_{g'} + \chi_g (1-\beta)\sum_{g'=1}^{G}\nu_{g'}\Sigma_{f_{g'}}\phi_{g'} + \chi_g\sum_{i=1}^{6}\lambda_i C_i$$

$$\cancelto{0}{\frac{\partial C_i}{\partial t}} = -\lambda_i C_i + \beta_i \sum_{g'=1}^{G}\nu_{g'}\Sigma_{f_{g'}}\phi_{g'}$$

$$0 = D_1 \frac{\partial^2\phi_1}{\partial x^2} - \Sigma_{a_1}\phi_1 \overbrace{- \Sigma_{s_1}\phi_1 + \Sigma_{s_{11}}\phi_1}^{\text{net} = -\Sigma_{s_{12}}\phi_1} + \cancelto{0}{\Sigma_{s_{21}}\phi_2} \quad \text{no upscatter}$$

$$+ \underset{④}{(1-\cancel{\beta})}(\nu_1\Sigma_{f_1}\phi_1 + \nu_{f_2}\phi_2) + \sum_{i=1}^{6}\cancel{\lambda_i} C_i \; ③$$

④ ← 3 + 4 cancel →

$$\chi_2 = 0$$

these cancel

$$0 = D_2\frac{\partial^2\phi_2}{\partial x^2} - \Sigma_{a_2}\phi_2 - \cancel{\Sigma_{s_2}\phi_2} + \Sigma_{s_{12}}\phi_1 + \cancel{\Sigma_{s_{22}}\phi_2} + 0$$

$$0 = \underset{①}{-\lambda_i C_i} + \beta_i\,(\nu_1\underset{②}{\Sigma_{f_1}\phi_1} + \nu_2\Sigma_{f_2}\phi_2)$$

b. Discuss briefly why the steady state flux is not a function of the precursor concentrations. State what is happening both mathematically and physically.

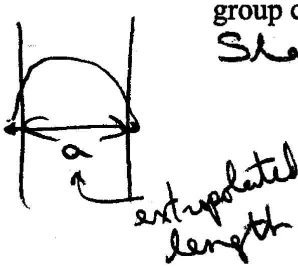
In the steady state, the delayed neutrons all show up eventually. Their energy is lower than prompt neutrons but they are high enough to be in the fast group (group 1).

Mathematically: $\lambda_i C_i = \beta_i \sum_{g'=1}^{G} \nu_{g'}\Sigma_{f_{g'}}\phi_{g'}$

& since $\sum_{i=1}^{6}\beta_i = \beta$ terms ③ & ④ cancel, leaving no precursor terms in the flux equations.

c. What are the shapes of the thermal and fast neutron fluxes for this steady state 2 group case?

Slab, not reflected $\Rightarrow \phi_g \sim \cos\left(\frac{\pi x}{a}\right)$, $g = 1, 2$.



d. What is the steady state spatial distribution of the delayed precursors for this steady state 2 group case?

Since $C_i = \dfrac{\beta_i}{\lambda_i}\left(\nu_1 \Sigma_{f_1} \phi_1 + \nu_2 \Sigma_{f_2} \phi_2\right)$

$C_i = \dfrac{\beta_i}{\lambda_i}\left(\cdots\cdots \right) \cos\left(\frac{\pi x}{a}\right)$ as well.

4. [40 marks] Consider a cubic homogeneous reactor of dimension a x a x a in an ocean of water. For a 1-group neutron diffusion model in steady state:

a. State the relevant neutron balance equations for the reactor and the ocean.

$$0 = D^c\left(\frac{\partial^2\phi^c}{\partial x^2} + \frac{\partial^2\phi^c}{\partial y^2} + \frac{\partial^2\phi^c}{\partial z^2}\right) + \left(\nu\Sigma_f^c - \Sigma_a^c\right)\phi^c \Leftarrow \text{Reactor core}$$

$$0 = D^w\left(\frac{\partial^2\phi^w}{\partial x^2} + \frac{\partial^2\phi^w}{\partial y^2} + \frac{\partial^2\phi^w}{\partial z^2}\right) - \Sigma_a^w \phi^w \Leftarrow \text{water}$$

b. State the required boundary conditions for an analytical solution.

12 BC's req'd. + $\phi(0,0,0)$ to set amplitude

(3) $\phi^c\Big|_{x=a/2} = \phi^w\Big|_{x=a/2}$ ← also for $y = a/2$ & $z = a/2$

faces at $\pm a/2$

(3) $J_x^c = -D\dfrac{\partial\phi^c}{\partial x}\Big|_{x=a/2} = J_x^w = -D\dfrac{\partial\phi^w}{\partial x}\Big|_{x=a/2}$ ← also for $y = \pm a/2$ & $z = \pm a/2$

and (3) $\phi^w\Big|_{x=\infty} = 0$ same for $y = \infty$ & $z = \infty$

(3) also $\dfrac{\partial\phi^c}{\partial x}\Big|_{x=0} = 0$ same for y & z

c. If you were solving this numerically, what boundary conditions would you use?

For numerical solution, just set $\phi\Big|_{x=MAX} = 0$, same for y & z, where MAX is the outside edge of the numerical grid.

d. Why are the B.C.s different for the numerical and analytical cases?

Analytically, you have 2 equations to solve, each with its own parameters & you have to match up the solutions at the interface – ie you have more boundaries to consider.

Numerically, you just have one eqn: $0 = \nabla \cdot D(\underline{r}) \nabla\phi(\underline{r}) + \left(\nu\Sigma_f(\underline{r}) - \Sigma_a(\underline{r})\right)\phi(\underline{r})$

The only "boundaries" are the outside edges.

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