ENGINEERING PHYSICS 4D3/6D3

DAY CLASS Dr. Wm. Garland
DURATION: 50 minutes
McMASTER UNIVERSITY MIDTERM EXAMINATION November 10, 1999

Special Instructions:

1. Closed Book. All calculators and up to 8 single sided 8 ½" by 11" crib sheets are permitted.
2. Do all questions.
3. The value of each question is as indicated. TOTAL Value: 50 marks

THIS EXAMINATION PAPER INCLUDES 2 PAGES AND 3 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

1. [20 marks] The general multigroup neutron diffusion equations with delayed precursors are given by:

$$\frac{1}{v_g}\frac{\partial \phi_g}{\partial t} = \nabla\cdot D_g \nabla \phi_g - \Sigma_{a_g}\phi_g - \Sigma_{s_g}\phi_g + \sum_{g'=1}^{G}\Sigma_{s_{g'g}}\phi_{g'}$$
$$+ \chi_g(1-\beta)\sum_{g'=1}^{G}\nu_{g'}\Sigma_{f_{g'}}\phi_{g'} + \chi_g^C\sum_{i=1}^{N}\lambda_i C_i + S_g^{ext}$$

$$\frac{\partial C_i}{\partial t} = -\lambda_i C_i + \beta_i \sum_{g'=1}^{G}\nu_{g'}\Sigma_{f_{g'}}\phi_{g'}$$

Note that ϕ_g and C_i are functions of $\underline{r}$ and t but the notation has been dropped for clarity. The poison equations are:

$$\frac{\partial I(\underline{r},t)}{\partial t} = \gamma_I \sum_{g'=1}^{G}\Sigma_{f_{g'}}\phi_{g'}(\underline{r},t) - \lambda_I\, I(\underline{r},t)$$

$$\frac{\partial X(\underline{r},t)}{\partial t} = \gamma_X \sum_{g'=1}^{G}\Sigma_{f_{g'}}\phi_{g'}(\underline{r},t) + \lambda_I\, I(\underline{r},t) - \lambda_X\, X(\underline{r},t) - \sum_{g'=1}^{G}\sigma^X_{a_{g'}}\phi_{g'}(\underline{r},t)\; X(r,t)$$

and the fuel depletion equation is:

$$\frac{\partial N_f}{\partial t} = -\, N_f(\underline{r},t)\sum_{g'=1}^{G}\sigma^f_{a_{g'}}\phi_{g'}(\underline{r},t)$$

Define each variable. Explain the significance of each term. Be brief; a few words per variable or term is sufficient.

2. [10 marks] What is the obvious error in the following expressions? Explain briefly.

a) Steady state one-group neutron balance equation:
$D(r)\nabla^2\phi(r) - \Sigma_a(r)\phi(r) = \nu\Sigma_f(r)\phi(r)$

b) For neutron group g, $\Sigma_{removal} < \Sigma_{absorption}$

c) The gradient of the flux is continuous at an interface

d) $\rho = 2$

e) For a reactor operating at constant power, as the fuel is burned up, the flux remains constant over time.

3. [20 Marks] In reference to the governing equations as set out in question 1:

a. Assuming two neutron groups (fast and thermal), no upscatter, no fast fissions, and no neutrons born in the thermal region, what are the transient flux and precursor equations?

b. Ignoring fuel depletion for the moment, what are the steady state xenon and iodine concentrations at a given flux (which is steady in time but may vary in space)? Which terms in the flux and precursor equations of (a) are dependent on these poison concentrations?

c. For the two-group approximation, what is the steady state precursor concentration, C_i, given the flux (which is steady in time but may vary in space)?

d. What do the two-group steady state flux equations look like if the steady state value of C_i is substituted in?

e. To numerically solve the transient fluxes, precursors, poisons, etc, a controller is introduced to keep the flux at some prescribed set point (which may be steady or may vary in time). This controller alters the absorption terms in the flux equations. Yet in the steady state algorithms, we used a fudge factor, k, in the fission terms. Explain the rational behind the two different schemes.

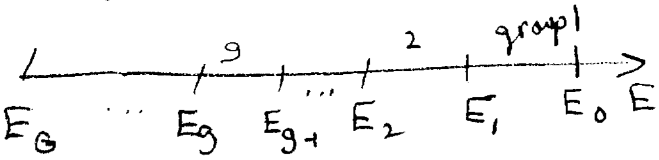
---The End---

Midterm 1999 - Solutions

1. <u>Variables for flux + precursor eqn's.</u>

ϕ_g - neutron flux in energy group g $(g = 1, 2 \dots G)$

v_g = neutron velocity in group g



t = time

D = diffusion coeff. [cm]

Σ = cross section [cm^{-1}] (subscripts a = absorption, s = scattering, f = fission)

$\Sigma_{sg'g}$ = Σ_s from group $g' \to g$

χ_g = fission partition function, χ_g^c refers to delayed prec.

β_i = delayed fraction ($\beta = \sum_{i=1}^{N} \beta_i$, N = # of delayed groups)

ν = # of neutrons produced per fission

S_g^{ext} = external source of neutrons.

C_i = delayed precursors.

λ_i = decay const. for C_i

<u>Term by term</u>

flux: ϕ_g — rate of change of neutron density in group g = net diffusion - absorption - scattering out + scattering in + prompt fission + delayed fission (all w.r.t. group g)

delayed precursors C_i: rate of change of precursor i = decay + fission source.

Variables for poison eqn's

I = iodine concentration

X = Xenon concentration

γ = production fraction

λ = decay constants

σ = microscopic crossections

Term by term

Iodine: rate of change of I concentration = fission source − decay

Xenon: rate of change of Xe concentration = fission source + I decay − Xe decay − burnoff

Fuel

Variables: N_f = concentration of fuel isotope

Term by term

rate of change of fuel concentration = absorption rate (or interaction rate) $\sim \Sigma_a \phi \sim \sigma_a N \phi$

2. a) The 1 speed eqn is:

$$\frac{1}{v}\frac{\partial \phi}{\partial t} = \nabla \cdot D \nabla \phi - \Sigma_a \phi + \nu \Sigma_f \phi$$

in steady state: $\nabla \cdot D \nabla \phi - \Sigma_a \phi + \nu \Sigma_f \phi = 0$

or $\nabla \cdot D \nabla \phi - \Sigma_a \phi = -\nu \Sigma_f \phi$

so the signs of the terms are incorrect.

<u>And</u> $D = D(r)$ ∴ can't take outside of ∇ operator.

∴ $D(r)\nabla^2\phi(r)$ should be $\nabla \cdot D(r)\nabla\phi$

b) $\Sigma_{removal} > \Sigma_a$ since removal includes outscattering & absorption.

c) The flux is continuous at an interface, not the current.

d) $\rho \equiv \frac{k-1}{k}$. $k \in (0, \infty)$.

∴ ρ min is $\frac{-1}{0} = -\infty$

ρ max is $\frac{\infty}{\infty} = 1$

∴ ρ cannot be 2

e) at Power constant, as $\Sigma_{fuel} \downarrow$, $\phi \uparrow$.

3.

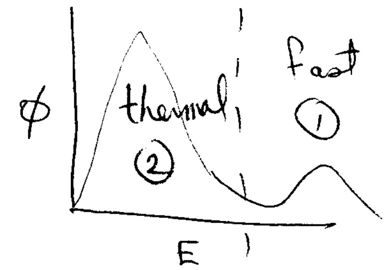
a) – 2 group approximation $\therefore G = 2$

– no upscatter $\therefore \Sigma_{s21} = 0$

– no fast fissions $\therefore \Sigma_{f_1} = 0$

– no thermal births $\therefore \chi_2 = 0 \Rightarrow \chi_1 = 1$

$\chi_2^c = 0 \Rightarrow \chi_2^c = 1$



$$\therefore \frac{1}{v_1}\frac{\partial \phi_1}{\partial t} = \nabla \cdot D_1 \nabla \phi_1 - \underbrace{\Sigma_{a_1}\phi_1 - \Sigma_{s_1}\phi_1 + \Sigma_{s_{11}}\phi_1}_{= -\Sigma_{R_1}} + \cancel{\Sigma_{s_{21}}\phi_2}^{\,0}$$

$$+ (1-\beta)\nu_2 \Sigma_{f_2}\phi_2 + \sum_{i=1}^{6} \lambda_i C_i + S_1^{ext}$$

↑ forget this for convenience. No relevant to the issues at hand.

$$\frac{1}{v_2}\frac{\partial \phi_2}{\partial t} = \nabla \cdot D_2 \nabla \phi_2 \overbrace{- \Sigma_{a_2}\phi_2 - \Sigma_{s_2}\phi_2 + \Sigma_{s_{22}}\phi_2}^{\equiv -\Sigma_{R_2}\phi_2} + \Sigma_{s_{12}}\phi_1$$

$$+ 0 + 0 + S_2^{ext} \leftarrow \text{ignore}$$

These 2 terms cancel since no upscatter

$$\frac{\partial C_i}{\partial t} = -\lambda_i C_i + \beta_i \nu_2 \Sigma_{f_2}\phi_2$$

b) S.S. ~~Iodine~~: $0 = \gamma_I \Sigma_{f_2}\phi_2 - \lambda_I I$

$$\Rightarrow I_\infty(r) = \frac{\gamma_I \Sigma_{f_2}\phi_2(r)}{\lambda_I}$$

fuel regions only

S.S. Xenon: $0 = \gamma_X \Sigma_{f_2}\phi_2 + \lambda_I I - \lambda_X X$

$$- X\underbrace{(\sigma_{a_1}^X \phi_1 + \sigma_{a_2}^X \phi_2)}_{\equiv \langle \sigma_{a_2}\phi \rangle}$$

$$\therefore X_\infty(r) = \frac{(\gamma_X + \gamma_I)\Sigma_{f_2}\phi_2(r)}{\lambda_X + \langle \sigma_{a_2}\phi \rangle}$$

↖ fn(r)

These poisons affect the Σ_a coefficients mainly & to a lesser extent D & Σ_s

3. (cont'd)

c) <u>SS C_i</u>

$$0 = -\lambda_i C_i + \beta_i \nu_2 \Sigma_{f_2} \phi_2$$

$$\therefore C_i(r) = \frac{\beta_i \nu_2 \Sigma_{f_2} \phi_2(r)}{\lambda_i}$$

fuel regions <u>only</u>.

d) Plugging in to flux equations of (a):

$$\frac{1}{v_1}\cancelto{0}{\frac{\partial \phi_1}{\partial t}} = \nabla \cdot D_1 \phi_1 - \Sigma_{R_1} \phi_1 + (1-\beta) \nu_2 \Sigma_{f_2} \phi_2 + \sum_{i=1}^{6} \frac{\lambda_i \beta_i \nu_2 \Sigma_{f_2} \phi_2}{\lambda_i}$$

$$= \nabla \cdot D_1 \phi_1 - \Sigma_{R_1} \phi_1 + \nu_2 \Sigma_{f_2} \phi_2 \qquad \text{since } \beta = \sum_{i=1}^{6} \beta_i$$

$$\frac{1}{v_2}\cancelto{0}{\frac{\partial \phi_2}{\partial t}} = \nabla \cdot D_2 \nabla \phi_2 - \Sigma_{R_2} \phi_2 + \Sigma_{S_{12}} \phi_1$$

(unchanged from before)

e) The S.S. fudge factor was only introduced as a means of artificially finding that particular combination of variables ~~that~~ precisely balanced the neutron gains and losses. The term in the flux equation ~~that~~ was most uncertain <u>and</u> affected the balance significantly was the $\nu \Sigma_f \phi$ term. Hence the term was fudged to $\frac{\nu \Sigma_f \phi}{K}$ and K was adjusted to give criticality. We could have adjusted Σ_a just as easily.

For the transient, we want to track the dynamic response to perturbations, controller actions and the like as poisons grow in, etc. So it is best to use a realistic controller that mimics the actual control rods.