Reactor Physics: Point Kinetics

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Summary:

Delayed precursors are introduced into the general neutron balance equations to account for this delayed neutron source. These delayed neutrons are crucial in determining the transient response of a reactor to reactivity perturbations. The model is drastically simplified to yield the classic point kinetics equations, which are investigated to reveal the kinetic behaviour of typical reactor systems.

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1 Introduction

1.1 Overview

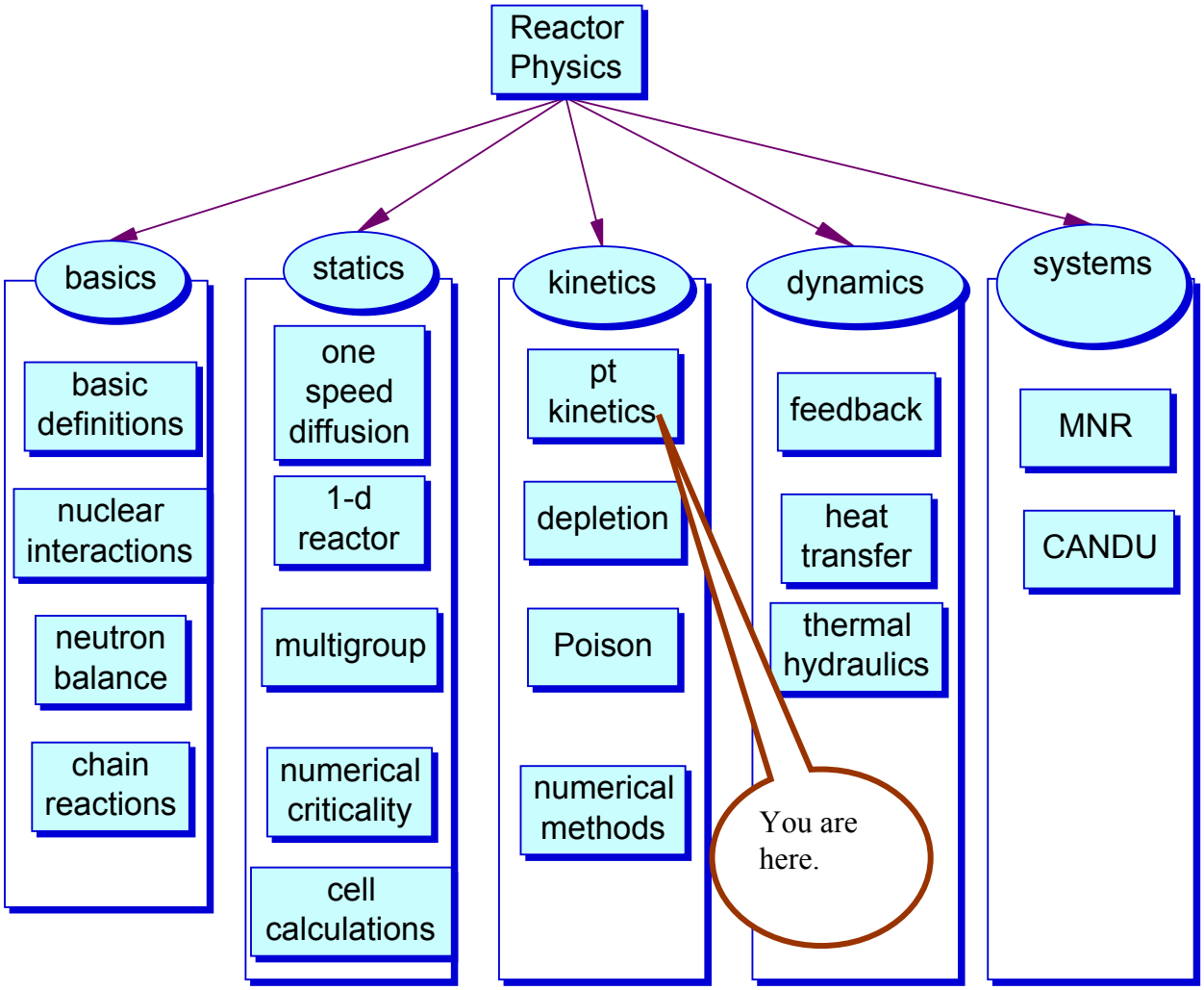


Figure 1 Course Map.

1.2 Learning Outcomes

The goal of this chapter is for the student to understand:

- The derivation and limitations of the point kinetics equations
- How to determine the neutron response to reactivity perturbations
- The important role played by delayed neutrons
- The inverse method and how it might be used

2 Point Kinetics Equations

We had the general multigroup neutron diffusion equations:

$$\frac{1}{v_g}\frac{\partial}{\partial t}\phi_g(\underline{r},t) = \underbrace{\nabla\cdot D_g(r)\nabla\phi_g(\underline{r},t)}_{\text{leakage}} - \underbrace{\Sigma_{ag}(r)\phi_g(\underline{r},t)}_{\text{loss by absorption}} - \underbrace{\Sigma_{sg}(r)\phi_g(\underline{r},t)}_{\text{removal by scattering}} + \underbrace{\sum_{g'=1}^{G}\Sigma_{sg'g}(r)\phi_{g'}(\underline{r},t)}_{\text{scattering into group g}}$$
$$+ \underbrace{\chi_g}_{\substack{\text{fraction}\\ \text{appearing}\\ \text{in group g}}}\underbrace{\sum_{g'=1}^{G}\nu_{g'}\Sigma_{fg'}(\mathbf{r})\phi_{g'}(\underline{r},t)}_{\substack{\text{total fission}\\ \text{production}}} \underbrace{+S_g^{ext}}_{\text{external source}} \tag{2.1}$$

and in one-speed form, this simplifies to

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(\mathbf{r},t) = \nabla\cdot D(\mathbf{r})\nabla\phi(\mathbf{r},t) - \Sigma_a(\mathbf{r})\phi(\mathbf{r},t) + \nu\Sigma_f(\mathbf{r})\phi(\mathbf{r},t) \tag{2.2}$$

In both those forms, the neutrons are assumed to all appear as soon as fissioning occurs. The term $\nu\Sigma_f(\mathbf{r})\phi(\mathbf{r},t)$ in equation 2.2 has dimensions of neutrons / cm^3-s, representing the rate of neutron production. But, in reality, a small fraction of the neutrons come, not from the fission directly, but from the subsequent decay of fission products (termed the delayed precursors). We need to mathematically account for this delayed source of neutrons is we are to model the transient behaviour of neutron flux correctly.

In this chapter we focus on the transient behaviour of the flux and we will virtually ignore the spatial effects and the energy effects. Obviously this is an approximation that has its limitations but it does serve admirably to illustrate reactor kinetics.

2.1 Delayed Precursors

We modify equation 2.2 by acknowledging that a fraction, β, of the neutrons produced in fission are the result of the decay of the delayed precursors:

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(\mathbf{r},t) = \nabla\cdot D(\mathbf{r})\nabla\phi(\mathbf{r},t) - \Sigma_a(\mathbf{r})\phi(\mathbf{r},t) + (1-\beta)\nu\Sigma_f(\mathbf{r})\phi(\mathbf{r},t) + \sum_{i=1}^{6}\lambda_i C_i$$
$$\frac{\partial}{\partial t}C_i(r,t) = -\lambda_i C_i(r,t) + \beta_i\nu\Sigma_f(r)\phi(r,t) \tag{2.3}$$

There are a large number of fission product isotopes that decay by neutron emission and, thus, are members of the delayed precursor family. For the purposes of modeling their effect on neutron kinetics, it is sufficient to group them into 6 groups ($C_1 \ldots C_6$) according to their half-life. The delayed precursors are assumed to not diffuse in solid fuel but the delayed precursor densities, C_i will be space dependent since the flux is space dependent. Typical values are:

Group #→	1	2	3	4	5	6
$T_{1/2}$	54.51	21.84	6.0	2.23	0.496	0.179
λ_I	0.0127	0.031	0.1155	0.310	1.397	3.871
β_i/β	.038	0.213	0.188	0.407	0.128	0.026
β_I	0.0002641	0.00148035	0.0013066	0.00282865	0.0008896	0.0001807

Table 1 Typical precursor coefficients.

For the values in table 1, $\beta \equiv \sum_{i=1}^{6} \beta_i = 0.0065$. So the delayed precursors only account for 0.65% of the neutrons. However, the long time constants (up to a minute) compared to the short fission and slowing down times (about 10 μsec to 1 msec or so, depending on the reactor design), means that these delayed neutrons have a large effect on the system time constants, as we shall see.

2.2 Derivation of the Point Kinetics Equations

So now, with the addition of the delayed precursors, we have 7 equations to solve for every point in space and time. To focus on the time behaviour, we will assume that the media is homogeneous and that the flux shape is known, ie:

$$\nabla^2\phi + B_g^2\phi = 0 \tag{2.4}$$

which, when used in equation 2.3 to replace the diffusion term, gives:

$$\frac{1}{v}\frac{\partial\phi}{\partial t} = -DB_g^2\phi - \Sigma_a\phi + (1-\beta)\nu\Sigma_f\phi + \sum_{i=1}^{6}\lambda_i C_i \tag{2.5}$$

We employ the "adiabatic" approximation that the flux can be factored into an amplitude part and a spatial part:

$$\begin{aligned} \phi(\underline{r},t) &= v\,n(t)\psi(\underline{r}) \\ C_i(\underline{r},t) &= \mathbb{C}_i(t)\psi(\underline{r}) \end{aligned} \tag{2.6}$$

giving:

$$\begin{aligned} \frac{\partial n}{\partial t} &= \left(-DB_g^2 - \Sigma_a + (1-\beta)\nu\Sigma_f\right)vn + \sum_{i=1}^{6}\lambda_i\mathbb{C}_i \\ \frac{\partial}{\partial t}\mathbb{C}_i &= -\lambda_i\mathbb{C}_i + \beta_i\nu\Sigma_f\, vn \end{aligned} \tag{2.7}$$

Defining:

$$k = \frac{\nu\Sigma_f/\Sigma_a}{1+L^2B_g^2}, \qquad \ell = \frac{1}{v\Sigma_a\left(1+L^2B_g^2\right)} = \text{neutron lifetime} \tag{2.8}$$

we can rearrange equation 2.7:

$$\begin{aligned}\frac{\partial n}{\partial t} &= -\Sigma_a\left(\frac{DB_g^2}{\Sigma_a}+1\right)vn+(1-\beta)\underbrace{v\Sigma_f v}_{k/\ell}n+\sum_{i=1}^{6}\lambda_i\mathbb{C}_i \\ \frac{\partial}{\partial t}\mathbb{C}_i &= -\lambda_i\mathbb{C}_i+\beta_i\underbrace{v\Sigma_f v}_{k/\ell}n\end{aligned} \tag{2.9}$$

Noting that $\frac{DB_g^2}{\Sigma_a}=L^2B_g^2$ and that $\frac{k}{\ell}=vv\Sigma_f$ we get this simplified form:

$$\begin{aligned}\frac{\partial n(t)}{\partial t} &= \left[\frac{k(1-\beta)-1}{\ell}\right]n(t)+\sum_{i=1}^{6}\lambda_i\mathbb{C}_i(t) \\ \frac{\partial}{\partial t}\mathbb{C}_i(t) &= -\lambda_i\mathbb{C}_i(t)+\beta_i\frac{k}{\ell}n(t)\end{aligned} \tag{2.10}$$

We make two further substitutions, $\rho\equiv\frac{k-1}{k}=$ reactivity and $\Lambda\equiv\frac{\ell}{k}=$ mean generation time to arrive (finally) at the **Point Kinetics Equations**:

$$\boxed{\begin{aligned}\frac{\partial n(t)}{\partial t} &= \frac{(\rho-\beta)}{\Lambda}n(t)+\sum_{i=1}^{6}\lambda_i\mathbb{C}_i(t) \\ \frac{\partial}{\partial t}\mathbb{C}_i(t) &= -\lambda_i\mathbb{C}_i(t)+\frac{\beta_i}{\Lambda}n(t),\ i=1...6\end{aligned}} \tag{2.11}$$

These are the classical form of the space independent kinetics equations, derived early on in the history of reactor design. The parameters ρ, β, etc, are the embodiment of the net effects of the more fundamental and physically based parameters such as cross sections, etc. that we see in the general neutron balance equation. So it is more difficult to see the direct effect on the reactor system of changes in cross sections, etc. But the Point Kinetics Equations are wonderfully succinct and allow us to get an analytical solution that we can use to investigate the nature of the reactor system, specifically its response to changes in reactivity, ρ.

2.3 Solution of the Point Kinetics Equations

In general, $\rho=\rho\left(\underbrace{n(t)}_{\text{dynamics}},\underbrace{t}_{\text{kinetics}}\right)$. The dynamic effects includes feedback effects due to temperature, power, fuel depletion, etc. The kinetic effect includes direct control rod action, etc. For the moment, we will assume only kinetic effects exist, ie, we impose a prescribed $\rho=\rho(t)$.

The case of constant ρ can be readily solved analytically but the solution is a bit messy. We start by assuming an exponential nature for the flux and delayed precursor concentrations:

$$\begin{aligned} n &= Ae^{\omega t} \\ \mathbb{C}_i &= \mathbb{C}_{i0}e^{\omega t} \end{aligned} \tag{2.12}$$

Exponential behaviour would be expected since the Point Kinetics Equations are proportional to themselves. Substituting into the precursor part of equation 2.11

$$\mathbb{C}_{i0}\omega \cancel{e^{\omega t}} = -\lambda_i \mathbb{C}_{i0} \cancel{e^{\omega t}} + \frac{\beta_i}{\Lambda} A \cancel{e^{\omega t}} \tag{2.13}$$

which gives

$$\mathbb{C}_{i0} = \frac{\beta_i A}{\Lambda(\omega + \lambda_i)} \tag{2.14}$$

Just as you would expect, the precursor concentrations are proportional to the neutron density (or flux). We substitute this back into the neutron equation to give:

$$\begin{aligned} \cancel{A}\omega \cancel{e^{\omega t}} &= \frac{(\rho - \beta)}{\Lambda} \cancel{A} \cancel{e^{\omega t}} + \sum_{i=1}^{6} \lambda_i \frac{\beta_i \cancel{A}}{\Lambda(\omega + \lambda_i)} \cancel{e^{\omega t}} \\ &\Downarrow \\ \omega &= \frac{(\rho - \beta)}{\Lambda} + \sum_{i=1}^{6} \lambda_i \frac{\beta_i}{\Lambda(\omega + \lambda_i)} \end{aligned} \tag{2.15}$$

The is a transcendental equation, ie, ω depends on itself. We can solve this graphically. For a given physical setup, λ_i, β_i, etc, are set. The ρ is variable and Λ is a function of ρ. It makes sense, then to plot ρ against ω. Physically, this is plotting the input (ρ) against the response (ω). But before we can do that, we need to get rid of the ρ dependency of Λ. We rewrite equation 2.15:

$$\begin{aligned} \Lambda\omega &= \rho + \sum_{i=1}^{6} \left(\lambda_i \frac{\beta_i}{(\omega + \lambda_i)} \right) - \beta \\ &\Downarrow \\ \Lambda\omega &= \rho + \sum_{i=1}^{6} \left(\frac{\lambda_i \beta_i}{(\omega + \lambda_i)} - \beta_i \right) \\ &\Downarrow \\ \Lambda\omega &= \rho + \sum_{i=1}^{6} \left(\frac{\lambda_i \beta_i - (\omega + \lambda_i)\beta_i}{(\omega + \lambda_i)} \right) \\ &\Downarrow \\ \Lambda\omega &= \rho - \sum_{i=1}^{6} \left(\frac{\omega \beta_i}{(\omega + \lambda_i)} \right) \end{aligned} \tag{2.16}$$

But

$$\Lambda \equiv \frac{\ell}{k} = \ell(1 - \rho) \tag{2.17}$$

$$\omega\ell\left(1-\rho\right)=\rho-\sum_{i=1}^{6}\left(\frac{\omega\beta_i}{\left(\omega+\lambda_i\right)}\right)$$
$$\Downarrow$$
$$\rho\left(1+\omega\ell\right)=\omega\ell+\sum_{i=1}^{6}\left(\frac{\omega\beta_i}{\left(\omega+\lambda_i\right)}\right) \tag{2.18}$$
$$\Downarrow$$

Dividing through we arrive at **The Inhour Equation**:

$$\rho=\frac{\omega\ell}{\left(1+\omega\ell\right)}+\frac{1}{\left(1+\omega\ell\right)}\sum_{i=1}^{6}\left(\frac{\omega\beta_i}{\left(\omega+\lambda_i\right)}\right) \tag{2.19}$$

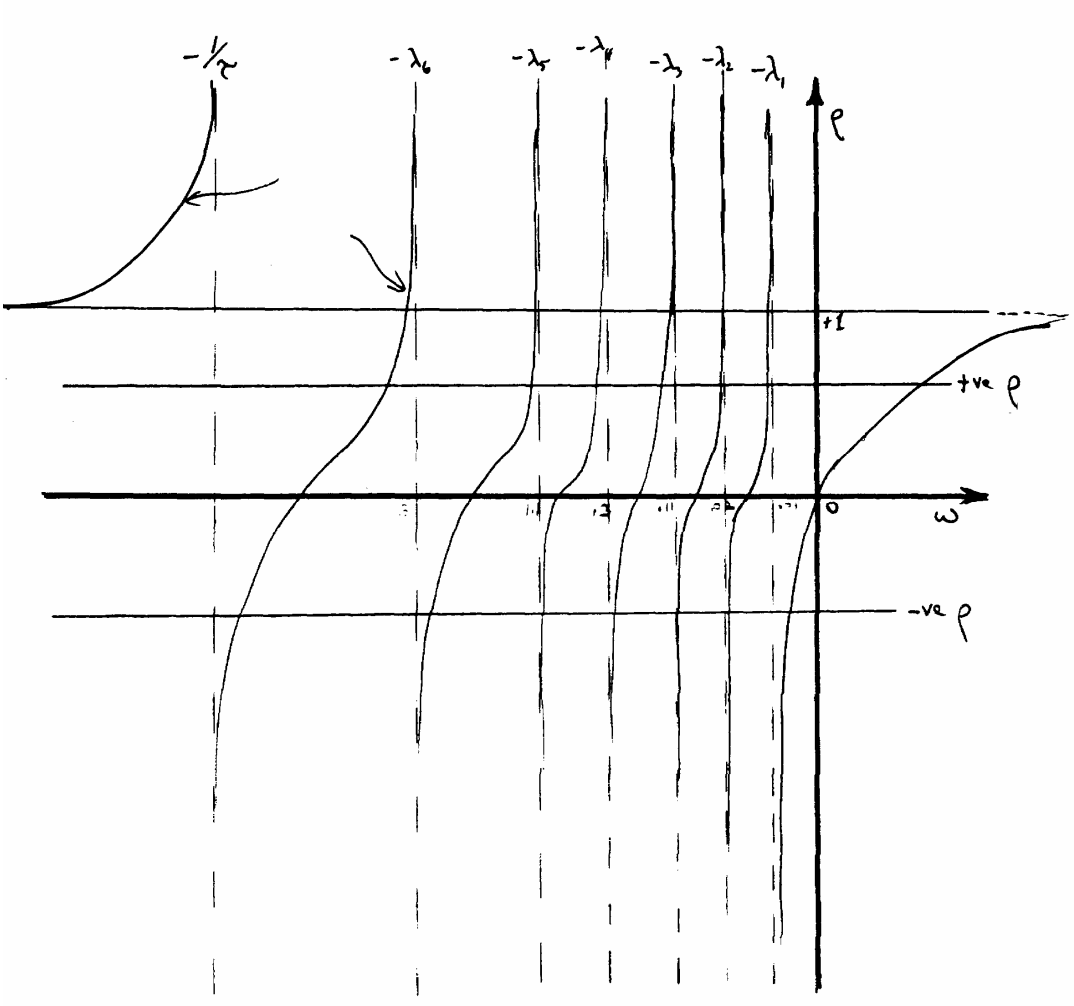


Figure 2 The roots of the inhour equation.

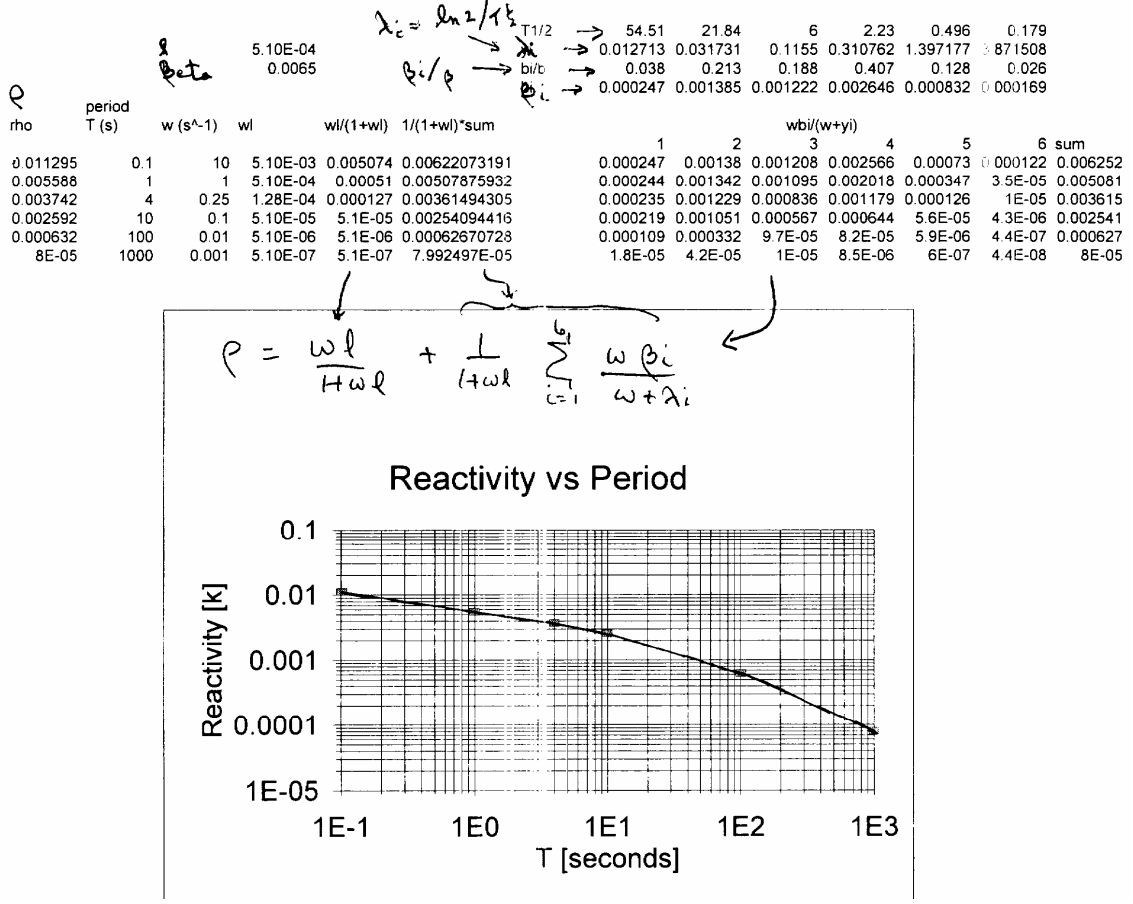
Note that there are 7 roots, ie there are 7 values of ω for a given value of ρ. That means we should have written the solution in the form:

$$n = n_0 \sum_{j=1}^{7} A_j e^{\omega_j t}$$
$$\mathbb{C}_i = \mathbb{C}_{i0} \sum_{j-1}^{7} B_{ij} e^{\omega_j t} \qquad (2.20)$$

We could go through the drudgery of solving for the coefficients but there is really no need. All we were really after was the relationship between the rate that the flux responds, ie the ω's, to the input ρ. Usually ρ is ρ(t) and we solve the point kinetics equations numerically. So we will move on.

Notice, in figure 2, that for ρ>0, there is only one ω>0. All the other ω's are <0, ie, they represent dieing components. If we plug in a ω into the inhour equation, we can directly calculate the ρ that must have caused it. Figure 3 shows such a calculation done with a spreadsheet. This brings us to two questions to contemplate:

1. How might this be used to calibrate a regulating rod?
2. How can the calibrated regulating rod be used to calibrate the control rods in MNR?



						T1/2 →	54.51	21.84	6	2.23	0.496	0.179	
			5.10E-04			→	0.012713	0.031731	0.1155	0.310762	1.397177	3.871508	
		Beta	0.0065			bi/b →	0.038	0.213	0.188	0.407	0.128	0.026	
						βi →	0.000247	0.001385	0.001222	0.002646	0.000832	0.000169	
ρ	period								wbi/(w+yi)				
rho	T (s)	w (s^-1)	wl	wl/(1+wl)	1/(1+wl)*sum		1	2	3	4	5	6	sum
0.011295	0.1	10	5.10E-03	0.005074	0.00622073191		0.000247	0.00138	0.001208	0.002566	0.00073	0.000122	0.006252
0.005588	1	1	5.10E-04	0.00051	0.00507875932		0.000244	0.001342	0.001095	0.002018	0.000347	3.5E-05	0.005081
0.003742	4	0.25	1.28E-04	0.000127	0.00361494305		0.000235	0.001229	0.000836	0.001179	0.000126	1E-05	0.003615
0.002592	10	0.1	5.10E-05	5.1E-05	0.00254094416		0.000219	0.001051	0.000567	0.000644	5.6E-05	4.3E-06	0.002541
0.000632	100	0.01	5.10E-06	5.1E-06	0.00062670728		0.000109	0.000332	9.7E-05	8.2E-05	5.9E-06	4.4E-07	0.000627
8E-05	1000	0.001	5.10E-07	5.1E-07	7.992497E-05		1.8E-05	4.2E-05	1E-05	8.5E-06	6E-07	4.4E-08	8E-05

Figure 3 Reactivity for a given period.

Notice how the period rapidly decreases as the reactivity increases. What are the control ramifications? Is there some limit we should avoid? (More about this next.)

3 Prompt Effects

3.1 The Prompt Jump

Consider a step change in ρ from a steady state situation. Before the change, the point kinetics equations in steady state are:

$$0 = \frac{(\rho_0 - \beta)}{\Lambda} n(0) + \sum_{i=1}^{6} \lambda_i \mathbb{C}_i(0) \tag{3.1}$$

This gives the steady state value of the summation term just prior to the ρ insertion.

After the ρ insertion, the ϕ (or n) changes quickly but the precursors take some time to react. Thus we can use the steady state value for the summation term. Thus:

$$\frac{\partial n(t)}{\partial t} = \frac{(\rho - \beta)}{\Lambda} n(t) - \frac{(\rho_0 - \beta)}{\Lambda} n(0) \tag{3.2}$$

Solving gives:

$$n(t) = n(0) e^{\frac{\rho-\beta}{\Lambda} t} + \frac{\rho_0 - \beta}{\rho - \beta} n(0) \left[1 - e^{\frac{\rho-\beta}{\Lambda} t} \right] \tag{3.3}$$

For $\rho < \beta$,

$$n(t) \to \frac{\rho_0 - \beta}{\rho - \beta} n(0) \tag{3.4}$$

thus:

$$\frac{n(t)}{n(0)} \to \frac{\rho_0 - \beta}{\rho - \beta} = \frac{\beta - \rho_0}{\beta - \rho} \tag{3.5}$$

For $\rho > \rho_0$, $\frac{n(t)}{n(0)} > 1$, ie, the neutron flux jumps, as sketched in figure 4. Note that in the prompt jump, the quickly decaying components of the solution (associated with the 6 negative ω's) generate a short lived spat of extra neutrons that induce addition fissions. This brings the power to a new and higher level But these components decay away, leaving the neutron level at a plateau. Eventually the one $\omega > 0$ causes the flux level to increase exponentially. This rapid jump to a new neutron level is called the "prompt jump".

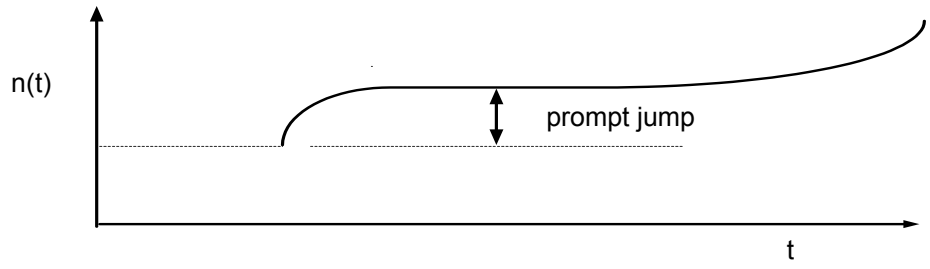


Figure 4 The prompt jump.

3.2 Prompt Critical

Looking once again at the point kinetics equations:

$$\begin{aligned} \frac{\partial n(t)}{\partial t} &= \frac{(\rho - \beta)}{\Lambda} n(t) + \sum_{i=1}^{6} \lambda_i \mathbb{C}_i(t) \\ \frac{\partial}{\partial t} \mathbb{C}_i(t) &= -\lambda_i \mathbb{C}_i(t) + \frac{\beta_i}{\Lambda} n(t), \; i = 1...6 \end{aligned} \tag{3.6}$$

notice that, in a critical reactor, the L.H.S. of both equations = 0. The delayed precursors provide a small number of neutrons, topping up those provided by the prompt generation term. The prompt term has a characteristic response time of Λ sec (typically about 100 μsec) while the delayed term has a characteristic response time of seconds to minutes. The reactor is actually sub-critical on prompt neutrons alone. If the reactor were critical on prompt neutrons, then the response time of the reactor system would be too fast for a physical control system to handle – the reactor would be uncontrollable. It is only because of the delayed neutrons that the average response time is slow enough for the system to be controllable.

In mathematical terms, the prompt term, $\frac{(\rho - \beta)}{\Lambda} n(t)$, is typically < 0 (that is, $\rho \sim 0$, $\beta \sim 0.0065$), the delayed term $\sum_{i=1}^{6} \lambda_i \mathbb{C}_i(t)$ is >0 and the two balance each other out. But what would happen if $\rho \geq \beta$? In that case, the reactor would be critical or super-critical on prompt neutrons alone! This condition, called **prompt criticality** is to be avoided above all else.

$$\frac{\rho - \beta}{\Lambda} = 0 \Rightarrow \rho = \beta = \approx 0.0065 = 6.5 \text{ milli-k (Canada)} = 1\$ \text{ (US)} = 100 \text{ cents} \tag{3.7}$$

This is the **prompt critical** condition.

4 Inverse Method

Here is a rather interesting, mind expanding way of looking at neutron dynamics. As we shall see, it gives us a way to determine ρ(t), given a n(t) history.

In time dt′ at time t′, the change in precursor density due to production is:

$$dC_i = \frac{\beta_i}{\Lambda} n(t')dt' \tag{4.1}$$

At some later time, t, there will remain

$$(dC_i)e^{-\lambda_i(t-t')} = \frac{\beta_i}{\Lambda} n(t')e^{-\lambda_i(t-t')}dt', \text{ where } t > t' \tag{4.2}$$

Thus, at any given time, t, the total precursor concentration is the sum (or integral) of the little bits produced in the past, accounting for decay, ie:

$$C_i(t) = \int_{-\infty}^{t} \frac{\beta_i}{\Lambda} n(t')e^{-\lambda_i(t-t')}dt' \tag{4.3}$$

where the implicit assumption is that, at t = - ∞, no precursors existed.

Letting $\tau = t - t'$,

$$C_i(t) = \int_{\infty}^{0} \frac{\beta_i}{\Lambda} n(t-\tau)e^{-\lambda_i\tau}d(-\tau) \tag{4.4}$$

ie:

$$C_i(t) = \int_{0}^{\infty} \frac{\beta_i}{\Lambda} n(t-\tau)e^{-\lambda_i\tau}d\tau \tag{4.5}$$

Note that t = now and t′ is some time in the past, therefore $\tau = t - t'$ = time from now to then, ie, τ = 0 is now and τ=1 is 1 second ago. Likewise, t-τ = t′, thus, n(t-τ) = neutron density back then, τ seconds ago.

Now that we have the delayed precursor concentration, we can plug it into the neutron balance equation to give:

$$\begin{aligned}
\frac{\partial n(t)}{\partial t} &= \frac{(\rho-\beta)}{\Lambda} n(t) + \int_0^{\infty} \left[\sum_{i=1}^{6} \lambda_i \frac{\beta_i}{\Lambda} e^{-\lambda_i\tau} \right] n(t-\tau)d\tau \\
&= \frac{(\rho-\beta)}{\Lambda} n(t) + \frac{\beta}{\Lambda} \int_0^{\infty} \left[\underbrace{\sum_{i=1}^{6} \lambda_i \frac{\beta_i}{\beta} e^{-\lambda_i\tau}}_{\text{delayed neutron kernal } \overbrace{D(\tau)}} \right] n(t-\tau)d\tau \\
&\equiv \frac{(\rho-\beta)}{\Lambda} n(t) + \frac{\beta}{\Lambda} \int_0^{\infty} D(\tau)\, n(t-\tau)d\tau
\end{aligned} \tag{4.6}$$

The delayed neutron kernal, $D(\tau)$, is the probability that a delayed neutron will be emitted in time $d\tau$ at time τ after a fission.

Solving equation 4.6 for ρ:

$$\begin{aligned}\rho &= \beta + \frac{\Lambda}{n(t)}\frac{dn(t)}{dt} - \beta\int_0^\infty D(\tau)\frac{n(t-\tau)}{n(t)}d\tau \\ &= \beta + \Lambda\frac{d\ln n(t)}{dt} - \beta\int_0^\infty D(\tau)\frac{n(t-\tau)}{n(t)}d\tau\end{aligned} \tag{4.7}$$

Thus we can determine $\rho(t)$ given $n(t)$.

This is useful for:

1. Control design
2. Diagnostics.

5 Multigroup Diffusion with Delayed Precursors

Now that the point kinetics equations have been explored to illustrate the transient nature of the neutron balance equations when delayed precursors have been included, we can appreciate the full blown multigroup neutron diffusion equations with delayed precursors. Starting with the multigroup equations that we have seen before, we simply make the adjustment for the proper accounting of the prompt and delayed neutrons:

$$\frac{1}{v_g}\frac{\partial}{\partial t}\phi_g(\underline{r},t) = \underbrace{\nabla\cdot D_g(\underline{r})\nabla\phi_g(\underline{r},t)}_{\text{leakage}} - \underbrace{\Sigma_{ag}(\underline{r})\phi_g(\underline{r},t)}_{\text{loss by absorption}} - \underbrace{\Sigma_{sg}(r)\phi_g(\underline{r},t)}_{\text{removal by scattering}} + \underbrace{\sum_{g'=1}^{G}\Sigma_{sg'g}(r)\phi_{g'}(\underline{r},t)}_{\text{scattering into group g}}$$
$$+ \underbrace{\chi_g^P}_{\substack{\text{fraction}\\ \text{appearing}\\ \text{in group g}}}\ \underbrace{(1-\beta)}_{\substack{\text{prompt}\\ \text{fraction}}}\ \underbrace{\sum_{g'=1}^{G}\nu_{g'}\Sigma_{fg'}(\underline{r})\phi_{g'}(\underline{r},t)}_{\substack{\text{total fission}\\ \text{production}}} + \underbrace{\chi_g^C}_{\substack{\text{fraction}\\ \text{appearing}\\ \text{in group g}}}\ \underbrace{\sum_{i=1}^{N}\lambda_i C_i(\underline{r},t)}_{\substack{\text{total}\\ \text{delayed}\\ \text{source}}} + \underbrace{S_g^{ext}}_{\substack{\text{external}\\ \text{source}}} \tag{5.1}$$

$$\frac{\partial}{\partial t}C_i(\underline{r},t) = -\lambda_i C_i(\underline{r},t) + \underbrace{\beta_i}_{\substack{i^{th}\text{ part}\\ \text{of delayed}\\ \text{neutrons,}\\ \text{split by}\\ \text{decay}\\ \text{constants}}}\ \underbrace{\sum_{g'=1}^{G}\nu_{g'}\Sigma_{fg'}(\underline{r})\phi_{g'}(\underline{r},t)}_{\substack{\text{total number of}\\ \text{fission neutrons}}} \tag{5.2}$$

Note that the energy spectrum of the prompt fission neutrons, χ_g^P, is not quite the same as the energy spectrum of the delayed neutrons, χ_g^C. The delayed neutrons tend to be of a slightly lower energy that the prompt neutrons.

It is instructive to have a look at the steady state version of these equations to see if they reduce to the steady state equations we have see before we introduced the delayed precursors into the picture. Were we justified in ignoring the delayed precursors in the steady state? Intuitively, we would say yes because, in the steady state, it shouldn't matter <u>when</u> a neutron was born, just how many.

Equation 5.2 in the steady state is:

$$\lambda_i C_i = \beta_i \sum_{g'=1}^{G}\nu_{g'}\Sigma_{fg'}\phi_{g'} \tag{5.3}$$

which we can substitute into equation 5.1 to give:

$$\begin{aligned} 0 = \quad & \nabla \cdot D_g \nabla \phi_g - \Sigma_{a\,g}\phi_g - \Sigma_{s\,g}\phi_g + \sum_{g'=1}^{G} \Sigma_{s\,g'g}\phi_{g'} \\ & + \chi_g^P (1-\beta) \sum_{g'=1}^{G} \nu_{g'} \Sigma_{f\,g'} \phi_{g'} + \chi_g^C \beta \sum_{g'=1}^{G} \nu_{g'} \Sigma_{f\,g'} \phi_{g'} + S_g^{ext} \end{aligned} \tag{5.4}$$

Grouping the fission terms we have:

$$\begin{aligned} 0 = \quad & \nabla \cdot D_g \nabla \phi_g - \Sigma_{a\,g}\phi_g - \Sigma_{s\,g}\phi_g + \sum_{g'=1}^{G} \Sigma_{s\,g'g}\phi_{g'} \\ & + \left[\chi_g^P + \left(\chi_g^C - \chi_g^P \right) \beta \right] \sum_{g'=1}^{G} \nu_{g'} \Sigma_{f\,g'} \phi_{g'} + S_g^{ext} \end{aligned} \tag{5.5}$$

The factor $\left(\chi_g^C - \chi_g^P \right)$ will only be = 0 if the two energy spectrums are equal. If a coarse grouping model were chosen (say a 4 group model with two thermal groups, an epithermal group and one fast group, then all the neutrons that are born, whether by prompt fission or by decay from the precursors, would be born in the fast group. In that case, $\chi_1^C = \chi_1^P = 1$ and all the other χ's = 0. If this is true, then the steady state neutron balance equation has no mention of delayed precursors and we were justified in not mentioning them in previous chapters. But, if a fine energy structure is being used, then we must use the steady state form in equation 5.4 or 5.5 so that the neutrons are born in the correct neutron group.

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